THE SUPER-KAMIOKANDE SOLAR ANALYSIS χ^2

To determine what oscillation parameters the Super-Kamiokande solar neutrino data set favors, a χ^2 minimization is performed on the observed recoil electron spectrum and the expected Monte Carlo simulated spectrum with oscillations. The total BP2004 SSM flux [1] is assumed ($\Phi_{^8\mathrm{B,SSM}} = 5.79 \times 10^6$ cm^{-2} s^{-1}, $\Phi_{hep,\mathrm{SSM}} = 7.88 \times 10^3$ cm^{-2} s^{-1}) and the χ^2 is minimized with respect to ^{8}B and *hep* flux scaling factors to achieve the best possible match between spectra. The general χ^2 is given as

$$\chi^2_{\mathrm{SK}}(\beta,\eta) = \sum_{i=1}^{N} \frac{\left(d_i - (\beta b_i + \eta h_i) \times f(E_i, \delta_B, \delta_S, \delta_R)\right)^2}{\sigma_i^2} + \left(\frac{\delta_B}{\sigma_B}\right)^2 + \left(\frac{\delta_S}{\sigma_S}\right)^2 + \left(\frac{\delta_R}{\sigma_R}\right)^2 + 2\Delta\log(\mathcal{L}),$$

where d_i is the data spectrum, N is the number of energy bins, and β and η are the unit-less flux scaling factors for the ^{8}B (b_i) and *hep* (h_i) simulated spectra respectively. The term $f(E_i, \delta_B, \delta_S, \delta_R)$ and related constraints are also chosen to minimized the χ^2 to take into account any effects from energy correlated systematic uncertainties. The last term is the unbinned time-variation with an extended maximum likelihood fit and is described in [2].

1 Flux Scaling Factor Uncertainties

Dropping the time-variation term, the spectrum contribution to the χ^2 can be written as

$$\chi^2(\beta,\eta) = \chi^2_{\mathrm{m}}(\beta_{\mathrm{m}},\eta_{\mathrm{m}}) + \begin{pmatrix}\beta-\beta_{\mathrm{m}}\\ \eta-\eta_{\mathrm{m}}\end{pmatrix}^T \mathbf{C} \begin{pmatrix}\beta-\beta_{\mathrm{m}}\\ \eta-\eta_{\mathrm{m}}\end{pmatrix},$$

for any given combination of δ_B, δ_S, and δ_R. The χ^2_{m} term is the minimized χ^2 value with factors β_{m} and η_{m}. The curvature matrix $\mathbf{C}$, written explicitly as

$$\mathbf{C} = \alpha \times \sum_{i=1}^{N} \begin{pmatrix} \frac{b_i^2}{\sigma_i^2} & \frac{b_i \cdot h_i}{\sigma_i^2} \\ \frac{b_i \cdot h_i}{\sigma_i^2} & \frac{h_i^2}{\sigma_i^2} \end{pmatrix},$$

includes the modifier α to reflect the energy uncorrelated systematic error on the total flux ($\sigma_{\rm sys}$). It is defined as

$$\alpha = \frac{\sigma_0^2}{\sigma_0^2 + \sigma_{\rm sys}^2}, \qquad \text{where} \qquad \sigma_0^2 = \left(\sum_{i=1}^{N} \frac{1}{\sigma_{{\rm stat},i}^2} \right)^{-1}.$$

The χ^2 is minimized in its matrix form and the uncertainties of the determined values of β and η are recognized to be the inverse elements of $\mathbf{C}$:

$$\sigma_\beta^2 = \alpha \times \sum_{i=1}^{N} \frac{\sigma_i^2}{b_i^2}, \qquad \sigma_\eta^2 = \alpha \times \sum_{i=1}^{N} \frac{\sigma_i^2}{h_i^2}, \qquad \sigma_{\rm c.t.}^2 = \alpha \times \sum_{i=1}^{N} \frac{\sigma_i^2}{b_i \cdot h_i},$$

where $\sigma_{\rm c.t.}^2$ is the cross-term correlation of the fluxes.

2 Energy Correlated Systematic Uncertainties

To account for uncertainties in the ^{8}B spectrum shape and the detector's energy scale and resolution, the simulated spectrum $\beta b_i + \eta h_i$ is shifted by the energy shape factor $f(E_i, \delta_B, \delta_S, \delta_R)$ which depends on the assigned energy correlated systematic errors. The shape factor can be written as

$$f(E_i, \delta_B, \delta_S, \delta_R) = f_B(E_i, \delta_B) \times f_S(E_i, \delta_S) \times f_R(E_i, \delta_R),$$

where δ_B describes the ^{8}B spectrum shape uncertainty, δ_S describes the energy scale uncertainty, and δ_R describes the energy resolution uncertainty. Each δ is varied until the χ^2 reaches a minimum. This is done by a simplex search with the constraints $(\delta_B/\sigma_B)^2$, $(\delta_S/\sigma_S)^2$, and $(\delta_R/\sigma_R)^2$ assuring sensible one sigma region values. The final simulated spectra is

$$(\beta b_i + \eta h_i) \times f_B(E_i, \delta_B) \times f_S(E_i, \delta_S) \times f_R(E_i, \delta_R),$$

when β, η, δ_B, δ_S, and δ_R are at their minimizing values. The ^{8}B neutrino spectrum is taken from [3] and its uncertainties from [4].

3 SK-I and SK-II Combined Analysis

Both SK-I and SK-II data sets employ the same χ^2 as explained above but with their respective binning (21 bins in SK-I and 17 bins in SK-II), time-variation thresholds (5.0 MeV and 7.5 MeV), systematic errors and response functions. The combined SK-I and SK-II χ^2 is a sum of the two separate χ^2s with common factors β and η. Care must be taken though when considering the ^{8}B spectrum shape uncertainty since both SK-I and SK-II analyses utilize the same distribution. All but δ_B are repeated in a combined χ^2 making a total of five constraints on the energy correlated systematic uncertainties. The total SK χ^2 is

$$\chi^2_{\text{SK}}(\beta,\eta) = \chi^2_{\text{SK-I}}(\beta,\eta) + \chi^2_{\text{SK-II}}(\beta,\eta) =$$

$$\sum_{i=1}^{21} \frac{\left(d_i - (\beta b_i + \eta h_i) \times f(E_i, \delta_B, \delta_{S,\text{SK-I}}, \delta_{R,\text{SK-I}})\right)^2}{\sigma_i^2}$$

$$+\sum_{i=1}^{17} \frac{\left(d_i - (\beta b_i + \eta h_i) \times f(E_i, \delta_B, \delta_{S,\text{SK-II}}, \delta_{R,\text{SK-II}})\right)^2}{\sigma_i^2}$$

$$+\left(\frac{\delta_B}{\sigma_B}\right)^2 + \left(\frac{\delta_S}{\sigma_S}\right)^2_{\text{SK-I}} + \left(\frac{\delta_R}{\sigma_R}\right)^2_{\text{SK-I}} + \left(\frac{\delta_S}{\sigma_S}\right)^2_{\text{SK-II}} + \left(\frac{\delta_R}{\sigma_R}\right)^2_{\text{SK-II}}$$

$$+2\Delta\log\left(\mathcal{L}_{\text{SK-I}}\right) + 2\Delta\log\left(\mathcal{L}_{\text{SK-II}}\right)$$

Additional references are [5] and [6].

4 The SK χ^2 Map

A χ^2 map of selected quantities from the SK-I and SK-II combined analysis as explained above can be found at `http://www-sk.icrr.u-tokyo.ac.jp/sk/lowe/sk2_data/`. The following list relates the column labels in the map with those described in this text.

`tan^2(th)`	$\tan^2\theta$
`delta^m2`	Δm^2
`chi^2`	$\chi^2_{\rm SK}$
`^8B`	$\beta\cdot\Phi_{^8{\rm B,SSM}}$
`hep`	$\eta\cdot\Phi_{hep,{\rm SSM}}$
`^8B un`	$\sigma_\beta\cdot\Phi_{^8{\rm B,SSM}}$
`hep un`	$\sigma_\eta\cdot\Phi_{hep,{\rm SSM}}$
`cross`	$\sigma_{\rm c.t.}\cdot(\Phi_{^8{\rm B,SSM}}\Phi_{hep,{\rm SSM}})^{1/2}$
`^8B sh`	$(\delta_{\rm B}/\sigma_{\rm B})$
`es-I`	$(\delta_{\rm S}/\sigma_{\rm S})_{\rm SK\text{-}I}$
`er-I`	$(\delta_{\rm R}/\sigma_{\rm R})_{\rm SK\text{-}I}$
`es-II`	$(\delta_{\rm S}/\sigma_{\rm S})_{\rm SK\text{-}II}$
`er-II`	$(\delta_{\rm R}/\sigma_{\rm R})_{\rm SK\text{-}II}$

When using these quantities in an oscillation χ^2 analysis, it is recommended that free fitting parameters describing the ^{8}B and *hep* fluxes be constrained to SK values and that the SK χ^2 minimum for each combination of oscillation parameters be added to the total χ^2. For example,

$$\chi^2=\chi^2_x(\beta,\eta)+\chi^2_{\rm SK}+\frac{(\beta-\beta_{\rm SK})^2}{\sigma^2_{\beta,{\rm SK}}}+\frac{(\eta-\eta_{\rm SK})^2}{\sigma^2_{\eta,{\rm SK}}}+\frac{2(\beta-\beta_{\rm SK})(\eta-\eta_{\rm SK})}{\sigma^2_{\rm c.t.,SK}},$$

where the SK-subscripted terms represent those values in the SK χ^2 map and χ^2_x is a separate analysis sensitive to the ^{8}B and *hep* fluxes. The total χ^2 is then minimized with respect to β and η.

When one wants to do a global solar analysis, i.e. combine SK results with SNO and other experiments, the correlation of the ^{8}B neutrino spectrum shape may need to be considered. To incorporate this uncertainty, one may perform a spectrum fit with the SK-I and SK-II recoil (total) electron energy spectra. The SK-II spectrum and errors can also be found on `http://www-sk.icrr.u-tokyo.ac.jp/sk/lowe/sk2_data/` with other information in the pre-print arXiv:0803.4312. SK-I data can be found in [6].

Lastly, when including the SK χ^2 map information in an analysis, please cite as *The Super-Kamiokande Collaboration (J.P. Cravens* et al.*), arXiv:0803.4312*.

References

[1] J.N.Bahcall and M.H.Pinsonneault, Phys. Rev. Lett. **92** (2004) 121301.

[2] S. Fukuda *et al.*, Phys. Rev. Lett. **86** (2001) 5651-5655.

[3] C.E. Ortiz *et al.*, Phys. Rev. Lett. **85** (2000) 2909.

[4] J.N.Bahcall *et al.*, Phys. Rev. **C54** (1996) 411.

[5] M.B.Smy, arXiv:hep-ex/0202020.

[6] J. Hosaka *et al.*, Phys. Rev. **D73** (2006) 112001.