Dissertation
Constraints of the neutrino oscillation parameters from 1117 day observation of solar neutrino day and night spectra in Super-Kamiokande

Nobuyuki Sakurai

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Abstract

The Super-Kamiokande (SK) measurement of the $^8$B solar neutrino flux and energy spectrum are presented based on 1117 days of data taken from May 31 1996 to April 24 2000.

The ratio of the measured $^8$B solar neutrino flux to the prediction by the standard solar model (SSM) of Bahcall and Pinsonneault is: $0.465 \pm 0.005\,(\text{stat.})^{+0.015}_{-0.013}\,(\text{syst.})$. Previous solar neutrino experiments have also shown deficits in the observed solar neutrino flux. This is called the solar neutrino problem.

The most likely solution for the solar neutrino problem is neutrino oscillations. There are four possible solutions which are obtained by combining the flux from the various solar neutrino experiments. They are often called the SMA, LMA, LOW, and VAC solutions. In order to constrain these solutions, an analysis using the daytime and nighttime recoil electron energy spectra is performed.

The analysis with the absolute normalization taken as a free parameter shows that the VAC solution, most of the SMA solution and the lower half of the LMA solution are disfavored at the 95% confidence level (C.L.). Even when the Hep neutrino flux is treated as an additional free parameter, the result is almost the same as that using the Hep neutrino flux fixed at the predicted value in the SSM.

By constraining the absolute flux to the SSM prediction, allowed regions are obtained using only SK results. The allowed regions have large mixing angle ($\sin^2 2\theta > 0.65$), and overlap with the LMA and LOW solutions.

The analysis considering the flux deficits observed by all solar neutrino experiments together with the SK day/night spectra shows that the LMA and LOW solutions have higher C.L. than the SMA and VAC solutions, which are allowed only at 90 ~ 95% C.L.
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Chapter 1

Introduction

The sun radiates $3.8 \times 10^{26}$ J per second as it shines. The most likely source which has been generating such a huge energy over 4.6 billion years is nuclear fusion. However, we can not look into the solar interior directly by traditional optical observations. The neutrinos which are created in the core can provide direct observation of the solar interior.

The neutrino, denoted as $\nu$, is a particle whose existence was proposed by W. Pauli in 1930 to explain the apparent energy deficit in $\beta$ decay. It was discovered in the 1950s by F. Reines and C.L. Cowan who observed neutrinos radiated from a reactor.

A neutrino is a lepton which has a spin of (1/2), no electric charge, and possible very small mass. There are three types of neutrinos. Each type is associated with charged massive leptons, such as electrons, muons, and tau leptons. A neutrino interacts with other particles only via the weak interaction, so that the cross section of interaction is very small. This is why the detection of neutrinos is so difficult. However, owing to this small interaction cross section, one can observe the solar core by the solar neutrino.

The first solar neutrino detection experiment was carried out by R. Davis and his collaborators using neutrino capture by $^{37}$Cl. This experiment operated until 1998 and the latest result shows that the observed flux is 33% of the expected flux [1]. Successive observations have reported significant solar neutrino deficits, too. This is known as the solar neutrino problem.

The most likely solution for the deficits is neutrino oscillations, which is a phenomenon where a neutrino changes its flavor while propagating. However, as shown later, the analysis using only the observed fluxes cannot produce a unique solution for neutrino oscillations, but only gives several parameter regions where the neutrino oscillation hypothesis can be valid.

Super-Kamiokande (SK) measures the energy of recoil electrons via elastic scattering of the solar neutrinos and electrons in real-time. Therefore, it has the ability to observe both a distortion of the energy spectrum and daytime-nighttime variation of the flux. In SK, solar neutrino data has been accumulated since 1996. The observed solar neutrino energy
spectrum is unique in the world so far. In this thesis, neutrino oscillations between two active neutrinos, $\nu_e$ and $\nu_{\mu,\tau}$, is examined by using daytime and nighttime recoil electron energy spectra.

A detailed description about the theoretical and experimental background is given in Chapter 2. In Chapter 3, SK and its current status are explained. Chapter 4 is about how to reconstruct the vertex, direction, and energy of an event. Calibrations of SK are explained in Chapter 5. Especially the energy calibration is essential for this analysis. In Chapter 6, the data reduction steps are explained. Chapter 7 is about the results from the 1117 day observation. The observed flux and daytime and nighttime recoil electron energy spectra are given in this chapter. The solutions of the neutrino oscillations are examined in Chapter 8. In order to constraint the allowed region obtained by the flux-only analysis, the daytime and nighttime recoil electron energy spectra are used. Chapter 9 is the conclusion of this thesis.
Chapter 2

The Standard Solar Model and Solar Neutrino Observations

In this chapter, the theory and other solar neutrino experiments are explained. Deficits in the observed solar neutrino flux, which is widely known as the solar neutrino problem, are also explained. Lastly, as the most likely solution of the solar neutrino problem, neutrino oscillation is explained.

2.1 The Standard Solar Model (SSM)

The sun has been generating energy by thermonuclear fusion in its core over the past 4.7 billion years. In order to predict the current temperature and composition profile of the solar core, standard solar models (SSMs) have been developed. There are several assumptions in the SSMs:

- The sun was convective and uniform in composition at the beginning of its evolution. Especially, the surface abundance of heavy elements does not change by evolution.

- Energy is generated by nuclear fusion in the core.

- The sun evolves maintaining a local balance between the gravitational force and the pressure gradient.

- Energy is transported by radiation and convection. In the core region, radiation is dominant, and the solar envelope is convective.

The models are constrained to reproduce the current solar radius, mass, and luminosity. In Table 2.1, the neutrinos created in the sun and its fluxes are summarized. A description of the solar neutrinos is given in the next section.
In this analysis, the adopted SSM is a model developed by Bahcall, Basu, and Pinsonneault [2]. It is called “BP98” from now on. Input parameters of the SSM are as follows. The solar radius and mass are \( R_\odot = 6.96 \times 10^8 \) m and \( M_\odot = 1.99 \times 10^{30} \) kg, respectively. The luminosity of the sun \( L_\odot \) is measured by several satellite experiments which are summarized in Ref.[3]. The average of the results is \( L_\odot = 3.844(1 \pm 0.004) \times 10^{26} \) J/sec. The uncertainty of \( L_\odot \) corresponds to a ±3% uncertainty of the total solar neutrino flux. The temperature in the core \( T_c \) is calculated to be \( 1.5 \times 10^7 \) K from these quantities assuming solar hydrostatic equilibrium. The age of the sun \( t_\odot \) is obtained to be \( t_\odot = (4.57 \pm 0.02) \times 10^9 \) year from the measurements of the abundance of isotopes with long lifetime such as \(^{206}\text{Pb}/^{207}\text{Pb}\) in meteorites [4], which was formed by melting and crystallization of a small planetary body. The estimated uncertainty of the total neutrino flux caused by the uncertainty of \( t_\odot \) is less than 1%. The abundances of heavy elements \( (Z > 5) \) in the primordial sun are determined using meteorites and the solar photosphere [5]. The heavy element mass fraction of the hydrogen mass \( (Z/X) \) is obtained as 0.02765 by Grevesse [5] and 0.02739 by Aller [6]. The uncertainty of \( (Z/X) \) is obtained from the variation of the measured value over the past decade: ±6.1% [3]. This uncertainty introduces a \(^8\text{B}\) neutrino flux uncertainty of 4.2%. The radiative opacities influence the evolution and the structure of the sun. It is computed by the OPAL group [7] using the heavy element abundances determined by Grevesse and Noels[5]. The uncertainty of solar neutrino flux caused by the uncertainty of radiative opacity is estimated by the comparison between the flux calculated using the OPAL opacities and the flux calculated using the older Los Alamos opacities. The uncertainty of the \(^8\text{B}\) neutrino flux caused by the uncertainty of the radiative opacities is estimated to be 5.2%. The heavy element and helium diffusion in the sun are also considered. This consideration increases the predicted \(^8\text{B}\) flux by about 14%. The uncertainty of the \(^8\text{B}\) neutrino flux caused by the uncertainty of heavy element and helium diffusion is estimated to be 4.0%.

Of course the physics of nuclear fusion reactions are considered in the SSM. It is explained in the next section.

The validity of the SSM is checked by the comparison of the observed helioseismological sound speed with the prediction of the SSM [8]. From that reference, BP98 prediction agrees with the observation to within 0.1 %. That agreement provides the validity of the temperature. The relation between the uncertainty of sound speed \( c \) and the uncertainty of the temperature \( T \) and the uncertainty of the mean molecular weight \( \mu \) is given as follows:

\[
\frac{\Delta c}{c} \simeq \frac{1}{2} \left( \frac{\Delta T}{T} - \frac{\Delta \mu}{\mu} \right) \quad (2.1.1)
\]

From this relation, it is expected that these three uncertainties are of similar magnitude [8].
<table>
<thead>
<tr>
<th>source</th>
<th>flux (cm$^{-2}$s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp chain</td>
<td></td>
</tr>
<tr>
<td>pp</td>
<td>$5.94 \times 10^6$ (1.00 ± 0.01)</td>
</tr>
<tr>
<td>pep</td>
<td>$1.39 \times 10^6$ (1.00 ± 0.01)</td>
</tr>
<tr>
<td>hep</td>
<td>$2.10 \times 10^8$</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>$4.80 \times 10^7$ (1.00 ± 0.09)</td>
</tr>
<tr>
<td>$^8$B</td>
<td>$5.15 \times 10^6$ (1.00$^{+0.19}_{-0.14}$)</td>
</tr>
<tr>
<td>CNO cycle</td>
<td></td>
</tr>
<tr>
<td>$^{13}$N</td>
<td>$6.05 \times 10^8$ (1.00$^{+0.19}_{-0.13}$)</td>
</tr>
<tr>
<td>$^{15}$O</td>
<td>$5.32 \times 10^8$ (1.00$^{+0.22}_{-0.15}$)</td>
</tr>
<tr>
<td>$^{17}$F</td>
<td>$6.33 \times 10^8$ (1.00$^{+0.12}_{-0.11}$)</td>
</tr>
</tbody>
</table>

Table 2.1: Solar neutrino fluxes predicted by the SSM (BP98)

2.2 Nuclear fusion in the sun and the solar neutrinos

The SSM predicts that over 98% of solar energy is produced from the pp-chain reaction, which is the conversion of four protons into $^4$He.

$$4p \rightarrow ^4He^* + 2e^+ + 2\nu_e$$  \hspace{1cm} (2.2.1)

The energy produced per pp-chain reaction is 26.7 MeV. Figure 2.1 shows the reaction tree of the pp-chain. As described above, the temperature in the core is about $1.5 \times 10^7$ K. Thus the protons in the core do not have enough energy to overcome the Coulomb repulsion. The solar nuclear fusion occurs via only the quantum mechanical tunneling effect. The energy dependence of the fusion cross section is written as follows:

$$\sigma(E) = \frac{S(E)}{E} \exp(-2\pi\eta(E)),$$  \hspace{1cm} (2.2.2)

where $E$ is the center-of-mass energy and $S(E)$ is called the S-factor. The Sommerfeld parameter $\eta(E)$ is defined as follows:

$$\eta(E) = \frac{Z_1 Z_2 e^2}{\hbar} \sqrt{\frac{\mu}{2E}},$$  \hspace{1cm} (2.2.3)

where $Z_1$ and $Z_2$ are the charge numbers of the nuclei. $\mu$ is the reduced mass of the system:

$$\mu = \frac{A_1 A_2}{A_1 + A_2},$$  \hspace{1cm} (2.2.4)
Figure 2.1: The pp-chain.

Figure 2.2: The CNO cycle.
Figure 2.3: The solar neutrino energy spectrum.

where $A_1$ and $A_2$ are the masses of the nuclei. In the condition in which the WKB approximation is valid, $S(E)$ is continuous and varies slowly. Then $S(E)$ can be expanded as follows:

$$S(E) = S(0) + S'(0)E + \frac{1}{2}S''(0)E^2$$  \hspace{1cm} (2.2.5)

The $S(0)$ for the solar nuclear fusion reactions are summarized in Tab.2.2.

The neutrino emission reactions of the pp-chain reaction are as follows:

$$p + p \rightarrow D + e^+ + \nu_e \quad \text{pp neutrino}$$  \hspace{1cm} (2.2.6)

$$E_\nu \leq 0.420\text{MeV}$$

<table>
<thead>
<tr>
<th>reaction</th>
<th>$S(0)$</th>
<th>notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp$</td>
<td>$4.00 \times 10^{-25}$ $\left(1 \pm 0.007^{+0.020}_{-0.011}\right)$ MeV b</td>
<td>$S_{11}$</td>
</tr>
<tr>
<td>$^3\text{He}(^3\text{He},2p)^4\text{He}$</td>
<td>$5.4 \pm 0.05$ MeV b</td>
<td>$S_{33}$</td>
</tr>
<tr>
<td>$^3\text{He}(^4\text{He},\gamma)^7\text{Be}$</td>
<td>$0.53 \pm 0.05$ keV b</td>
<td>$S_{34}$</td>
</tr>
<tr>
<td>$^7\text{Be}(p,\gamma)^8\text{B}$</td>
<td>$19^{+4}_{-2}$ eV b</td>
<td>$S_{17}$</td>
</tr>
<tr>
<td>$^3\text{He}(p,e^+ + \nu_e)^4\text{He}$</td>
<td>$2.3 \times 10^{-20}$ keV b</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: $S(0)$ for the solar nuclear fusion reactions.
\[ p + e^- + p \rightarrow D + \nu_e \quad \text{pep neutrino} \quad (2.2.7) \]

\[ E_\nu = 1.442 \text{MeV} \]

\[ ^7\text{B} + e^- \rightarrow ^7\text{Li} + \nu_e \quad ^7\text{Be neutrino} \quad (2.2.8) \]

\[ E_\nu = 0.861 \text{MeV}(90\%), \; 0.383 \text{MeV}(10\%) \]

\[ ^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e \quad ^8\text{B neutrino} \quad (2.2.9) \]

\[ E_\nu < 15 \text{MeV} \]

\[ ^3\text{He} + p \rightarrow ^4\text{He}^* + e^+ + \nu_e \quad \text{HeP neutrino} \quad (2.2.10) \]

\[ E_\nu < 18.77 \text{MeV} \]

The energy spectra and the fluxes of these neutrinos predicted by the SSM (BP98) are shown in Fig.2.3 and Tab.2.1, respectively.

There is another reaction series called the CNO cycle. Fig.2.2 shows the schematic overview of the CNO cycle. High temperature \( (T_c > 10^8 \text{ K}) \) is required for the nuclear fusion of heavy elements such as C, N, and O. So, in the current core temperature \( (T_c \sim 10^7 \text{K}) \), the contribution to total energy generation is suppressed to 2%. The neutrino emission reactions of the CNO cycle are as follows:

\[ ^{13}\text{N} \rightarrow ^{13}\text{C}^* + e^+ + \nu_e \quad ^{13}\text{N neutrino} \quad (2.2.11) \]

\[ E_\nu < 1.2 \text{MeV} \]

\[ ^{15}\text{O} \rightarrow ^{15}\text{N}^* + e^+ + \nu_e \quad ^{15}\text{O neutrino} \quad (2.2.12) \]

\[ E_\nu < 1.73 \text{MeV} \]

\[ ^{17}\text{F} \rightarrow ^{17}\text{F}^* + e^+ + \nu_e \quad ^{17}\text{F neutrino} \quad (2.2.13) \]

\[ E_\nu < 1.2 \text{MeV} \]

The energy spectra and the fluxes of CNO neutrinos predicted by the SSM (BP98) are also shown in Fig.2.3 and Tab.2.1, respectively.
2.3 The \(^8\text{B}\) neutrino flux and energy spectrum

More than 99\% of the neutrino events in SK are from \(^8\text{B}\) neutrinos. In this section, the uncertainties of the flux and the energy spectrum of the \(^8\text{B}\) neutrino are described.

The \(^8\text{B}\) neutrino flux has a large uncertainty compared to other fluxes such as for pp neutrino and \(^7\text{Be}\) neutrino. The uncertainty in the prediction originate from the following sources [9]:

\[
\begin{align*}
L_{\odot} & : \ 2.8 \ % \\
t_{\odot} & : \ 0.6 \ % \\
\text{Opacities} & : \ 5.2 \ % \\
\text{Diffusion} & : \ 4.0 \ % \\
Z/X & : \ 4.2 \ % \\
S_{11}(0) & : \ 4.0 \ % \\
S_{33}(0) & : \ 2.1 \ % \\
S_{34}(0) & : \ 7.5 \ % \\
S_{17}(0) & : \ 10.5 \ %
\end{align*}
\]

The uncertainties from \(L_{\odot}\) to \(Z/X\) are explained in Sec.2.1. \(S(0)\) is the first term in Eq. (2.2.5) and the subscript represents the atomic number of the interacting nuclei. For example, \(S_{11}(0)\) is \(S(0)\) for proton-proton nuclear fusion. The uncertainty in \(S_{17}(0)\), namely the cross section of \(^7\text{Be}(p, \gamma)^8\text{B}\), is the dominant uncertainty for the \(^8\text{B}\) flux. \(S_{17}(0)\) is measured in three different ways: direct \(^7\text{Be}(p, \gamma)^8\text{B}\) measurement, measurement using the reaction \(^7\text{Li}(p, \gamma)^8\text{Li}\), and an indirect measurement involving the dissociation of \(^8\text{B}\) in the Coulomb field. But these measurements are carried out with higher energy than the the energy of proton in the solar core. Moreover, these results are not consistent with each other as summarized in Ref.[10]. So Bahcall and his collaborators calculate the recommended value from these results [10]:

\[
S_{17}(0) = 19_{-2}^{+4}.
\] (2.3.1)

This value is used in the SSM (BP98).

As shown in (2.2.9), a \(^8\text{B}\) neutrino is produced via the \(\beta^+\) decay of \(^8\text{B}\). The energy levels of \(^8\text{B}\) decay are shown in Fig.2.4. The energy spectrum of \(^8\text{B}\) neutrinos is obtained by measurements of \(^8\text{B}\) \(\beta\) decay spectrum and the \(\alpha\) spectrum from the subsequent \(^8\text{Be}\) decay. The \(^8\text{B}\) \(\beta\) decay spectrum with the momentum above 9 MeV/c is measured using a \(\beta\)-spectrometer [12]. The error of the absolute momentum calibration is estimated using the \(^{12}\text{B}\) \(\beta\) spectrum shape \((Q_{\beta^+} = 13.4\ \text{MeV})\) and is obtained to be \(\pm 0.090\ \text{MeV}\). The delayed \(\alpha\) spectrum of \(^8\text{Be}\) is measured by several experimental groups [13] [14] [15]. These \(\alpha\) decay data are compared and used to fit the \(\beta\) decay data. The best-fit \(\alpha\) spectrum and its uncertainty are obtained from this overconstrained comparison. The \(^8\text{B}\) neutrino spectrum is determined using the best-fit \(\alpha\) spectrum. This is shown in Fig.2.5. The
Figure 2.4: Energy levels in the $^8$B ($\beta^+$)$^8$Be(2$\alpha$) decay chain. The gray zone shows the $2^+$ states of $^8$Be$^*$. 

Figure 2.5: The $^8$B neutrino energy spectrum is presented by the solid line. The broken line and the dotted line show the spectra allowed by $\pm 3\sigma$ theoretical and experimental uncertainties.
spectra allowed by ±3σ uncertainties are also shown in the same figure. The uncertainty of the $^{8}$B neutrino energy spectrum is caused by the experimental uncertainty of the α spectrum measurement. A more detailed explanation is given in Ref.[3].

2.4 Hep neutrino

SK can also detect Hep neutrinos. As shown in Tab.2.1, the Hep neutrino flux is expected to be be much smaller than that of $^{8}$B neutrino. So the contribution to the neutrino flux measurement of SK should be very small. However, in energy spectrum measurement, the contribution of the Hep neutrino may be not negligible. The endpoint of the Hep neutrino energy spectrum is higher than that of $^{8}$B neutrino as shown in Fig.2.3. So SK may observe the Hep neutrino as an excess in the high energy side of the recoil electron energy spectrum.

The uncertainty in the Hep flux mainly comes from the cross section for the $^{3}$He($p,e^{+} + \nu_{e}$)$^{4}$He reaction. The relation between the measured thermal neutron cross section and the low energy cross section factor for the production of Hep neutrinos is so complicated that many authors obtained many different conclusions [16] [17]. From the various estimations,

$$S(0) = 2.3 \times 10^{-20} \text{ keV b} \quad (2.4.1)$$

is used in the SSM (BP98) [3]. But no uncertainty in the Hep flux is provided.

From the analysis using the recoil electron energy spectrum from 18 MeV to 25 MeV in 1117 days of data, the preliminary upper limit of the Hep neutrino is obtained to be 13.2 times the Hep flux predicted by the SSM (BP98) [18].

2.5 Solar neutrino flux observations

2.5.1 Homestake

The first solar neutrino observation was carried out by R. Davis and his Brookheaven collaborators in 1968. Their detector consisted of a horizontal steel tank, 6.1 m diameter and 14.6 m long, containing 615 m$^{3}$ of C$_{2}$Cl$_{4}$. It was located in the Homestake Gold Mine in Lead, South Dakota, United States. The average overburden for the detector was 4200 ± 100 meter water equivalent (m.w.e).

In order to detect solar neutrinos, they used the inverse β reaction $^{37}$Cl($\nu_{e},e^{-}$)$^{37}$Ar. At the end of each exposure of about eight months, $^{37}$Ar was collected by purging the C$_{2}$Cl$_{4}$ with He gas. He, carrying with it Ar, was cleaned by a condenser (−40°C) and a molecular sieve trap before a chaoal trap. This chaoal trap was cooled by liquid N$_{2}$ (−196°C) to freeze and absorb Ar (freezing point of Ar is −189°C). Then the trap was warmed (+200°C) and Ar was transferred to a small system to measure the volume. After
Table 2.3: Solar neutrino detection rates of flux measurements. For SK, the ratio to the total flux is shown.

that, Ar was transferred to a gas chromatographic system. It separated Ar from the heaver gases, such as Kr, Xe and Rn. Then volume of Ar was measured and transferred to the proportional counter. Ar was mixed with CH$_4$ whose volume was 7% of the Ar volume. The number of $^{37}$Ar was determined by observing their decay electrons. The decay electrons with an energy of 2.8 keV represents 81.5% of all decays. An additional 8.7% of the decays involves a K orbital electron capture together with the emission of a \( \gamma \)-ray (2.6 \sim 2.8 \text{ keV}). 10% of these \( \gamma \)-rays converts and deposits an energy of 2.8 keV. The reaction threshold of \( ^{37}\text{Cl} \) neutrino absorption is 0.814 MeV, so the dominant components of neutrino sources were $^7$Be neutrinos and $^8$B neutrinos. The contribution of pep neutrinos and neutrinos from the CNO-cycle were small because their fluxes are small. The energy of $^7$Be neutrinos is 0.861 MeV, so it can excite just the Gamow-Teller transition to the $^{37}$Ar ground state. $^8$B neutrinos can excite $^{37}$Cl to numerous states. The interaction cross section is calculated from the $\beta$ decay $^{37}\text{Ca}(\beta^+)^{37}\text{K}$, which is an isospin mirror of $^{37}\text{Cl}(\nu_e,e^-)^{37}\text{Ar}$. The number of events predicted by the SSM is \[ R_{C_{F98}}^{B_{B98}} = 7.7_{-1.0}^{+1.2} \text{ SNU}. \] Tab.2.3 summarizes the contributions of neutrino sources. The measured capture rate \cite{1} is

\[ R_{C_{F98}}^{\nu_{e}} = 2.56 \pm 0.16 \pm 0.16 \text{ SNU}, \]

where “SNU” is the number of interactions per $10^{36}$ targets per second. The ratio of the measured rate to the SSM prediction is $0.33 \pm 0.06$. 

<table>
<thead>
<tr>
<th>source</th>
<th>Chlorine (SNU)</th>
<th>Gallium (SNU)</th>
<th>SK</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp</td>
<td>0.0</td>
<td>69.6</td>
<td>0.0</td>
</tr>
<tr>
<td>pep</td>
<td>0.22</td>
<td>2.84</td>
<td>0.0</td>
</tr>
<tr>
<td>hep</td>
<td>0.01</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>1.15</td>
<td>34.4</td>
<td>0.0</td>
</tr>
<tr>
<td>$^8$B</td>
<td>5.87</td>
<td>12.4</td>
<td>0.999</td>
</tr>
<tr>
<td>$^{13}$N</td>
<td>0.10</td>
<td>3.65</td>
<td>0.0</td>
</tr>
<tr>
<td>$^{15}$O</td>
<td>0.36</td>
<td>6.05</td>
<td>0.0</td>
</tr>
<tr>
<td>$^{17}$F</td>
<td>0.0</td>
<td>0.07</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td>$7.7_{-1.0}^{+1.2}$</td>
<td>$129_{-6}^{+8}$</td>
<td>1.0</td>
</tr>
</tbody>
</table>
2.5.2 Kamiokande

Kamiokande was a water Cherenkov detector which was located near SK. Purified water of 3000 m$^3$ was contained in a stainless steel tank of 15.6 m diameter and 16.1 m height. The inner detector had 946 50 cm-PMT viewing 2140 m$^3$ of purified water.

The observation of solar neutrinos started in 1987 using recoil electrons via neutrino-electron elastic scattering. The Cherenkov photons emitted from recoil electrons are detected by the PMTs. The vertex, direction, and energy of each event were reconstructed using the Cherenkov photons. Solar neutrino events could be extracted using the direction correlation to the sun.

The energy threshold was $\sim 7.0$ MeV. Therefore, $^8$B neutrinos and Hep neutrinos could be observed. The measured flux of solar neutrinos over 2079 days is:

$$\phi_{Kam}^{obs} = 2.80 \pm 0.19 \text{ (stat.)} \pm 0.33 \text{ (syst.)} (\times 10^6 \text{ cm}^{-2}\text{s}^{-1}), \quad (2.5.2)$$

while the SSM prediction is $5.15(1.00^{+0.19}_{-0.14}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$ [19]. The ratio of the measured flux to the SSM prediction is $0.55^{+0.13}_{-0.13}$.

2.5.3 GALLEX and SAGE

GALLEX and SAGE were radiochemical experiments using Ga. A solar neutrino was detected by an inverse $\beta$ interaction $^7$Ga($\nu_e,e^-)^{71}$Ge. The energy threshold of absorption is $232.69 \pm 0.15$ keV. So pp neutrinos could be detect by these detectors. The K and L Auger electrons from $^{71}$Ge decay have energies of 10.4 keV and 1.2 keV, respectively. The lifetime for $^{71}$Ge electron capture has been measured as $\tau_{1/2} = 11.43 \pm 0.03$ day [20].

The GALLEX detector was located at Gran Sasso Underground Laboratory at the depth of 3800 m.w.e. The detector observed solar neutrino from 1991 to 1997. The target consisted of 101 tons (53.5 m$^3$)of GaCl$_3$, of which 12 tons were $^{71}$Ga. At the end of each 20 day exposure, about 1 mg of Ge carriers were added to extract $^{71}$Ge chemically. The GeCl$_4$ was volatile and purged with N$_2$ gas. Then it converted to GeH$_4$ and filled a proportional counter with Xe. The proportional counter was enclosed in a NaI veto counter surrounded by lead shielding and counted the K and L Auger electrons for a period of 90 days. The detector was calibrated twice with a neutrino source ($^{51}$Cr) of known intensity [21]. $^{51}$Cr emits neutrinos with energies of 428 keV (9.0%), 747 keV (81.63%) and 752 keV (8.5%), which is similar to the energies of pp and $^7$ Be solar neutrinos. The source was introduced inside of the detector. The source emitted about $6 \times 10^{13}$ neutrinos per second just after the installation. This flux was 10 times larger than that of solar neutrinos. After 3 months calibration, the flux was reduced to 1/10 of the initial one. The $^{51}$Cr calibration showed that the observed neutrino detection efficiency was consistent with the estimated value.
The result of \(^{51}\text{Cr}\) calibration is:

\[
\frac{\sigma^{(51}\text{Cr})_{\text{measured}}^{\text{GALLEX}}}{\sigma^{(51}\text{Cr})_{\text{BUS}}^{\text{ss}}} = 0.93 \pm 0.08, \tag{2.5.3}
\]

where \(\sigma^{(51}\text{Cr})_{\text{measured}}^{\text{GALLEX}}\) is the absorption cross section of \(^{71}\text{Ga}\) measured by the \(^{51}\text{Cr}\) calibration at GALLEX. \(\sigma^{(51}\text{Cr})_{\text{BUS}}^{\text{ss}}\) is the calculated cross section by Bahcall and Ulrich [22] and it is \(59.2 \times 10^{-46} (1 \pm 0.1) \text{cm}^{-2}\). Another test was performed at the end of experiment by adding \(^{71}\text{As}\) to the target solution [24]. \(^{71}\text{As}\) decays \((\beta^+ 32\% \text{ and electron capture 68\%})\) with a the half-life of 2.72 days into \(^{71}\text{Ge}\). By using the \(^{71}\text{Ge}\) from \(^{71}\text{As}\) decay, the efficiency of extraction was studied and it was found to be consistent with expectation.

The measured solar neutrino capture rate by GALLEX is [23]:

\[
R_{\text{GALLEX}}^{\text{obs}} = 77.5 \pm 6.2_{-4.3}^{+4.7} \text{SNU}. \tag{2.5.4}
\]

The SAGE detector is situated in a tunnel in the northern Caucasus mountains in southern Russia at the depth of about 4700 m.w.e. Solar neutrino observation began in 1990. The target is 50 tons of metallic \(^{68}\text{Ga}\). At the end of an exposure period (typically one month), a small amount (a few hundred mg) of Ge carrier is added to each detector. Then \(^{71}\text{Ge}\) is extracted chemically by a solution of dilute HCl and H\(_2\)O\(_2\). The GeCl\(_4\) is separated from the metallic \(^{68}\text{Ga}\) emulsion by bubbling the extracts with Ar. GeCl\(_4\) is then transformed into GeH\(_4\) and mixed with Xe to be filled in proportional counters to count the K auger electrons. Each sample counted for a period of 2 ~ 3 months. \(^{51}\text{Cr}\) calibration was performed in SAGE too. The result is [26]:

\[
\frac{\sigma^{(51}\text{Cr})_{\text{measured}}^{\text{SAGE}}}{\sigma^{(51}\text{Cr})_{\text{BUS}}^{\text{ss}}} = 0.95 \pm 0.12, \tag{2.5.5}
\]

The measured solar neutrino capture rate of SAGE is [25]:

\[
R_{\text{SAGE}}^{\text{obs}} = 66.6_{-7.1}^{+6.8} + 4.3_{-4.0}^{+4.6} \text{SNU}. \tag{2.5.6}
\]

The SSM prediction for Gallium experiments is [2]:

\[
R_{\text{Ga}}^{\text{SSM}} = 129_{-6}^{+8} \text{SNU}. \tag{2.5.7}
\]

The contributions of neutrino sources are summarized in Tab.2.3. The ratio of measured rate to the SSM prediction is \(0.60 \pm 0.07\) (GALLEX) and \(0.52 \pm 0.07\) (SAGE), respectively.

### 2.6 The solar neutrino problem

As described above, the observed solar neutrino fluxes are significantly smaller than the SSM expectation. Because of the different energy threshold of those experiments, the
fluxes for some neutrino sources can be estimated roughly. The relative fluxes to the SSM prediction for each experiment is written as follows:

\[ R_{Cl} = 0.0\alpha_{ip} + 0.15\alpha_{Be} + 0.76\alpha_{B8}, \quad (2.6.1) \]
\[ R_{Ga} = 0.56\alpha_{pp} + 0.27\alpha_{Be} + 0.1\alpha_{B8}, \quad (2.6.2) \]
\[ R_{SK} = 0.0\alpha_{pp} + 0.0\alpha_{Be} + 0.999\alpha_{B8}, \quad (2.6.3) \]

where \( \alpha \) is relative neutrino flux from each source written in the subscript. Small flux components are ignored.

Measurements give \( R_{Cl} = 0.33 \), \( R_{Ga} = 0.56 \), and \( R_{SK} = 0.465 \). Then, the following relations are obtained:

\[ \phi(pp) \sim 0.9 \times \phi_{SSM}(pp) \]
\[ \phi(7\text{Be}) \sim 0 \times \phi_{SSM}(7\text{Be}) \]
\[ \phi(8\text{B}) \sim 0.5 \times \phi_{SSM}(8\text{B}) \]

So, lack of \( 7\text{Be} \) neutrino and half flux of \( 8\text{B} \) neutrino are concluded from flux measurements.

To explain the deficit in the framework of the SSM, some possibilities are discussed such as the temperature in the core \( T_c \) and the cross section of \( 7\text{Be} \) proton capture. The temperature in the core influences the rate of nuclear fusion. If the \( T_c \) is much lower than the value used in the SSM, then all of the neutrino flux is decreased. The temperature dependence of the \( 8\text{B} \) neutrino flux is \( \phi(8\text{B}) \sim T_{c}^{18} \), although that of the \( 7\text{Be} \) neutrino flux is \( \phi(7\text{Be}) \sim T_{c}^{8} \) [9]. Thus, a large reduction in the \( 7\text{Be} \) neutrino flux results in a much larger reduction in the \( 8\text{B} \) neutrino flux in the SSM. In addition, as described at the end of Sec.2.1, the uncertainty of the temperature is estimated to be less than 1% from helioseismology.

The ratio of proton capture by \( 7\text{Be} \) is 0.1% of that of electron capture by \( 7\text{Be} \). So, if \( S_{17}(0) \) larger than expected by 2 orders of magnitude, then the results can be explained. The estimated error of \( S_{17}(0) \) is \( \sim 25\% \) at \( 1\sigma \), so this increase is excessive. The uncertainty of \( 7\text{Be} \) electron capture rate is also studied and found to be less than 2%.

In summary, in order to solve the solar neutrino problem by changing the SSM, impractical assumptions are required. However, the SSM itself is confirmed by precise temperature measurement by helioseismology. The problem, therefore, cannot be explained by uncertainties of the SSM. In next section, solar neutrino oscillations are introduced as a candidate of solution.

### 2.7 Neutrino oscillations

In this section, the theory of neutrino oscillations is described. It is regarded as the most likely solution of the solar neutrino problem. Here I consider just two neutrino flavors to simplify the discussion.
2.7.1 Vacuum oscillations

The neutrino states of definite mass $|m\rangle$ can be written as a linear combination of states of flavor $|f\rangle$ as follows:

$$|m\rangle = U|f\rangle. \quad (2.7.1)$$

A unitary matrix $U$ is chosen as follows:

$$U = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta 
\end{pmatrix}, \quad (2.7.2)$$

where $\theta$ is the mixing angle in vacuum.

Here, the mass eigenstates are defined as $|\nu_1\rangle$ and $|\nu_2\rangle$, and the flavor eigenstates are defined as $|\nu_e\rangle$ and $|\nu_\mu\rangle$. Then a particle which is produced at $t = 0$ as an $\nu_1$ evolves in time as:

$$|\nu_l(t)\rangle = \sum_i U_{li} \exp (-iE_it)|\nu_i(0)\rangle$$

$$= \sum_i U_{li} |\nu_i\rangle \exp (-iE_it)\langle\nu_i|\nu_l(0)\rangle \quad (2.7.3)$$

where $E_i$ is the neutrino energy. $i$ denotes the initial mass eigenstate of the neutrino, which is 1 or 2. $l$ denotes lepton flavor, such as $e$, $\mu$, or $\tau$.

From Eq. (2.7.3), the probability amplitude for a lepton $l$ to change to a lepton $l'$ is calculated as follows:

$$\langle \nu_l|\nu_l(t)\rangle = \sum_{i,i'} U_{li} U_{l'i}^{-1} \exp (-iE_it)\delta_{ii'}$$

$$= \sum_{i} \sum_{i'} U_{li} U_{l'i}^{-1} \exp (-iE_it)$$

$$= \sum_{i} U_{li} U_{l'i}^{-1} \exp (-iE_it) \quad (2.7.4)$$

If the neutrino does not interact with other particles, and the neutrino momentum $p$ is much larger than the electron mass $m_e$, then $E_i$ can be written as follows:

$$E_i = \sqrt{p^2 + m_i^2}$$

$$\simeq p + \frac{m_i^2}{2p}$$

$$\simeq E + \frac{m_i^2}{2E} \quad (2.7.5)$$
The probability that a neutrino created as $\nu_i$ at $t = 0$ is in the state $\nu_{i'}$ is calculated from Eq. 2.7.4 and Eq. 2.7.5:

$$P(\nu_i \to \nu_{i'}) = |\langle \nu_i | \nu_{i'} \rangle|^2 = \sum_i \sum_{i'} U_{ii'}^* U_{i'i} \exp\{-i(E_i - E_{i'})t\} = \sum_i \sum_{i'} U_{ii'}^* U_{i'i} (2 - 2 \cos(E_i - E_{i'})t)$$ \hspace{1cm} (2.7.6)

From Eq. (2.7.6), the probability of $\nu_e - \nu_x$ oscillation is calculated as follows:

$$P(\nu_e \to \nu_x) = \cos^2 \theta \sin^2 \theta \cos(E_1 - E_2)t = \frac{1}{2} \sin^2 2\theta \left(1 - \cos \frac{m_1^2 - m_2^2}{2E}t \right) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos \frac{m_1^2 - m_2^2}{2E}t \right) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4E}L\right)$$ \hspace{1cm} (2.7.7)

where $\Delta m^2 = m_1^2 - m_2^2$ and $L = t$ in natural units ($c=1$).

Therefore, the probability of $\nu_e - \nu_e$ oscillation is obtained as:

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4E}L\right) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\pi L}{L_V}\right).$$ \hspace{1cm} (2.7.8)

where the oscillation length in vacuum $L_V$ is defined as:

$$L_V = \frac{4\pi E}{\Delta m^2}$$ \hspace{1cm} (2.7.9)

### 2.7.2 Neutrino oscillations in matter (the MSW effect)

When neutrinos propagate thorough matter, potentials acting on neutrinos are different for each neutrino flavor. If there is no flavor changing neutral current, the neutral current interaction ($Z^0$ exchange) is identical for all neutrino flavors. However, the charged current interaction ($W^\pm$ exchange) can occur only with electron neutrinos. This causes the potential for $\nu_e$ to be different from that for $\nu_\mu$ and $\nu_\tau$. This mechanism was pointed out by Mikheyev and Smirnov based on the theory advocated by Wolfenstein, and is called the MSW effect [27] [28].
In the potential $V$, the hamiltonian $H$ can be written as:

$$
H = \sqrt{p^2 + M^2} + V
= p + \frac{1}{2p}(M^2 + 2pV).
$$

(2.7.10)

Here I consider the oscillation between $\nu_e$ and $\nu_x, (x = \mu$ or $\tau)$. Then $V$ is:

$$
V = \begin{pmatrix}
V_{NC} + V_{CC} & 0 \\
0 & V_{NC}
\end{pmatrix},
$$

(2.7.11)

where $V_{NC}$ is the potential via the neutral current and $V_{CC}$ is the potential via the charged current. $V_{CC}$ was calculated by Bethe to be $\sqrt{2}G_FN_e$, where $G_F$ is the Fermi coupling constant and $N_e$ is the electron number density.

From the analogy between Eq.( 2.7.5) and Eq.( 2.7.10), $(M^2 + 2pV)$ is treated as an effective mass due to the potential $V$. For convenience, the coordinate is changed from the mass eigenstates to the flavor eigenstates:

$$
M_{\text{eff}}^2 = UM^2U^{-1} + 2pV
= \frac{1}{2}(m_1^2 + m_2^2 + A)I
+ \frac{1}{2} \begin{pmatrix}
A - \Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\
\Delta m^2 \sin 2\theta & -A + \Delta m^2 \cos 2\theta
\end{pmatrix},
$$

(2.7.13)

where $A$ is defined as follows:

$$
A = 2pV_{CC} = 2\sqrt{2}pG_FN_e
$$

(2.7.14)

The time development equation of the flavor eigenstates in matter is written as follows using Eq.( 2.7.10) and Eq.( 2.7.11) :

$$
\frac{d}{dt} \begin{pmatrix}
\nu_e \\
\nu_x
\end{pmatrix}
= \begin{pmatrix}
p & 0 \\
0 & p
\end{pmatrix} U^{-1} + \frac{1}{2p} M_{\text{eff}}^2 \begin{pmatrix}
\nu_e(t) \\
\nu_x(t)
\end{pmatrix}
$$

(2.7.15)

$$
= \frac{1}{4} (m_1^2 + m_2^2 + A)I
+ \frac{1}{4p} \begin{pmatrix}
A - \Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\
\Delta m^2 \sin 2\theta & -A + \Delta m^2 \cos 2\theta
\end{pmatrix} \begin{pmatrix}
\nu_e(t) \\
\nu_x(t)
\end{pmatrix}
$$

(2.7.16)
Terms which are common between each flavor are dropped from Eq. (2.7.15) and Eq. (2.7.16). The eigenvalues of \( M_{\text{eff}} \) are obtained as follows:

\[
m_{\text{eff},i}^2 = \frac{1}{2} \left( m_1^2 + m_2^2 + A \right) \\
\pm \frac{1}{2} \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2)^2 \sin^2 2\theta}. \quad (i = 1, 2)
\]

(2.7.17)

The eigenstates in matter are:

\[
\begin{pmatrix}
\nu_e \\
\nu_x
\end{pmatrix} = \begin{pmatrix}
\cos \theta_M & \sin \theta_M \\
\sin \theta_M & \cos \theta_M
\end{pmatrix} \begin{pmatrix}
\nu_{1m} \\
\nu_{2m}
\end{pmatrix},
\]

(2.7.18)

where \( \theta_M \) is:

\[
\tan 2\theta_M = \frac{\tan 2\theta}{1 - \frac{A}{\Delta m^2 \cos 2\theta}}.
\]

(2.7.19)

The mixing length in matter \( L_M \) is

\[
L_M = \frac{4\pi E}{m_{\text{eff},i}^2 - m_{\text{eff},1}^2}
\]

(2.7.20)

\[
= \frac{4\pi E}{\Delta m^2} \frac{1}{\left( \frac{A}{\Delta m^2} - \cos 2\theta \right)^2 + \sin^2 2\theta}.
\]

(2.7.21)

In order to compute the oscillation probability, the electron density \( N_e \) obtained by the SSM is used. Fig.2.8 shows the electron density profile of the sun. The density decreases with increasing radius, so there could be a condition where \( N_e \) and \( r \) causes \( \tan 2\theta_M \) to be infinite (resonance), namely the mixing of neutrinos is maximum. \( N_e \) at the resonance is represented as \( N_{e,\text{crit}} \). The minimum neutrino energy that can be resonant is:

\[
E_{\text{crit}} = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} G_F N_e(\text{core})},
\]

(2.7.22)

where \( N_e(\text{core}) \) is the electron number density in the core.

Fig.2.6 illustrates how a neutrino can completely change its flavor. If \( \nu_e \) is created in the region \( N_e > N_{e,\text{crit}} \) and propagates into the region \( N_e < N_{e,\text{crit}} \), the state follows the upper path and \( \nu_e \) is fully converted into \( \nu_\mu \). This phenomenon occurs when the density gradient is sufficiently small.

\[
\frac{1}{N_e} \frac{dN_e}{dr} \ll \frac{\Delta m^2 \sin^2 2\theta}{2E \cos 2\theta}.
\]

(2.7.23)

This is called the adiabatic condition.
Figure 2.6: Level crossing of the mass eigenstates. As the electron density falls, a \( \nu_e \) can oscillate almost completely to \( \nu_\mu \).

If the adiabatic condition is not satisfied, the state in the upper path of Fig.2.6 undergoes a transition to the lower path while passing through the crossing point with a probability given by:

\[
P_f = \exp \left(-\frac{\pi}{2} \gamma \right),
\]

where

\[
\gamma = \frac{\Delta m^2 \sin^2 2\theta}{2E \cos 2\theta} \frac{1}{N_e} \frac{dN_e}{dr}
\]

(2.7.25)

The \( \nu_e \rightarrow \nu_e \) transition probability, taking \( P_f \) into account, is:

\[
P(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \left( \frac{1}{2} - P_f \right) \cos 2\theta \cos 2\theta_M.
\]

(2.7.26)

2.8 Possible results from the day and night energy spectrum

In this section I briefly explain the possible solar neutrino oscillation parameters based on experiments and expected results from the observation of daytime and nighttime recoil electron energy spectra.
Figure 2.7: Neutrino production as a function of the solar radius

Figure 2.8: The electron number density in the sun as a function of the solar radius.
Figure 2.9: The electron density in the Earth as a function of Earth’s radius.

Fig. 2.10 shows the allowed parameter regions of neutrino oscillations obtained from the observed flux from solar neutrino experiments. Combined results are also shown in the same figures. The MSW region and the Vacuum region are shown in the upper and the lower figures, respectively. A detailed explanation about these figures is given in Chapter 8. Each allowed region is called the following:

- **Small mixing angle solution (SMA)**  
  The region around $\sin^2 2\theta = 5 \times 10^{-3}$, $\Delta m^2 = 3 \times 10^{-6}$ eV$^2$.

- **Large mixing angle solution (LMA)**  
  The region around $\sin^2 2\theta = 0.9$, $\Delta m^2 = 3 \times 10^{-5}$ eV$^2$.

- **Low solution (LOW)**  
  The region at $\sin^2 2\theta = 0.9$, $\Delta m^2 = 1 \times 10^{-7}$ eV$^2$.

- **Vacuum solution (VAC)**  
  The region at $\sin^2 2\theta > 0.6$, $\Delta m^2 < 4 \times 10^{-10}$ eV$^2$.

The expected daytime and nighttime recoil electron energy spectra for typical parameter values in each MSW region are presented in Fig. 2.11. From the figures, it is found that the survival probability increases with energy in the case of the SMA solution, while it is almost flat for the LMA and LOW solutions. So the spectrum shape can be used to distinguish between possible solutions.
A $\nu_\mu$ converted from $\nu_e$ in the sun may oscillate back to $\nu_e$ via the MSW effect in the earth [29]. So the daytime and the nighttime results might be different for certain parameter regions. This is called “neutrino regeneration”. In the earth, the resonance condition is represented as follows [30]:

$$\Delta m^2 \approx 4 \times 10^{-6} \text{ eV}^2 (\rho/5 Y_e \text{ g cm}^{-3}),$$  \hspace{1cm} (2.8.1)

where the density of the Earth $\rho = 3.5 \sim 13 \text{ g cm}^{-3}$, the neutrino energy $E_\nu \approx 10 \text{ MeV}$ and the electron number per atomic number $Y_e = 1/2$. Fig.2.9 shows the electron density profile of the earth [31]. The oscillation length of the resonance is $L_{M,\text{res}} \approx 6.8 \times 10^6 \text{ cm} / \sin 2\theta$. It is of the order of the diameter of the Earth, $(1.3 \times 10^9 \text{ cm})$, even if $\sin 2\theta$ is large.

The differences between the daytime and the nighttime recoil electron energy spectra are also shown in Fig.2.11. For the SMA solution, there is no difference between the day time and night time spectra. For the LMA solution, the difference is $1 \sim 10\%$ depending upon parameters. For the LOW solution, the daytime flux is slightly smaller than nighttime.

For the VAC solution, there is variety of solutions. However, it is possible to be excluded by using information from both the spectrum shape and daytime-nighttime asymmetry.
Figure 2.10: The neutrino oscillation allowed regions at 95% C.L. from the flux measurements. Thick lines show the allowed regions obtained from the measured flux of SK. Gray lines and thin lines show the allowed regions for the Gallium experiment and the Chlorine experiment, respectively. The shaded regions show the allowed regions obtained from all flux measurements.
Figure 2.11: The expected daytime and nighttime energy spectra at each MSW region
Chapter 3

Super-Kamiokande

In this chapter, the detection principle and the components of SK are described in detail. The cross section for $\nu_e$-$e$ elastic scattering and Cherenkov radiation are explained in Sec.3.1. From Sec.3.2, the detector components, the DAQ system, and the detector simulation are explained.

3.1 Detection method

3.1.1 Cross section

In SK, solar neutrinos are detected through neutrino-electron elastic scattering:

$$\nu_{e,\mu,\tau} + e^- \rightarrow \nu_{e,\mu,\tau} + e^-.$$  \hfill (3.1.1)

Feynman diagrams of these interactions are given in Fig.3.1.

The differential cross section of $\nu_e$-$e$ elastic scattering is calculated as follows [33] :

$$\frac{d\sigma}{dy} = \frac{2G_F^2 m_e}{2} \left\{ g_L^2(T) \left[ 1 + \frac{\alpha}{\pi} f_-(z) \right] 
+ g_R^2(T)(1-z)^2 \left[ 1 + \frac{\alpha}{\pi} f_+(z) \right] 
+ g_R(T)g_L(T) \frac{m_e}{E_\nu} z \left[ 1 + \frac{\alpha}{\pi} f_{+-}(z) \right] \right\},$$  \hfill (3.1.2)

where $\sigma$ is the cross section of $\nu_e$-$e$ elastic scattering, $E_\nu$ is the energy of the incident neutrino, $T_e$ is the kinetic energy of recoil electron, $m_e$ is the electron mass, $z = T/E_\nu$, and $G_F = 1.16639 \times 10^{-11}$ MeV$^{-2}$ is the fermi coupling constant.
Figure 3.1: The Feynman diagrams for the elastic scattering of a neutrino against an electron.

Figure 3.2: The Feynman diagrams for the electroweak radiative corrections for $\nu_e$-$e$ elastic scattering.

Figure 3.3: The Feynman diagram for the electroweak radiative correction for $\nu_\mu$-$e$ elastic scattering.
\( g_L \) and \( g_R \) in Eq. (3.1.2) are defined as follows:

\[
g_L^{(\nu,e)}(T) = \rho^{(\nu)}_{NC} \left[ \frac{1}{2} - \kappa^{\nu,e}(T) \sin^2 \theta_w(m_Z) \right] - 1, \quad (3.1.3)
\]

\[
g_R^{(\nu,e)}(T) = -\rho^{(\nu)}_{NC} \kappa^{\nu,e}(T) \sin^2 \theta_w(m_Z), \quad (3.1.4)
\]

where \( \theta_w \) is the Weinberg angle, which is obtained to be \( \sin^2 \theta_w = 0.2317 \) from experiments. \( \rho^{(\nu)}_{NC} \) is obtained as follows:

\[
\rho^{(\nu)}_{NC} = 1.0126 \pm 0.0016. \quad (3.1.6)
\]

The function \( \kappa^{\nu,e}(T) \) is:

\[
\kappa^{\nu,e}(T) = 0.9791 + 0.0097 I(T) \pm 0.0025, \quad (3.1.7)
\]

where

\[
I(T) \equiv \frac{1}{6} \left\{ \frac{1}{3} + (3 - y^2) \left[ \frac{1}{2} y \ln \left( \frac{y + 1}{y - 1} \right) - 1 \right] \right\} \quad (3.1.8)
\]

and \( y = \sqrt{1 + 2m_e/T} \).

Eq. (3.1.2) includes the effects of radiative correction, Quantum Chromodynamics effects (QCD), and Quantum Electrodynamics (QED) effects. The recoil electron energy dependence of \( \kappa \) comes from the interactions whose Feynman diagrams are shown in Fig.3.2.

For \( \nu_\mu-e \) scattering, the coefficients \( g_L^{(\nu_\mu,e)}(T) \) and \( g_R^{(\nu_\mu,e)}(T) \) are given as follows:

\[
g_L^{(\nu_\mu,e)}(T) = \rho^{(\nu)} \left[ \frac{1}{2} - \kappa^{\nu_\mu,e}(T) \sin^2 \theta_w(m_Z) \right], \quad (3.1.9)
\]

\[
g_R^{(\nu_\mu,e)}(T) = -\rho^{(\nu)} \kappa^{\nu_\mu,e}(T) \sin^2 \theta_w(m_Z), \quad (3.1.10)
\]

where

\[
\kappa^{\nu_\mu,e}(T) = 0.9970 - 0.00037 I(T) \pm 0.0025, \quad (3.1.12)
\]

and \( \rho^{(\nu)} \) is same as above.

The difference between the \( \kappa^{\nu,e}(T) \) and \( \kappa^{\nu_\mu,e}(T) \) comes from the fact that the first diagram in Fig.3.2 is replaced by the diagram in Fig.3.3. The functions \( f_-(z), f_+(z), f_+-(z) \) describe QED effects. They are described in Ref. [33]. The total \( \nu-e \) scattering cross section as a function of the neutrino energy is shown in Fig.3.4.

The energy distribution of recoil electrons is calculated as follows:

\[
F(T_e)dT_e = \int_{E_{\nu,\text{min}}}^{E_{\nu,\text{max}}} \phi(E_\nu) \frac{d\sigma}{dT_e} dE_\nu, \quad (3.1.13)
\]
Figure 3.4: The total electron-neutrino scattering cross section as a function of neutrino energy.

Figure 3.5: The differential electron-neutrino scattering cross section for neutrinos with an incident energy of 8 MeV.
Figure 3.6: The total energy distribution of electrons scattered by $^8$B neutrinos.

where $\phi(E_\nu)$ is the neutrino flux as a function of $E_\nu$. The distribution of $d\sigma/dT_e$ for neutrinos with an incident energy of 8 MeV is shown in Fig.3.5. Fig.3.6 shows the total energy distribution of electrons scattered by $^8$B neutrinos.

The scattering angle of a recoil electron with respect to the incident neutrino direction is obtained as follows:

$$\cos \theta = \frac{1 + \frac{m_e}{E_\nu}}{\sqrt{1 + \frac{2m_e}{T_e}}} \quad (3.1.14)$$

The scattering angle of recoil electrons is calculated with this equation and the mean is obtained to be approximately 9° for electrons with energy above 5 MeV.

From Eq. (3.1.13), the expected number of solar neutrino events in SK is calculated as follows:

$$N_{\text{event}} = N_e^{SK} \times \int_0^{T_e^{max}} F(T_e)dT_e \quad (3.1.15)$$

where $N_e^{SK} = 1.08 \times 10^{34}$ is the number of electrons within the inner detector, and $T_e^{max} = \frac{2E_\nu^2}{(2E_\nu + m_e)}$. $N_{\text{event}}$ is obtained to be 287.8 events/day.

### 3.1.2 Cherenkov radiation

A charged particle moving in a dielectric medium radiates Cherenkov photons if the particle velocity $v$ exceeds the speed of light in the medium $c/n$, where $n$ is the refractive index of
the medium and $c$ is the light speed in vacuum. Cherenkov photons are emitted in a cone defined by the angle $\theta_{ch}$ with respect to the direction of the particle motion. Fig.3.7 is a schematic view of the definition of $\theta_{ch}$. $\theta_{ch}$ is obtained as follows:

$$
\cos \theta_{ch} = \frac{1}{n(\lambda) \beta},
$$

(3.1.16)

where $n(\lambda)$ is the refractive index of the medium that depends on the wavelength $\lambda$ and $\beta$ is equal to $v/c$. In water, the refractive index is about 1.34. So, the maximum opening angle is 42°.

The minimum total energy required for a particle to produce Cherenkov photons is obtained from Eq. (3.1.16):

$$
E_{thr} = \frac{n \times m}{\sqrt{n^2 - 1}},
$$

(3.1.17)

where $m$ is mass of the particle. For an electron in water, $E_{thr}$ is 0.767 MeV.

The number of emitted Cherenkov photons $N_{\text{photon}}$ per unit length $dL$ is given as follows:

$$
\frac{d^2 N_{\text{photon}}}{dLd\lambda} = \frac{2 \pi \alpha}{\lambda^2} \left( 1 - \frac{1}{n^2 \beta^2} \right),
$$

(3.1.18)

where $\alpha$ is the fine structure constant. For $\lambda = 300 \sim 660$ nm, the number of Cherenkov photons is about 370 per centimeter.

A Cherenkov imaging detector such as SK can determine the energy of a particle by the number of detected Cherenkov photons, the location of interaction using the information of timing when the PMTs detect the Cherenkov photons, and the direction of the particle using the Cherenkov ring image.
3.2 General detector description

SK is a water-Cherenkov detector located underground in the Kamioka mine in Gifu Prefecture, Japan. The average rock overburden above SK is 2800 m.w.e. A schematic view of the detector is given in Fig.3.8.

A cylindrical stainless steel tank forms the SK detector. The tank, whose total height is 41 m and diameter is 39 m, holds 50,000 m$^3$ of ultra-pure water. SK is split into two optically separate detectors: inner and outer. The separation between the inner and outer detectors is provided by a stainless steel structural grid, Tyvek, and polyethylene black sheets. Although the inner and outer detectors are optically separated, water can flow between the two detectors.

3.2.1 The inner detector (ID)

The inner detector is a cylinder of 36.2 m in height and 33.8 m in diameter. It encloses 32,481 m$^3$ of water, which is viewed by 11,146 inward-facing 50 cm diameter photomultiplier tubes (PMTs), which will be explained in Sec.3.3. All of the PMTs are mounted on stainless steel frames. A stainless steel frame to support twelve of the inner detector PMTs and two of the outer detector PMTs constitutes one super-module. Fig.3.10 is a schematic view of a super-module. The photocathode coverage by these PMTs is about 40% . The remaining area is covered by black polyethylene sheets to suppress the reflection of Cherenkov light.
Figure 3.9: Local coordinate system of SK.

Figure 3.10: A super-module supports twelve ID PMTs and two OD PMTs.
Figure 3.11: Schematic view of a 50 cm photomultiplier tube (HAMAMATSU R3600-05) and the leakage of light from or to the outer detector.

3.2.2 The outer detector (OD)

The outer detector surrounds the inner detector and provides an almost uniformly thick (1.95 ~ 2.2 m) 4π active veto. The purpose of the outer detector is to actively veto cosmic ray muons, and to decrease the detection of γ-rays from the surrounding rock.

The outer detector is viewed by 1885 outward-pointing 20 cm diameter PMTs (HAMAMATSU R1408) with 60 cm square wavelength shifter plates [34]. All other surfaces in the outer detector are covered with a reflective material, Tyvek, to enhance light collection in the OD.

3.3 Photomultiplier tube (PMT)

We use 50 cm diameter PMTs (HAMAMATSU R3600-05) in the ID. A schematic view of a PMT is given in Fig.3.11. It was originally developed by the HAMAMATSU Photonics Company and Kamiokande collaborators and improved for use in SK [35]. The improved points are the following:

- The 1 photo-electron(p.e.) peak has become clearer by optimizing the structure of the dynode as shown in Fig.3.12.
• A larger area for the dynode has reduced the influence of the residual geomagnetic field. Hence, the timing resolution (i.e. the transit time spread) at the 1 p.e. level has become better (5 nsec → 3 nsec).

The glass body of the PMT is made of Pyrex glass of 5 mm thickness which is transparent to light down to 330 nm in wavelength. The photocathode area is coated with a Bialkali material (Sb-K-Cs). The quantum efficiency as a function of light wavelength is shown in Fig.3.13. It is 22% at 390 nm.

The dynode has a Venetian blind structure with 11 stages, which provides a large photosensitive area. The dark noise rate caused by thermal electrons from the photocathode is shown in Fig.3.14. The average dark rate is stable around 3.4 kHz after 2 months from the start of SK. The value of supplied voltage which gives a gain of $10^7$ is 2000 V. The average transit time is about 100 nsec for multi p.e. light. For single p.e. light, the variation of the transit time of PMTs is about 10 nsec in a supplied voltage range of 1500 to 2000 V.

The large size of the PMTs makes them susceptible to the geomagnetic field. To obtain a uniform response from the PMTs, the geomagnetic field (450 mG in natural conditions) must be reduced to less than 100 mG [35]. Compensation coils are used to accomplish this in SK and the residual magnetic field is kept at less than 100 mG in every position of the detector.

Some of the PMTs are dead by short circuit or some other unknown reasons. The time variation in the number of dead PMTs is shown in Fig.3.15. The fraction of dead PMTs is about 1.4% at present.

### 3.4 The water purification system

Impurities in the SK water can cause strong attenuation and scattering of Cherenkov light. Moreover, if they are radioactive, like $^{222}$Rn, they could become a possible source of background to solar neutrino events. The decay chain of $^{222}$Rn is shown in Fig.3.17. None of the electrons or $\gamma$-rays from the decay chain have energy above the threshold for this analysis (5.0 MeV). However, because of finite energy resolution of SK, such low energy electrons are sometimes observed as electrons with energy above 5.0 MeV. So the purity of the water is essential for solar neutrino observation in SK.

The source of water in SK is an underground aquifer in the Kamioka mine. The water purification system makes highly purified water from the mine water. Fig.3.16 is a schematic of the water purification system. The water passes through the following components:

- **1 $\mu$m normal filter**:
  Removes relatively large particles. Some of Rn also rejected with dust.

- **Heat exchanger**:
Figure 3.12: The Single-p.e. distribution from 50 cm PMTs. There is a clear peak around 400 ADC counts. A spike near the lowest ADC channel is caused by photo-electrons that miss the first dynode.

Figure 3.13: The quantum efficiency of the PMTs and the relative Cherenkov spectrum as a function of light wavelength.
Figure 3.14: The time variation of dark rate from the beginning of SK. The dashed line shows the elapsed day when SK started the solar neutrino analysis.

Figure 3.15: The time variation of the number of dead PMTs from the beginning of SK. The dashed line shows the elapsed day when SK started the solar neutrino analysis.
Figure 3.16: Schematic view of the water purification system.

Figure 3.17: The decay chain of $^{222}\text{Rn}$. 
<table>
<thead>
<tr>
<th>impurities</th>
<th>reduction efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{222}\text{Rn}$</td>
<td>$\sim 99.9%$</td>
</tr>
<tr>
<td>dust ($&gt;0.1\mu\text{m}$)</td>
<td>$\sim 99.7%$</td>
</tr>
<tr>
<td>bacteria</td>
<td>$\sim 100%$</td>
</tr>
<tr>
<td>ion</td>
<td>$\sim 99%$</td>
</tr>
</tbody>
</table>

Table 3.1: The impurity reduction efficiency of the water purification system

Water pumps increase the water temperature. The heat exchanger decreases the temperature to about 12 °C to inhibit bacterial growth.

- **Ion exchanger**:  
  Removes metal ion (Fe$^{2+}$, Ni$^{2+}$, Co$^{2+}$) impurities in the water. It can also remove $^{218}\text{Po}$ which is a daughter nuclei in the decay of $^{222}\text{Rn}$ and easily ionizes.

- **UV sterilizer**:  
  Reduces bacteria.

- **Rn-less-air dissolving system**:  
  Dissolves Rn-less-air in water to improve the Rn removal efficiency of the vacuum de-gasifier.

- **Reverse osmosis filters**:  
  Reverse osmosis by a high performance membrane which removes even organisms on the order of 100 molecular weight.

- **Vacuum de-gasifier**:  
  Removes gases (Rn, oxygen, etc.) dissolved in water; about 96% of the dissolved radon gas is removed at this stage. The output water is put back into the main stream.

- **Cartridge polisher**:  
  High performance ion exchangers.

- **Ultra filter**:  
  Removes sub-$\mu$m contamination.

Usually, the water purification system supplies the purified water from the bottom of the tank and removes it from the top of the tank. The flow rate is 30 ~ 70 ton/hour. $^{222}\text{Rn}$ concentration in the SK tank water is less than 5 mBq/m$^3$. A summary of the reduction efficiency is shown in Table 3.1 [36].
3.5 The air purification system

SK is situated in a mine, and the $^{222}$Rn concentration in the mine air is $50 \sim 2000$ mBq/m$^3$. Therefore the detector is literally surrounded by sources of Rn gas. The rock dome above the water tank and the hallway are covered with a Mineguard polyurethane material, which prevents Rn gas emanation from rocks. Double doors at all entrances also limit the amount of mine air that enters the detector area. Furthermore, purified fresh air is pumped directly into the experiment area from outside of the mine with a flow rate of 7 m$^3$/min. Fig.3.18 is a schematic view of the air purification system. The components of the air purification system are as follows:

- **Compressor**:
  Takes in air from outside the mine and pressurizes it to $7 \sim 8.5$ atm.

- **0.3,0.1,0.01 μm air filter**:
  Removes dusts in air.

- **Air drier**:
  Absorbs moisture, since the radon reduction efficiency of the carbon column depends on the humidity of air.

- **Carbon column**:
  CO absorbs Rn.

- **Active charcoal**:
  Active charcoal cooled to $-41^\circ C$ traps the Rn.

The $^{222}$Rn reduction efficiency is $\sim 99.98\%$ [36].
3.6 The data acquisition system

3.6.1 The inner detector

Fig. 3.19 gives a schematic view of the ID data acquisition system. Each PMT is connected to a front-end electronics module called the Analog Timing Module (ATM) [37], which digitizes the charge and timing information of PMTs. Each ATM accepts 12 input channels and 20 ATMs are usually included in each TKO (Tristan-Kek-Online) box [38]. A total of 934 ATMs are distributed in 48 TKO boxes. They are housed in 4 electronics hut on top of the detector. A TKO Super Control Header (SCH) module is also put into each TKO box. The SCH is responsible for sending ATM data to the Super Memory Partners (SMPs) and the GONG (Go NoGO) module. The SMPs are in a VME (Versa Module Europe) rack and store data temporary. The GONG module distributes incoming global trigger signals and event identification numbers to each ATM.

The PMT signal fed to the current splitter is divided into four separate signals. One is sent to a discriminator, which has a threshold level for each channel set to $-1$ mV, which corresponds to 1/4 p.e. When the PMT signal exceeds the threshold level, a HITSUM signal with 200 nsec width and about 11 mV/channel height is asserted on the ATM front panel. All HITSUM signals are summed up in each electronics hut with a summing module and sent to the trigger electronics in the central hut to generate a global trigger for SK. Fig. 3.20 shows a schematic view of the trigger generation.

At the same time, one of the parallel signals, which is sent to the QAC (Charge to Analog Converter) and the TAC (Timing to Analog Converter), starts to integrate a constant current. If the global trigger is issued within about 1.3 $\mu$s, the information in the QAC/TAC are digitized and stored in internal memory. Since the integration of the TAC is started by the PMT signal in each channel, the amount of charge integrated by the TAC is a measure of the timing of the signal. There are two TACs and QACs, which are called A and B, for each channel, so that two successive events, like a muon and a following decay electron, can be acquired.

The ADC/TDC data contains 12 bits. The ATM has $\sim 450$ pC dynamic range with a resolution of 0.2 pC and $\sim 1300$ nsec dynamic range with a resolution of 0.4 nsec. To obtain the timing and charge information with high accuracy, conversion tables are used to convert ADC and TDC counts to pC and nsec, respectively. The inaccuracies in the conversion using the tables are negligible for the solar neutrino analysis.

The room temperature dependence of the ADC and the TDC is less than 3 counts/°C (0.6 pC/°C) and 2 counts/°C (0.8 nsec/°C), respectively. In order to correct for the remaining temperature dependence, reference signals with a few constant timing and charge are regularly taken every 30 minutes. The measured values are used to convert ADC and TDC counts in each of these period. The temperature is around 23 °C and it is kept within $\pm 0.5$ °C of this temperature in the electronics huts. Possible variations coming from
Figure 3.19: Schematic view of the ID data acquisition system.

Figure 3.20: Schematic view of trigger generation.
the temperature dependence is less than 0.3 pC and 0.4 nsec for the charge and timing information.

3.6.2 The outer detector

The outer detector data are acquired by a different electronics system. The PMT cable is connected to a Charge-to-Time (QTC) module, which discriminates the signal from the PMT. If the pulse height exceeds 25 mV (\( \sim 0.5 \) p.e.), the QTC integrates the charge of the PMT signal in a 200 nsec window and makes an Emitter-Coupled-Logic (ECL) pulse. This pulse is sent to a LeCroy1877 Time-to-Digital converter (TDC). The TDC module records the timing of the leading and trailing edge from which we know the timing and charge information of the PMT signal. The width of the time window for the OD data is 16 \( \mu \)sec (10 \( \mu \)sec before and 6 \( \mu \)sec after the global trigger), and the resolution is \( \sim 0.5 \) nsec.

3.6.3 Trigger

The rectangular HITSUM signals from individual ATMs are summed up to generate the total HITSUM signal. When the HITSUM signal exceeds a discriminator threshold, a global trigger is issued.

There are three triggers for the ID: the High Energy trigger (HE trigger), the Low Energy trigger (LE trigger), and the Super Low Energy trigger (SLE trigger). For the solar neutrino data analysis, LE and SLE triggered data are used. (This data set contains HE triggered events.) The threshold of the LE trigger is \(-320\) mV, which corresponds to \( \sim 29 \) hits from the ID PMTs. The trigger efficiency of the LE trigger is measured by DT calibration to be 100% at 6.5 MeV. This is described in Sec.5.5. The LE trigger rate is usually stable at about 11Hz.

The threshold of the SLE trigger was changed 3 times so far in order to take data with lower energy.

<table>
<thead>
<tr>
<th>Start date</th>
<th>End date</th>
<th>Threshold</th>
<th>Trigger rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 May 1996</td>
<td>31 May 1997</td>
<td>(-320) mV(LE)</td>
<td>11Hz</td>
</tr>
<tr>
<td>31 May 1997</td>
<td>14 May 1999</td>
<td>(-260) mV(SLE)</td>
<td>10Hz</td>
</tr>
<tr>
<td>14 May 1999</td>
<td>17 Sep 1999</td>
<td>(-250) mV(SLE)</td>
<td>166Hz</td>
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<tr>
<td>17 Sep 1999</td>
<td>22 Dec 1999</td>
<td>(-222) mV(SLE)</td>
<td>526Hz</td>
</tr>
<tr>
<td>22 Dec 1999</td>
<td></td>
<td>(-212) mV(SLE)</td>
<td>535Hz</td>
</tr>
</tbody>
</table>

The energy threshold at 5.5 MeV is 100% efficiency for all SLE thresholds. 100% efficiency at 5.0 MeV is achieved only by the SLE threshold of \(-212\) mV. The SLE trigger efficiency is measured by DT calibration, which is explained in Sec.5.5.

For reference, the HE trigger threshold is \(-340\) mV, corresponding to about 31 hits.
Figure 3.21: Data flow of the online data taking and offline data processing.

The HITSUM signals from the QTCs are also summed separately to make a total HITSUM for the outer detector. The OD trigger occurs when there is a coincidence of 19 OD PMTs above the QTC threshold in a 200 nsec window.

All triggers are fed into the TRG module as shown in Fig.3.19. When at least one of the trigger signals is asserted, the TRG module records the trigger types, the trigger timing with 20 nsec accuracy, and the event number. The TRG also generates the global trigger signal and a 16 bits event identification number on its front panel. These are distributed to all of the electronics to trigger the current event. The trigger data stored in the TRG module are read by a separate on-line computer and sent to the on-line host computer to be merged with the PMT data.

### 3.7 Real time data processing

As shown in Fig.3.19, the digitized data is sent from the SMP to the online computers via FDDI. There are two work stations in each electronics hut. Each workstation reads out and collects the data from 6 SMP modules. This data are sent via FDDI to an online host computer, which concatenates all information from the ID and OD from one global trigger. Then, the data are sent to a workstation to reformat into the ZEBRA format.

Fig.3.21 shows the flow of data after reformatting. The reformatted data are sent to the offline host computer and stored in the magnetic tape library (MTL). Then the data are converted from ADC/TDC counts to p.e./nsec. After this conversion, all events are reconstructed and examined by various reduction programs. Data reduction is explained
in Chapter 7.

For events issuing only the SLE trigger (typically below 6.5 MeV), a software trigger which is called Intelligent Trigger (IT) is applied. This is a real-time fiducial volume cut procedure. Most very low energy events are due to $\gamma$-rays from the ID wall materials and the rock surrounding the tank, so the vertices are distributed near the wall. The IT eliminates those events, keeping only 8% of the incoming events at the SLE threshold = $-212$ mV.

The data stream of the IT diverges from the online host computer. The online host computer sends the data which issues only the SLE trigger to 12 CPUs (IT CPUs), which reformat the data and reconstruct the event vertex.

Fast vertex reconstruction by the IT is required to keep up with the high data rate. For this purpose, a fast vertex reconstruction algorithm (the Hayai fitter) is applied to the data before the usual one, which is described in Chapter 4. First, the fitter makes a list of all hit PMTs that has a neighboring hit PMT in space (distance within 10 m) and time (hit within 33.3 nsec). A timing distribution is made from this set of PMTs, and the number of PMTs in a sliding window of 16.7 nsec width is examined. The set of PMTs that belongs to the window with the largest number of hits is used in the vertex reconstruction. The event is rejected if the number of hit PMTs is less than 5.

The initial vertex for the subsequent grid search is obtained as the centroid of the selected hit PMTs. In order to prevent starting a vertex too close to the wall, the vertex is moved 2 meters away from the ID wall. Then the Hayai fitter searches the vertex which gives the maximum “goodness” which is defined as follows:

$$goodness = f(a) \frac{1}{N_0} \sum_{i=1}^{N_0} \exp \left[ -\frac{(t_i - t_i^{exp} - T_{offset})^2}{2\sigma_i^2} \right].$$

(3.7.1)

where $f(a)$ is a function which reflects the shape of the expected Cherenkov cone from the true vertex. $a$ is the magnitude of the anisotropy defined as follows:

$$a = |\vec{a}| = \frac{\sum_{i=1}^{N_0} q_i |\vec{d}_i|}{\sum_{i=1}^{N_0} q_i}.$$  

(3.7.2)

$N_0$ is the number of selected hit PMTs. $t_i$ is the detection time of $i$-th hit PMT. $T_{offset}$ is the time offset to make the center of $(t_i - t_i^{exp} - T_{offset})$ zero. $\vec{d}_i$ is the distance vector from the trial vertex to the $i$-th hit PMT. The trial vertices are uniformly distributed with step sizes of 1.86 m. $t_i^{exp}$ is the expected detection time of $i$-th PMT, defined as follows:

$$t_i^{exp} = T_0 + \frac{|\vec{d}_i|}{c/n},$$

(3.7.3)

where $n$ is the refractive index of water.
Figure 3.22: The vertex distribution of $^8$B MC before and after the IT cut.

When the vertex which maximizes the goodness is found, then the location of the vertex is examined. If the distance between the reconstructed vertex and the ID wall is less than 2 m, the event is rejected.

After this fast reduction, the normal vertex reconstruction is also applied. The vertex reconstruction method is explained in Sec.4.1. Events whose reconstructed vertex is located within 2 m of the ID wall is rejected. The reduction efficiency of the IT depends on the SLE trigger rate. For example, when the SLE trigger threshold $= -212$ mV, 92% of events are rejected. The event identification numbers of remaining events are sent to the reformat machine. Only the data for these events are reformatted and sent to the offline host.

Fig.3.22 shows the vertex distribution of $^8$B MC events which pass only the SLE trigger. The dashed histogram shows the distribution before IT cut, and the solid histogram shows that after the IT cut. 10% of events generated in the fiducial volume (described in Chapter 6) are rejected by the IT cut.

3.8 Detector simulation

In order to study the detector response for solar neutrino events, a Monte Carlo simulation (MC) of the detector is carried out. The SK detector simulation consists of 2 parts. The first stage is particle transport. To simulate interactions of photons and electrons with the detector materials, simulation package, GEANT [39] is used. GEANT is a system of detector description and simulation tools that is designed for high energy physics exper-
<table>
<thead>
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<th>note</th>
</tr>
</thead>
<tbody>
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</tr>
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<tr>
<td></td>
<td>Cherenkov radiation</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: The interaction of photons and electrons considered in GEANT

iments. It can simulate electromagnetic interaction with energy from 10 keV to 10 TeV. For SK detector simulation, electrons and $\gamma$-rays with energy down to 100 keV are considered. Tab.3.2 summarizes the interactions which are considered in GEANT. As shown in Tab.3.2, Cherenkov photon radiation is considered in GEANT, but light attenuation in water is simulated by programs developed for SK. Light attenuation is due to absorption and scattering. From the theory of optics, the attenuation factor of Rayleigh scattering is proportional to $\lambda^4$. So it is the dominant effect for photons of short wavelength ($\sim$ 400 nm). For photons of longer wavelength, absorption is the dominant effect. The magnitudes of these effects are tuned so that the MC reproduces the LINAC calibration results well. The light attenuation length in water is measured using laser, as described in Sec.5.3.1. Fig.5.9 shows the results of the measurement together with the attenuation length used in the SK detector simulation as a function of photon wavelength. The magnitude of light absorption in water depends on the purity of water, so it varies with water transparency. This relation is also considered in the simulation. The reflectivity of the PMT and the black sheet are measured and the results are used in the simulation. Fig.3.23 shows the reflectivity of the PMT and the black sheet as a function of incidence angle. The left figure shows the reflectivity of the PMT surface and the right one shows the reflectivity of the black sheet. Photon polarization is taken into account. The dotted lines in both figures are for p-wave photons. The curves for s-wave photons are presented by dashed lines. The solid lines show the average reflectivity of p-wave and s-wave.

The second stage of simulation is for the detector response. It consists of the following items:

- Quantum efficiency of the PMT:
Figure 3.23: The left figure shows the reflectivity of the surface of the 50 cm PMT. The three curves show the p-wave, the s-wave, and the average reflectivity. The right figure shows the reflectivity of the black sheet.

Figure 3.24: The left figure shows the TDC timing resolution used in the detector simulation. The resolution at 1 p.e. is 2.9 nsec. The right figure shows the timing shift by slewing used in the detector simulation. The timing shift at 1 p.e. is 0.7 nsec.
The photon wavelength dependence of the PMT quantum efficiency is considered. It is shown in Fig.3.13.

- **Dark current of the PMT:**
The dark noise rate is set as the observed average dark rate of the ID PMTs in the detector simulation. The charge of dark hit PMTs is treated as a single p.e. hit.

- **Resolution of the ADC and the TDC:**
  For solar neutrino events, most PMTs detect single p.e. The output charge of the PMTs for single p.e. is measured by the Ni-Cf calibration as described in Sec.5.1.2. The PMT output charge in the detector simulation is distributed according to the histogram in Fig.5.4. The spread of TDC counts is also taken into account. The dominant origin of TDC count resolution is the PMT transit time. The TDC count resolution used in the detector simulation is shown in the left figure of Fig.3.24. At 1 p.e., the resolution is 2.90 nsec. The TDC count shift due to slewing is also considered. The right figure of Fig.3.24 shows the TDC count shift as a function of charge. At 1 p.e., the shift is about 0.7 nsec.

- **After pulse:**
The after-pulse of PMT is considered in MC. It affects the tail of the TDC count distribution. The cause of the after pulse is surmised as follows: An electron which hits the dynode is sometimes scattered elastically back to the photo-cathode against the electric field in the PMT. The electron then turns and goes to the dynode again. This returning electron makes a delayed pulse, which is observed as the after-pulse.

  The time difference of the after-pulses is measured with LINAC calibration data and obtained to be about 36.5 and 107 nsec. The probability of each pulse is also measured and is obtained to be 1.9% and 1.15%, respectively.

After simulation, the timing and charge information of each PMT is stored in the same data format as real data. Then the same data reduction procedure as real data is applied to MC.
Chapter 4

Event Reconstruction

The event reconstruction algorithms used for the solar neutrino analysis is described in this chapter.

First, vertex reconstruction which uses the time information of hit PMTs is described. The second step is the reconstruction of the event direction using the pattern of hit PMTs. After these reconstructions, the effective number of hit PMTs, $N_{\text{eff}}$, is calculated. $N_{\text{eff}}$ is converted into energy by using a conversion function obtained from LINAC calibration, which is described in Sec. 5.4. Additionally, muon track reconstruction is described in the last section.

4.1 Vertex reconstruction

The timing information from hit PMTs is used to reconstruct the vertex position. The path length of an electron recoiling from the neutrino interaction is less than 10 cm. It is negligible in comparison with the vertex resolution of approximately 1 m. Therefore, the vertex is treated as a point in vertex reconstruction program.

Prior to the reconstruction, the PMT timing information is evaluated to remove hit caused by random noise and reflected light. A typical timing distribution of hit PMTs is shown in Fig.4.2. To select the PMTs which are used for vertex reconstruction, the following time window method is used:

1. Sort the hit PMTs in chronological order and locate the 200 nsec time window which contains the maximum number of hit PMTs ($N_{bg}$).

2. Estimate the number of dark noise hits, $N_{bg}$, by the relation:

$$N_{bg} \equiv \frac{(t_3 - t_2)}{(t_2 - t_1) + (t_4 - t_3)} (N_{\text{hit}}(t_1 \sim t_2) + N_{\text{hit}}(t_3 \sim t_4)) \quad (4.1.1)$$

$t_{1\sim4}$ are defined in Fig.4.2.
Figure 4.1: Event display of a typical low energy event.

Figure 4.2: Absolute time distribution of hit PMTs in the event in Fig.4.1.
Figure 4.3: The grid used in the initial stage of the vertex reconstruction. The left figure shows a projection on the XY plane and the right figure shows a projection on the XZ plane. Solid lines and broken lines represent the ID tank and the fiducial volume, respectively.

3. Reduce the width of the window in 19 nsec steps to optimize the significance of the signal. The significance, $S$, is defined as follows:

$$ S \equiv \frac{N_{hit}(t_2 \sim t_3) - N_{bg}}{\sqrt{N_{bg}}} \quad (4.1.2) $$

Only the hit PMTs within the time window with maximum $S$ are used for vertex reconstruction.

The vertex point is found by using a grid search method. The distance between the grid points used in the initial stage of vertex reconstruction is 397.5 cm as shown in Fig. 4.3. The vertex is placed on a grid point, and for the grid point, the “goodness” of vertex reconstruction (defined by Eq. 4.1.3) is calculated. The calculation is repeated for every point on the grid.

$$ goodness(x, y, z) \equiv \frac{1}{N_{hit}} \sum_{i=1}^{N_{hit}} \frac{1}{\sigma^2} \exp \left( -\frac{(t_{res,i}(x, y, z) - t_{center}(x, y, z))^2}{2\sigma^2} \right), \quad (4.1.3) $$

where $(x, y, z)$ is the position of the grid point. $N_{hit}$ is the number of hit PMTs selected by the time window method. $\sigma$ is the PMT time resolution, which is 5 nsec for all PMTs. $t_{res,i}$ is the residual time of i-th hit PMT which is defined as follows:

$$ t_{res,i} \equiv t_i - \frac{n}{c} \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}, \quad (4.1.4) $$
Figure 4.4: Projection of the reconstructed vertex distribution for LINAC calibration events at the position \(x = -388.9\text{cm}, \ y = -70.7\text{cm}, \ z = +27\text{cm}\) with the electron beam energy set to 8.35 MeV. The broken lines show the electron beam injection position.
Figure 4.5: The goodness distribution for LINAC calibration events at the position 
\((x = -388.9 \text{ cm}, y = -70.7 \text{ cm}, z = +27 \text{ cm})\) with the electron beam energy set to 8.35 MeV.

where \(t_i\) is the time of the \(i\)-th hit PMT, \(t_{center}\) is the mean time of the distribution of 
\(t_{res,i}\). \(n\) is the refractive index of water and \(c\) is the speed of light in vacuum. \((x_i, y_i, z_i)\) 
is the position of the \(i\)-th hit PMT. In an ideal case, all of the \(t_{res,i}\) are equal, and the 
goodness will be 1. The distance between the grid points is reduced to 5 cm as the trials converge upon the final vertex. Fig. 4.5 shows a distribution of the goodness using events from LINAC calibration. Fig. 4.4 shows the reconstructed vertex of the same events. The resolution of vertex reconstruction is measured by LINAC calibration system, and found to be \(\sim 75 \text{ cm}\) for electron energy of 10 MeV. A full explanation of the LINAC calibration 
is given in Chapter 5.

4.2 Direction reconstruction

Once the vertex position of an event has been determined, the direction can be recon-
structed by examining the pattern of hit PMTs created by the Cherenkov ring. In an ideal 

\begin{itemize}
  \item The event pattern is expected to be a clear ring with an opening angle of 42° about the 
  direction of the electron's path.
  \item However, photons can be scattered in the water, reflected off the PMTs and the black sheet.
  \item Also, the electron may undergo multiple scattering. All of these effects cause the Cherenkov ring to be smeared.
  \item The hit PMTs which are used to reconstruct the direction are selected using a 50 nsec timing window from the TOF (Time-Of-Flight) subtracted time distribution.
\end{itemize}

A maximum likelihood method is used to
Figure 4.6: The probability density function for the angle of Cherenkov photons relative to the recoil electron momentum.

Figure 4.7: The PMT acceptance function.
Figure 4.8: The direction cosine distribution from LINAC calibration data at the position $(x=-388.9\text{cm}, y=-70.7\text{cm}, z=+27\text{cm})$ with the electron beam energy of 8.35 MeV.

determine the direction. The likelihood function is defined as:

$$L(\vec{d}) = \sum_{i=1}^{N_{hit}} \log(P_i(\cos \theta_{dir})) \times \frac{\cos \theta_i}{a(\theta_i)}, \quad (4.2.1)$$

$$\theta_i \equiv \arccos \left( \frac{\vec{d}_i \cdot \vec{p}_i}{|\vec{d}_i||\vec{p}_i|} \right) \quad (4.2.2)$$

$P_i(\cos \theta_{dir})$ is a probability density function for the angle of Cherenkov photons relative to the recoil electron momentum. Fig.4.6 shows the distribution of $P(\cos \theta_{dir})$. $\theta_i$ is the angle of incidence of the Cherenkov photon on the i-th PMT. $\vec{p}_i$ is the vector from the reconstructed vertex to the position of the i-th hit PMT, $\vec{d}_i$ is the direction vector that the i-th hit PMT is facing. $a(\theta_i)$ is acceptance of the PMT photo cathode as function of $\theta_i$. The distribution of $a(\theta_i)$ is presented in Fig.4.7. A grid search method is used to find which direction has the maximum likelihood value. The step sizes used in this grid search are $20^\circ$, $9^\circ$, $4^\circ$, $1.6^\circ$. The mean value of the multiple scattering angle is about $27^\circ$ for electrons with an energy of 10 MeV. So the step sizes are small enough to reconstruct the direction of low energy electrons. Fig.4.8 shows the reconstructed direction of the same events as in Fig.4.5. The resolution of the direction reconstruction is measured with LINAC calibration data, and found to be $\sim 27^\circ$ for electron energy of 10 MeV. A full explanation of the LINAC calibration system is given in Chapter 5.

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4.3 Calculation of the number of the hit PMTs

The energy of a charged particle that is above the Cherenkov threshold is approximately proportional to the number of Cherenkov photons. In SK, the number of Cherenkov photons emitted by a particle is proportional to the total number of photo-electrons in the hit PMTs. However, the number of hit PMTs is used for the energy calculation instead of the total number of photo-electrons. The reasons for this are as follows:

- In low energy events, the number of emitted Cherenkov photons is small (about 2600 photons at 16 MeV). Thus, the mean number of Cherenkov photons in each hit PMT is about one.

- The charge resolution of the PMTs is not good at the single p.e. level.

- The number of hit PMTs does not strongly depend on the gain of PMTs, which is not the case with total p.e.

Only the hit PMTs with residual time within a 50 nsec window ($N_{50}$) are used for the energy calculation. This selection criteria removes accidental hits from dark noise in the PMTs. However, $N_{50}$ varies with several factors, including water transparency, acceptance of each hit PMT, etc. Therefore, some corrections must be applied to calculate the effective number of hits ($N_{eff}$).

$$
N_{eff} = \sum_{i=1}^{N_{50}} \left[ (X_i + \epsilon_{tail} - \epsilon_{dark}) \times \frac{N_{all}}{N_{alive}} \times \frac{R_{cover}}{S(\theta_i, \phi_i)} \times \exp\left(\frac{r_i}{\lambda}\right) \times G(i) \right] \tag{4.3.1}
$$

The corrections to calculate $N_{eff}$ are given as follows:

$X_i$: Multi-photo-electron hit correction

If an event occurs near the edge of the fiducial volume and is directed towards the nearest wall, then the observed number of hit PMTs will be small, since the Cherenkov cone will not have much distance to expand. When this happens, some PMTs may be hit with multiple photons. The expected number of photo-electrons hitting each PMT is estimated from the occupancy of the eight surrounding PMTs. When the number of hit PMTs and the number of functional PMTs surrounding the i-th PMT are $n_i$ and $N_i$ respectively, the correction factor is given by:

$$
X_i = \frac{\log \left( \frac{1}{1-\alpha_i} \right)}{\alpha_i}, \tag{4.3.2}
$$

where

$$
\alpha_i = \frac{n_i}{N_i}. \tag{4.3.3}
$$

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Figure 4.9: The effective photo cathode area \( R_{\text{cover}} / S(\theta_i, \phi_i) \)

\[ \epsilon_{\text{tail}} : \text{Correction for reflected Cherenkov photons} \]
This factor corrects for hits which occur later than the 50 nsec time window. When the number of hit PMTs in the 50 nsec window and the number in the 100 nsec window are \( N_{50} \) and \( N_{100} \) respectively, the correction factor is given by:

\[ \epsilon_{\text{tail}} = \frac{N_{100} - N_{50}}{N_{50}}. \]  

(4.3.4)

\[ \epsilon_{\text{dark}} : \text{Correction for dark noise hits} \]
This factor accounts for hits caused by dark noise in the PMTs. It is defined as follows:

\[ \epsilon_{\text{dark}} = \frac{N_{\text{alive}} \times R_{\text{dark}}}{N_{50}}. \]  

(4.3.5)

where \( N_{\text{alive}} \) is the number of all live PMTs in the ID and \( R_{\text{dark}} \) is the mean dark rate for a given run.

\[ \frac{N_{\text{tail}}}{N_{\text{alive}}} : \text{Dead PMT correction} \]
This factor accounts for the time variation in the number of dead PMTs.

\[ \frac{R_{\text{cover}}}{S(\theta_i, \phi_i)} : \text{Correction for effective photo cathode coverage} \]
\( R_{\text{cover}} \) is the photo coverage ratio of the ID, and it is equal to 0.4041. \( S(\theta_i, \phi_i) \) is the effective photo cathode area of the \( i \)-th hit PMT as viewed from the angles \( (\theta_i, \phi_i) \). Fig. 4.9 shows the distribution of \( R_{\text{cover}} / S(\theta_i, \phi_i) \) and the definition of \( (\theta_i, \phi_i) \).
Figure 4.10: The definition of parameters used in the goodness function for muon reconstruction.

\[ \exp\left(\frac{r_i}{\lambda(r_{\text{hit}})}\right) : \text{Water transparency correction} \]

This factor compensates for the change in the attenuation of Cherenkov light in water. \( r_i \) is the distance from the i-th hit PMT to the reconstructed vertex. \( \lambda \) is the water transparency measured using Cherenkov light from decay electrons produced when muons stop in the fiducial volume.

\[ G(i) : \text{Correction for the quantum efficiency of Pre-series PMT} \]

375 PMTs used in the ID were produced as a pre-series to the main PMTs. These PMTs have larger quantum efficiency than the other PMTs. The correction factor for these PMTs is given as follows:

\[
G(i) = \begin{cases} 
0.833, & \text{for 375 PMTs} \\
1.000, & \text{for other PMTs}
\end{cases}
\]

(4.3.6)

4.4 Muon track reconstruction

Cosmic ray muons enter the detector at a rate of about 3 Hz. Some of these muons break the nucleus of \(^{16}\text{O}\) in water and produce radioactive elements. When these decay, they emit electrons, positrons, or \(\gamma\)-rays with energy as high as 20 MeV. Event caused by such emissions are called "spallation events" and they are one of the major sources of background for the solar neutrino analysis. To reject spallation events, correlations in distance and time are made between the low energy event and the preceding muon events. The charge information of the muon is also used. To obtain these quantities, muon track reconstruction is performed.
Muon track reconstruction is performed in two steps. The first step makes use of the charge distribution of hit PMTs. The second step uses the timing information in each hit PMT.

(1) Fast reconstruction
In the first step of reconstruction, the entrance and exit positions of the muon are searched roughly. The entering position is determined by the earliest hit PMT in the ID, that has more than 2 neighboring hit PMTs within 5 nsec. The exit point is defined as the center of gravity of PMTs which detect more than 231 p.e. (the charge-saturated PMTs). To check the quality of fast reconstruction, two parameters are used: the distance from the entrance position to each saturated PMT ($L_{ent}$) and the distance from the exit point to each saturated PMT ($L_{exit}$). If $L_{ent} > 300$ cm and $L_{exit} < 300$ cm, then the fast reconstruction is considered to be successful. For muons which stop in the ID and multiple muon events, these conditions are not satisfied.

(2) TDC fit
If the first reconstruction is unsuccessful, reconstruction using the TDC information is applied. In order to evaluate the validity of the fitting, a goodness function is defined. It is similar to that for vertex reconstruction:

$$
goodness = \frac{1}{\sum_{i} \sigma_{i}^{2}} \times \sum_{i} \frac{1}{\sigma_{i}^{2}} \exp\left\{ -\frac{1}{2} \left( \frac{t_{i} - T}{1.5\sigma_{i}} \right)^{2} \right\}, \tag{4.4.1}$$

where

$$t_{i} = T_{i} - \frac{d_{\mu}(x_{exit})}{c} - \frac{d_{ph}(x_{exit})}{c/n} \tag{4.4.2}$$

$\sigma_{i}$ = Time resolution of the i-th hit PMT
$T$ = Time when the cosmic muon enters the ID
$T_{i}$ = Time when the i-th hit PMT detects Cherenkov photon,
$d_{\mu}(x_{exit})$ = Flight distance of the muon as a function of $x_{exit}$, which is defined in Fig.4.10.
$d_{ph}(x_{exit})$ = Flight distance of the Cherenkov photon as a function of $x_{exit}$
c = Light velocity in vacuum
$n$ = Refractive index of water.

Successful muon reconstruction is defined as that with $goodness > 0.8$.

Efficiency and resolution of reconstruction
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</tr>
<tr>
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<td>10</td>
<td>8</td>
</tr>
<tr>
<td>hard</td>
<td>41</td>
<td>2</td>
</tr>
<tr>
<td>edge clipper</td>
<td>58</td>
<td>19</td>
</tr>
<tr>
<td>multi</td>
<td>56</td>
<td>28</td>
</tr>
<tr>
<td>total</td>
<td>1000</td>
<td>62</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of muon fitter efficiency for various types of muons.

The efficiency of the fitter is estimated using 1000 events from data. The result is summarized in Tab.4.1. The definition of muon categories is given as follows:

- clear single = A muon with a clear Cherenkov ring
- stopping = A muon stopping in the ID
- hard = A hard interaction muon with ResQ (see Eq. (6.3.2))
  > \(2.5 \times 10^8\) p.e.
- edge clipper = A muon whose track length is less than 5 m
- multi = More than one muon in an event.

The fitter efficiency is found to be 94%. The resolution of the reconstruction is estimated by using Monte Carlo muon events. The reconstructed entrance and exit positions are compared with generated ones. A distribution of the difference between them is shown in Fig.4.11. From this figure, the 1σ resolution is measured to be 67 cm for a single clear muon.
Figure 4.11: The track resolution of the muon fitter for a single penetrating muon Monte Carlo. The resolution is estimated to be 67 cm.
Chapter 5

Calibration

In this chapter, calibration systems are described in detail. In order to check the gain of PMTs, the Xe calibration system and the Ni-Cf calibration system are used. They are described in Sec.5.1. PMT timing calibration is carried out using a laser system as described in Sec.5.2. Water transparency is important for a water Cherenkov detector such as SK. The wavelength dependence of the water transparency is measured using a laser, and long-term variations are measured using the energy spectrum of electrons from the decay of cosmic ray muons. They are described in Sec.5.3. In Sec.5.4, energy calibrations in SK are explained. The LINAC calibration system, the DT calibration system, and the results of these calibrations are described in this section. Trigger efficiency measurement is described in Sec.5.5.

5.1 PMT gain

5.1.1 Relative gain

In order to check the relative gain and the stability of uniform gain of PMTs, the Xe calibration system is used [40]. A schematic view of this system is shown in Fig.5.1. This system consists of the following components.

1. **Light source**
   A Xe lamp produces an intense flash of light. Pulse-by-pulse variation of intensity is less than 5%. It is better than other types of flash lamps like the Ar lamp. After operation of a few hours, the change in light intensity is less than 1%.

2. **Filters to control light intensity**
   Light from the lamp is filtered by a ultraviolet (UV) filter. This is because the scintillator ball absorbs the UV light. Neutral Density (ND) filters are employed to
Figure 5.1: The Xe calibration system.

Figure 5.2: Typical result from Xe calibration. The relative gain spread is obtained to be 7%.
change the light intensity independent of wavelength, so that the intensity of light from the scintillator ball can be controlled.

3. **Light transportation**
   After these filters, the light is split in two paths. One of them is fed to a photo-diode and a PMT to monitor the light intensity and trigger. The other is sent to the inner detector via an optical fiber.

4. **Scintillator ball in the inner detector**
   A scintillator ball is at the end of the optical fiber. This scintillator ball consists of acrylic acid resin, BBOT (50 ppm) and MgO (500 ppm). BBOT is a wavelength shifter which absorbs UV light and emits light near the wavelength of Cherenkov light in water. MgO is a powder that diffuses light.

In checking the relative gain of PMTs, the charge of each PMT should be corrected with the acceptance of each PMT, light attenuation in water and the non-uniformity of the scintillator ball. The corrected charge is defined by Eq. (5.1.1).

\[
Q_{\text{cor}} = \frac{Q \times r}{\exp\left(-\frac{r}{x}\right) f_{\text{acc}}(\theta_{\text{PMT}}) f_{\text{ball},\theta}(\theta) f_{\text{ball},\phi}(\theta, \phi)}.
\]

where

- \(Q\) Charge of a PMT
- \(r\) Distance from scintillator ball to a PMT
- \(f_{\text{acc}}\) Function for acceptance correction
- \(f_{\text{ball},\theta}\) Function to correct the zenith angle dependent non-uniformity of the scintillator ball
- \(f_{\text{ball},\phi}\) Function to correct the azimuth angle dependent non-uniformity of the scintillator ball

\(Q_{\text{cor}}\) of all PMTs is shown in Fig.5.2. The spread in relative gain is 7%. The stability of the relative gain is also studied by performing 5 measurements in a year. The time variation in the gain spread is found to be less than 2%.

5.1.2 **Absolute gain**

Light intensity of the Xe calibration system is much larger (100 ~ 200 p.e. per PMT) than that from a typical solar neutrino event (single p.e. level). In order to check the
Figure 5.3: A schematic view of the Ni-Cf calibration system. Cf is situated at the center of the container.

Figure 5.4: The charge distribution of a PMT after a single p.e. hit.
uniformity for single p.e. PMT response, the Ni-Cf calibration system is used [42]. A schematic view of the Ni-Cf calibration system is shown in Fig.5.3. This system uses $\gamma$-rays from the Ni($n,\gamma$)Ni reaction with energy up to 9 MeV. Compton electrons produced by the $\gamma$-rays emit Cherenkov light that results in at most 1 p.e. per PMT. The output charge distribution from Ni-Cf calibration is shown in the Fig.5.4. From this figure, the relation between the number of photo-electrons and the output charge is obtained as 2.055 pC/1 p.e.

5.2 PMT timing

It is essential to obtain accurate relative timing and timing resolution of PMTs for precise reconstruction of the vertex and direction. They could vary due to differences in transit time between PMTs, the difference of cable length between PMTs, and the different slewing behavior in each PMT. In order to study the charge dependence of the relative timing of PMTs, a laser system shown in Fig.5.5 is used [40]. The laser is a N$_2$ laser with light wavelength of 337 nm and pulse width of $\sim$ 3 nsec. Light from the laser is fed to a DYE laser to produce 384 nm light. After conversion of wavelength, light is filtered to control the intensity. Then light is split into 2 optical fibers. One is sent to a diffuser ball and the other is sent to the triggering system. The diffuser ball consists of two diffuser elements, TiO$_2$ and LUDOX. TiO$_2$ is a small tip ($\sim$ 3 mm) at the end of the optical fiber. LUDOX is a silica gel with 20 nm glass fragments. The TiO$_2$ tip is placed at the center of LUDOX as shown in Fig.5.5. The combination of the small TiO$_2$ tip and LUDOX provides diffused light without introducing additional timing spread. Fig.5.6 is a plot of the timing response of a PMT as a function of charge (“TQ-map”). The small points correspond to each measured value and each open circle shows the mean value with 1 $\sigma$ error bar. The timing resolution also depends on the charge. A typical value for single p.e. is 3 nsec. The TQ-maps are made for each PMT and used in event reconstruction.

5.3 Water transparency

Light attenuation and scattering directly affects the number of photons detected by PMTs. Water transparency in SK is measured using two methods. One is a direct measurement of water transparency. This measurement is also used to study the wavelength dependence of water transparency, which is important in simulating the propagation of Cherenkov photon in water. The other method measures long-term variation in the water transparency using electrons from the decay of the cosmic ray muons.
Figure 5.5: A schematic view of PMT timing calibration system.

Figure 5.6: The timing response of a PMT as a function of charge ("TQ-map").
5.3.1 Direct measurement

Fig. 5.7 is a schematic view of the system used in the direct water transparency measurement [41]. A combination of a N₂ laser and DYE produces monochromatic light with wavelength between 337 and 600 nm. Light is split in two. One is used to monitor the intensity and the other is sent to a diffuser ball via optical fiber. The diffuser ball is a spherical acrylic ball with MgO. The light radiated from the diffuser ball is detected by a CCD camera sitting at the water surface. By changing the distance from the diffuser ball to the CCD camera, light intensities from different distances can be measured. Light intensity is normalized by the pulse height from the monitor PMT. The effect of scattered light is removed by only using CCD pixels near the diffuser ball. The water transparency $\lambda$ is given by:

$$\lambda = \frac{L}{\ln \frac{I_{PMT}}{I_{CCD}(L)}}$$  \hspace{1cm} (5.3.1)

where $I_{PMT}$ is the intensity of the light source and $I_{CCD}(L)$ is the measured light intensity at a wavelength of L. Fig. 5.8 shows the measured light intensity as a function of distance at a wavelength of 420 nm. From this figure, the water transparency during this calibration is found to be 96.7 m at 420 nm. This measurement is also carried out at several other wavelengths (337, 400, 500 and 580 nm). The wavelength dependence of water transparency obtained from this calibration is given in Fig. 5.9 and it is used to tune the MC. The solid line in the figure shows the wavelength dependence of water transparency used in the MC.
Figure 5.8: The direct measurement of water transparency. Water transparency is obtained from the inverse of the slope of the fit line. In this figure, water transparency is obtained to be 97.7 m at a wavelength of 420 nm.

Figure 5.9: Wavelength dependence of water transparency. Line shows the wavelength dependence of water transparency used in the MC.
5.3.2 Time variation

Electrons from the decay of the cosmic ray muons are used to measure the relative change of water transparency [43],

\[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu, \]

or

\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu. \]

Even though SK is located deep underground, cosmic ray muons reach SK with a rate of about 3 Hz. About 6000 per day stop in the inner detector.

The criteria to select a pure sample of \( \mu \)-e decay electron events are as follows.

- Time difference between a stopping \( \mu \) event and a low energy event is 2.0~8 \( \mu \)sec.

- The distance from the reconstructed vertex of the low energy event to the nearest wall is larger than 200 m. This is same as the fiducial volume cut applied to the solar neutrino data sample.

- The number of effective PMT hits is larger than 50.

The number of the selected decay electron events is about 1500 events/day. In this analysis, to get rid of the scattered and reflected light effects, the relevant PMTs are selected by the following criteria: (1) PMTs with times that are within 50 nsec window after TOF subtraction, (2) PMTs are within a cone of opening angle 32° \( \sim \) 52° with respect to the electron direction. Water transparency is calculated from the relation between the charge of given hit PMT and the distance from that PMT to the reconstructed vertex.

\[ Q = \exp \left( \frac{r}{\lambda} \right) \times q(r) \quad (5.3.2) \]

Here, \( r \) is the distance from the hit PMT to the reconstructed vertex, \( \lambda \) is water transparency, and \( q(r) \) is the charge of hit PMTs at distance \( r \). From Eq.( 5.3.2), Eq.( 5.3.3) is obtained.

\[ \log(q(r)) = -\frac{r}{\lambda} + Q \quad (5.3.3) \]

Fig.5.10 shows a typical distribution of \( \log(q(r)) \) as a function of \( r \). This distribution is fit to a linear function, and the water transparency is calculated as the inverse of the slope. This measurement is repeated every week. In order to eliminate statistical fluctuation in the weekly measurement, the water transparency for a given week is defined as a running average over 5 weeks, centered on the given week. The result of this measurement is presented in Fig.5.11. The systematic error of this measurement is estimated from the stability of the measured values. The deviation of each data point from the 5-point running average is 0.5%. Therefore the systematic error is estimated to be 0.5%/\( \sqrt{5} = 0.22 \% \).
Figure 5.10: The \( \log(q(r)) \) distribution from a typical data sample.

Figure 5.11: Time variation of the measured water transparency
5.4 Energy calibration

For the energy calibration of SK, an electron linear accelerator and a neutron generator are used.

5.4.1 The electron linear accelerator (LINAC)

For a precise measurement of the absolute energy scale, an electron linear accelerator is set up in a shielded room near the SK tank as shown in Fig.5.12 [45]. The LINAC beam has monochromatic energy, which allows for the precise measurement of absolute energy scale. One can also use the LINAC to measure the resolution of the reconstructed vertex, direction and energy.

5.4.1.1 The LINAC calibration system

The linear accelerator (LINAC) is a medical grade accelerator (Mitsubishi ML-15III). It is a traveling-wave type, so electrons are accelerated as they travel with the microwave in the acceleration tube. The pulse width of the microwave generated in a klystron is 2μsec and the pulse rate is adjustable between 10 and 66 Hz. The average beam momentum can be changed between $5 \sim 16$ MeV/c by manipulating the input power and microwave frequency.

The number of electrons per bunch in the ID should not be much larger than one. For this purpose, the output current of the electron gun is controlled so that the number of electrons is $10^6$ per bunch at the end of the acceleration tube.

The beam is fed into the inner detector with the beampipe evacuated to $10^{-4} \sim 10^{-5}$ torr. The beampipe consists of a stainless steel tube of 10 cm diameter and 4 m length. Inside this tube, there is another tube made of μ-metal. This additional tube decreases strength of the geo-magnetic field to 1% of the natural value.

As given in Fig.5.12, the beam passes through 3 bending magnets(D1 ~ D3). Two 15 degree bending magnets (D1 and D2) and collimators just before these magnets provide monochromatic energy. The D3 magnet bends the beam into the detector. Energy loss by bremsstrahlung at the D3 magnet is about $1 \times 10^{-8}$ MeV for a beam energy of 10 MeV. There are quadrupole magnets (Q-magnets) just before and after the D3 magnet to focus the beam at the end of the beam pipe. The current of the Q-magnet is determined by beamline programs called “Decay-Turtle” and “Transport”. In order to guide the beam to the control position at the end of the beam pipe, the last magnets (XY steering magnets) after the Q-magnets are used.

To monitor the beam direction, there are 4 scintillation counters. Two of them are mounted along the beam line. One is at the end of the acceleration tube and the other is located just after the first 15 degree bending magnet. They are used to roughly check
Figure 5.12: Schematic view of the LINAC calibration system. The solid circles show the beam injection positions.

<table>
<thead>
<tr>
<th>No.</th>
<th>x (cm)</th>
<th>y (cm)</th>
<th>z (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-388.9</td>
<td>-70.7</td>
<td>1228.</td>
</tr>
<tr>
<td>B</td>
<td>-388.9</td>
<td>-70.7</td>
<td>27.</td>
</tr>
<tr>
<td>C</td>
<td>-388.9</td>
<td>-70.7</td>
<td>-1173.</td>
</tr>
<tr>
<td>D</td>
<td>-813.1</td>
<td>-70.7</td>
<td>1228.</td>
</tr>
<tr>
<td>E</td>
<td>-813.1</td>
<td>-70.7</td>
<td>27.</td>
</tr>
<tr>
<td>F</td>
<td>-813.1</td>
<td>-70.7</td>
<td>-1173.</td>
</tr>
<tr>
<td>G</td>
<td>-1237.</td>
<td>-70.7</td>
<td>1228.</td>
</tr>
<tr>
<td>H</td>
<td>-1237.</td>
<td>-70.7</td>
<td>27.</td>
</tr>
<tr>
<td>I</td>
<td>-1237.</td>
<td>-70.7</td>
<td>-1173.</td>
</tr>
</tbody>
</table>

Table 5.1: The injection position of the LINAC beam. The coordinates are defined in Fig.5.12.
the intensity and direction of the beam. The other two counters are at the end of the beam pipe, as shown in Fig.5.13. They are called the trigger counter and the veto counter, respectively. The veto counter consists of four scintillation counters around the beam line and is used for optimizing the beam direction at the end of the beam pipe. The trigger counter is a scintillation counter of 1 mm thickness, which is wrapped in aluminum foil (15 μm thickness). The window at the tip of the beam pipe is Ti with a thickness of 100 μm. The reflectivity of the Ti window is measured to be about 40 ± 10%, which depends on the photon incidence angle and wavelength.

The beampipe after the D2 magnet can be disassembled and reconstructed easily, so that the beam can be injected at various positions. The beam injection positions are summarized in Tab.5.1.

5.4.1.2 Beam energy measurement

The LINAC beam energy is measured with a Ge detector (SEIKO EG&G ORTEC GMX-35210-PEG) [46]. A schematic drawing of the Ge detector is presented in Fig.5.14. The Ge crystal used in the detector is a coaxial type hyper pure crystal with a diameter of 57.5 mm and a length of 66.4 mm. The window for incident particles is made of Be with a thickness of 500 μm. The energy resolution of this detector is 1.92 keV for 1.33 MeV γ-rays of 60Co. The output from the Ge detector is read out and digitized by a Multi-Channel-Analyzer (MCA, SEIKO EG&G Multi-Channel-Analyzer7070). The energy scale of the Ge detector
Figure 5.14: Schematic view of the Ge detector (SEIKO EG&G ORTEC GMX-35210-PEG).

<table>
<thead>
<tr>
<th>Type</th>
<th>air-core(\pi \sqrt{2}), current loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean track radius</td>
<td>(\rho = 75\text{cm})</td>
</tr>
<tr>
<td>Resolution</td>
<td>(\Delta p/p = 0.01%)</td>
</tr>
<tr>
<td>Solid angle</td>
<td>(\Omega = 8.7 \times 10^{-4}) at (\Delta p/p = 0.01%)</td>
</tr>
<tr>
<td>Maximum magnetic field</td>
<td>208G</td>
</tr>
</tbody>
</table>

Table 5.2: Features of the air-core \(\beta\) spectrometer at the Tanashi branch of KEK.

is calibrated by various \(\gamma\)-ray sources (e.g. 661 keV \(\gamma\)s from \(^{137}\text{Cs}\), \(~8.999\text{ MeV} \gamma\)s from Ni(n,\(\gamma\))Ni reaction). The relation between the MCA channel and the \(\gamma\)-ray energy is almost linear. The deviation from linearity is within 0.02% for \(\gamma\)-ray momentum above 5 MeV.

In measuring the energy of a charged particle with the Ge detector, one must take into account energy loss in the inactive layer surrounding the Ge crystal and the window of incidences. In order to measure these energy losses in the inactive region, the Ge detector is calibrated using an air-core \(\beta\) spectrometer at the Tanashi branch of KEK. A schematic view and a summary of features of the air-core \(\beta\) spectrometer are presented in Fig.5.15 and Tab.5.2, respectively.

The electron source used in this calibration is \(^{207}\text{Bi}\). \(^{207}\text{Bi}\) emits an electron with monochromatic energy by an internal conversion process [47]. The energies of electrons
used in this calibration are 975.6 keV and 1682.2 keV.

Fig.5.16 shows the result of this calibration. The upper figure in Fig.5.16 shows the observed energy spectrum for 975.7 keV electrons injected at the center of the Ge detector taken for 29.90 minutes. The cross marks indicate data with statistical error bars. The histogram shows the result from MC simulation in which the thickness of the inactive layer is assumed to be 41 μm. From this measurement, the mean energy loss in the inactive layer is estimated to be 27 keV. The peak position of the observed distribution and that from MC agree within 3.2 keV.

The lower figure in Fig.5.16 shows the observed energy spectrum for 1682.2 keV electrons taken for 590.93 minutes. The solid circles indicate data with statistical error bars. The MC for the 1682.2 keV electrons assuming the same inactive layer thickness is shown as a histogram.

In measuring the LINAC beam energy, the D3 magnet is disconnected and the endcap with a trigger counter is connected horizontally. Then the Ge detector is placed in front of the endcap. The observed energy distribution of the LINAC beam obtained from the Ge detector is presented in Fig.5.17. The solid circles represent data and the histogram represents MC simulation.

The spread and the tail of the energy distribution are caused by γ-rays which escape from the Ge crystal and electrons scattering backward on the surface of the Ge crystal. The beam energy is obtained by comparing the observed distribution with MC simulation.

The results of the beam energy measurements are summarized in Tab.5.3. The difference
Figure 5.16: Energy distributions from the air-core spectrometer calibration. The upper figure shows the observed energy spectrum for 975.7 keV electrons injected at the center of the Ge detector taken for 29.90 minutes. The lower figure shows the same for 1682.2 MeV.
Figure 5.17: The observed LINAC beam energy spectrum from the Ge detector. Solid circles show data with statistical error bars. Histograms show the MC. The number at the upper left side of each figure is the total MC electron energy.

between the “Ge energy” and “energy in tank” is the energy lost in the endcap. These energy losses are considered in the MC simulation. Typical energy loss values for 8 MeV electrons calculated by MC simulation are 185 keV in the trigger counter, 11 keV in the Ti window, 126 keV in the beryllium window and 27 keV in the inactive layer of the Ge crystal. As described above, the beam momentum is determined by the electric current of the D1 magnet, and the relation between them is a linear function. The systematic error in the beam energy measurement is estimated to be 20 keV using the deviation of measured energy from the linear function. The beam momentum spread is studied by comparing the observed energy distribution spread in data with that in MC simulation, and is found to be less than 0.4%.
<table>
<thead>
<tr>
<th>D1 current (A)</th>
<th>Ge energy (MeV)</th>
<th>beam momentum (MeV/c)</th>
<th>energy in tank (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>5.08</td>
<td>4.25</td>
<td>4.89</td>
</tr>
<tr>
<td>2.15</td>
<td>6.03</td>
<td>5.21</td>
<td>5.84</td>
</tr>
<tr>
<td>2.5</td>
<td>7.00</td>
<td>6.17</td>
<td>6.79</td>
</tr>
<tr>
<td>3.2</td>
<td>8.86</td>
<td>8.03</td>
<td>8.67</td>
</tr>
<tr>
<td>4.0</td>
<td>10.99</td>
<td>10.14</td>
<td>10.78</td>
</tr>
<tr>
<td>5.0</td>
<td>13.65</td>
<td>12.80</td>
<td>13.44</td>
</tr>
<tr>
<td>6.0</td>
<td>16.32</td>
<td>15.44</td>
<td>16.09</td>
</tr>
</tbody>
</table>

Table 5.3: Summary of the beam energy measurement. “Ge energy” is the original beam energy obtained from simulation. “Energy in tank” is the energy after the endcap.

5.4.1.3 SK energy calibration

In the energy calibration of SK, there are some backgrounds which are related to the LINAC beam itself.

One of them is due to γ-rays originating from the LINAC system. When the LINAC accelerates electrons, the klystron and the acceleration tube emit γ-rays. Electrons that diverge from the beam and hit the beampipe and the collimators also emit γ-rays. Most of those are absorbed in the OD and the rock between the LINAC and the SK tank, but some may enter the inner detector. These background events can be easily eliminated by requiring the “LINAC trigger”, which is issued when a trigger counter signal coincides with a LINAC microwave pulse. The logic for the trigger used in LINAC calibration is shown in Fig.5.18. The final type of background is due to extra electrons in the beam. It can be eliminated by checking the arrival time of those electrons. A typical TOF-subtracted timing distribution of the hit PMTs is shown in Fig.5.19. If the pulse height of the extra peak is more than 30% of expected signal and the time difference is more than 30 nsec, then the event is cut. The vertex distribution after these cut is shown in Fig.5.20. In this figure, the beam momentum is 16.32 MeV and the injection position is \((x, y, z) = (-388.9\text{cm}, -70.7\text{cm}, +27\text{cm})\). A clear peak is found at the endcap position.

The absolute energy scale (the relation between \(N_{\text{eff}}\) and the total electron energy) is obtained from MC simulation whose parameters are tuned to reproduce certain aspects of the LINAC data. For the determination of the absolute energy scale of SK, a simple comparison between the beam energy and \(N_{\text{eff}}\) is not used. This is because the direction of the beam is only downward and the injection position is limited.

The parameters which are used to tune the MC are the following:

- PMT timing resolution
Figure 5.18: The logic for the trigger used in LINAC calibration. In normal data taking, the “LINAC trigger” is used. The microwave trigger is used to take data used to estimate the background due to the LINAC instrumentation.

Figure 5.19: The TOF subtracted timing distributions of the hit PMTs for some typical LINAC events. The number of peaks corresponds to the number of electrons in the event.
Figure 5.20: Vertex position distribution from LINAC data taken at \((x, y, z) = (-388.9\text{cm}, -70.7\text{cm}, +27\text{cm})\). The beam momentum is 16.31 MeV/c. Projections of the scatter plot are shown to the right and underneath. The scatter plot limits correspond to the limits of the ID.
• PMT collection efficiency
• Scattering and absorption coefficient of water

Various sets of these parameters are checked by many MC simulations and the parameter set that best reflects observed distribution in data (not only the energy scale, but also the angular resolution) is selected. With the tuned MC, the relation between $N_{eff}$ and energy ($4 \sim 50$ MeV) is obtained from Eq. (5.4.1).

$$A4 = 1. - 0.001541420 \times N_{eff}$$
$$A3 = 1. - 0.006440886 \times A4 \times N_{eff}$$
$$A2 = 1. - 6.049027 \times 10^{-4} \times A3 \times N_{eff}$$
$$E = 0.7354543 + 0.1337601 \times A2 \times N_{eff}$$  (5.4.1)

Fig.5.21 is the energy distribution in data and the tuned MC for various beam momenta. The crosses indicate data and the histogram is for the MC simulation. Each data and MC distribution is fit to a Gaussian and the energy at the peak is taken to be the energy scale. The range of the fit is asymmetric ($-1\sigma \sim +2\sigma$) to reduce the influence of the background, which tends to populate the low energy tail of the distribution. The energy scale differences $\frac{MC}{DATA}$ between data and MC at various positions and energy are shown in Fig.5.22. The position-weighted average is performed at various energy points to get a global result. The weight is given by the ratio of the volume surrounding a given point to the fiducial volume. Fig.5.23 shows the position averaged $\frac{MC}{DATA}$. All position differences are less than $\pm 0.5\%$. The energy dependence of $\frac{MC}{DATA}$ is shown in Fig.5.24. All differences are within $\pm 0.5\%$, except at one position where the error is 0.7\%.

The systematic error for the absolute energy calibration include:

(A) The error in the beam energy measurement

(B) The error introduced by LINAC-originating background

(C) The error due to the uncertainty in the reflection from the endcap materials

(A) is estimated to be 20 keV as described above. In order to estimate (B), two kinds of empty trigger data are taken; microwave trigger data and clock trigger data. When the former data is taken, trigger is issued only by the microwave pulse from the LINAC, but the signal from the trigger counter is not required. 90% of events obtained with this trigger are empty. The latter contain only dark noise of SK electronics and ID PMTs. A MC simulation of LINAC events without dark noise simulation is added to both of these data event-by-event. By comparing the energy at the peak of the energy distribution of “microwave trigger data + MC” with “random trigger data + MC”, the systematic error in the absolute energy scale is conservatively estimated to be 0.16\%. (C) is the most serious

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Figure 5.21: Observed energy distribution from LINAC calibration. The beam injection position is at \((x = -1237 \, \text{cm}, y = -70.7 \, \text{cm}, z = +1228 \, \text{cm})\). The cross marks show data with statistical error bars. The histogram shows MC. The beam energy is written in each figure.
Figure 5.22: Energy scale difference between data and MC. A~I represent the beam injection positions which are summarized in Tab.5.1. The error bars represent only the statistical errors.
Figure 5.23: Energy dependence of the position-weighted average absolute energy scale difference between data and MC.

Figure 5.24: Position dependence of the absolute energy scale difference between data and MC.
Figure 5.25: Energy resolution difference between data and MC.
Figure 5.26: Energy dependence of the position-weighted average energy resolution difference between data and MC.

Figure 5.27: Position dependence of the energy resolution difference between data and MC.
<table>
<thead>
<tr>
<th>beam momentum (MeV/c)</th>
<th>fraction hitting endcap(%)</th>
<th>error due to reflectivity(%)</th>
<th>total systematic error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.08</td>
<td>4.7</td>
<td>±0.68</td>
<td>±0.71</td>
</tr>
<tr>
<td>6.03</td>
<td>3.3</td>
<td>±0.40</td>
<td>±0.55</td>
</tr>
<tr>
<td>7.00</td>
<td>2.8</td>
<td>±0.22</td>
<td>±0.44</td>
</tr>
<tr>
<td>8.86</td>
<td>1.3</td>
<td>±0.18</td>
<td>±0.33</td>
</tr>
<tr>
<td>10.99</td>
<td>0.88</td>
<td>±0.11</td>
<td>±0.27</td>
</tr>
<tr>
<td>13.65</td>
<td>0.67</td>
<td>±0.08</td>
<td>±0.24</td>
</tr>
<tr>
<td>16.31</td>
<td>0.51</td>
<td>±0.06</td>
<td>±0.21</td>
</tr>
</tbody>
</table>

Table 5.4: Summary of systematic errors from LINAC calibration at various beam momenta. The second column shows the fraction of Cherenkov photons which hit the endcap, the third column shows the systematic error due to the uncertainty in the reflectivity of the endcap, and the fourth shows the total systematic error for the absolute energy scale calibration.

uncertainty for the absolute energy scale calibration. At the momentum of 5 MeV/c, 4.7% of Cherenkov photons hit the endcap. Although the reflectivity of stainless steel and the titanium window is measured, the possible presence of a small bubble on the titanium window introduces the uncertainty. The total systematic errors are summarized in Tab.5.4.

The fractional energy resolution (Eq. (5.4.2)) is also studied with the LINAC.

\[
Resolution = \frac{\sigma_{\text{energy}}}{E_{\text{observed}}} \tag{5.4.2}
\]

Here, \(\sigma_{\text{energy}}\) is the width of the Gaussian function which is obtained by fitting and \(E_{\text{observed}}\) is the peak energy. Fig.5.25 shows the energy resolution difference \(\frac{\text{MC-DATA}}{\text{DATA}}\) at various points and energies. All position-weighted average differences agree within about 2.0% as shown in Fig.5.26. The position dependence is also within 2.0% as shown in Fig.5.27.

5.4.1.4 Angular and vertex resolution

The angular resolution of the recoil electron momentum in SK is measured with the LINAC downward-going beam. Fig.5.28 presents distributions of reconstructed direction from LINAC data at the injection position \((x, y, z) = (-388.9\text{cm}, -70.7\text{cm}, +1228\text{cm})\). The angular resolution is defined as the angle with respect to the direction of the beam that includes 68% of the reconstructed direction. The energy dependence of the angular resolution at various positions is shown in the upper figures of Fig.5.29. The difference between
data and MC is presented in the lower figures of Fig.5.29. Most points are within ±0.5°. This difference is included in the systematic error of the solar neutrino signal extraction.

The vertex resolution is also measured. Distributions of the distance from the endcap to the reconstructed vertex is presented in Fig.5.30. The vertex resolution is defined as the distance from the center of the Ti window of the endcap to the reconstructed distance which includes 68% of the events. Fig.5.31 shows the energy dependence of the difference in vertex resolution between data and MC at various positions. The timing resolution of PMTs for single p.e. are adjusted in the MC so that the measured angular resolution is consistent with MC at all 8 points. As a result of this tuning, the timing resolution is found to be 2.4 nsec, which is consistent with the measured value [35].

5.4.2 Time variation and directional dependence of the absolute energy scale

In order to check for time variation and directional dependence of the absolute energy scale, the energy spectrum of spallation events and that of electrons from muon decay are used. Spallation events are electrons and γ-rays from the decay of radioactive nuclei produced by cosmic ray muon interactions in the ID. A more detailed explanation is given in Sec.6.3. The selection criteria are as follows:

1. The time difference between a low energy event and a muon event is less than 0.1 sec

2. The distance from the reconstructed vertex of a low energy event to a reconstructed muon track is less than 3 m

3. The spallation likelihood explained in Sec.6.3 is larger than a threshold value

The time variation in the mean value of the energy spectrum is presented in Fig.5.32. The error bars indicate only statistical error. From this figure, the time variation is found to be less than 0.5% over the entire run time. Fig.5.34 presents the energy scale as a function of azimuthal angle and zenith angle. The error bars show only the statistical error. The directional dependence is within ±0.5%.

The time variation of the absolute energy scale is also studied by examining the energy spectrum of electrons from the decay of cosmic ray muons in the fiducial volume. The data sample used to obtain the spectrum is the same as that used to measure the water transparency (Sec.5.3.2). Fig.5.33 shows the time variation of the mean value of the energy spectrum. The error bars include only statistical error. From this figure, the time variation of the absolute energy scale is less than ±0.5%. This is consistent with the result obtained using spallation events.

90
theta distribution $x=-1237 \ y=-70.7 \ z=1228$

![Graphs showing angular distribution for different energies.](image)

**Figure 5.28:** The observed angular distribution from LINAC calibration. The beam injection position is at $(x = -1237 \text{cm}, y = -70.7 \text{cm}, z = +1228 \text{cm})$. The cross marks show data with statistical error bars. The histogram shows MC. The beam energy is written in each figure.
Figure 5.29: The angular resolution difference between data and MC.
Figure 5.30: Observed distance distribution from LINAC calibration. The beam injection position is at \((x = -1237 \text{ cm}, y = -70.7 \text{ cm}, z = +1228 \text{ cm})\). The cross marks show data with statistical error bars. The histogram shows MC. The beam energy is written in each figure.
Figure 5.31: The vertex resolution difference between data and MC.
Figure 5.32: The time variation of the energy scale from spallation events.

Figure 5.33: The time variation of the energy scale from decay electron events.
Figure 5.34: The directional dependence of the energy scale from spallation events. The left figure shows the zenith angle dependence. The right figure shows the azimuthal angle dependence.

5.4.3 The DT generator

In order to check for directional dependence of the energy scale and to cross-check the LINAC calibration results, the decay of $^{16}\text{N}$ is used [48]. $^{16}\text{N}$ is produced by an (n,p) reaction on $^{16}\text{O}$ in the water of SK. The neutron source is the deuterium-tritium neutron generator (DT generator, MF Physics Model A-211). It can produce about $10^6$ neutrons in each pulse at a rate up to 100 Hz. Fig.5.35 is a schematic drawing of the DT generator.

The DT generator is hung by a crane and is moved into position in the detector.

The reaction by which the DT generator produce neutrons is,

$$ ^3\text{H} + ^2\text{H} \rightarrow ^4\text{He} + n. $$

The generated neutron energy is 14.2 MeV. It is energetic enough to produce $^{16}\text{N}$ by an (n,p) reaction on $^{16}\text{O}$, which requires a neutron energy greater than $\sim 11$ MeV.

$$ ^{16}\text{O} + n \rightarrow ^{16}\text{N} + p $$

The $^{16}\text{N}$ decay with a Q-value of 10.4 MeV and a half-life of 7.12 sec produces an electron of maximum energy 4.3 MeV together with a 6.1 MeV $\gamma$-ray. These are useful for energy calibration of SK.

$$ ^{16}\text{N} \rightarrow ^{16}\text{O} + e^- + \gamma + \nu_e $$

96
Figure 5.35: The DT generator.

<table>
<thead>
<tr>
<th>Fraction (%)</th>
<th>$J_i^P \rightarrow J_f^P$</th>
<th>$\Delta I$</th>
<th>$E_\gamma$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.2</td>
<td>$2^- \rightarrow 3^-$</td>
<td>+1</td>
<td>6.129</td>
<td>GT allowed</td>
</tr>
<tr>
<td>28.0</td>
<td>$2^- \rightarrow 0^+$</td>
<td>-2</td>
<td>none</td>
<td>GT 1st forbidden</td>
</tr>
<tr>
<td>4.8</td>
<td>$2^- \rightarrow 1^-$</td>
<td>+1</td>
<td>7.116</td>
<td>GT allowed</td>
</tr>
<tr>
<td>1.06</td>
<td>$2^- \rightarrow 2^-$</td>
<td>+0</td>
<td>8.872</td>
<td>F+GT allowed</td>
</tr>
<tr>
<td>0.012</td>
<td>$2^- \rightarrow 0^+$</td>
<td>-2</td>
<td>6.049</td>
<td>GT 1st forbidden</td>
</tr>
<tr>
<td>0.0012</td>
<td>$2^- \rightarrow 1^-$</td>
<td>+1</td>
<td>9.585</td>
<td>GT allowed</td>
</tr>
</tbody>
</table>

Table 5.5: Summary of the beta decay of $^{16}$N. GT means Gamow-Teller transition and F means Fermi transition.
Figure 5.36: Energy distribution from DT calibration. The position of the DT generator is at \( (x = -388.9\text{cm}, y = -70.7\text{cm}, z = 0\text{cm}) \).

The modes of \(^{16}\text{N}\) decay are summarized in Tab.5.5.

During calibration, the DT generator is raised 2 m from the neutron production point in order to minimize reflection by the housing. Calibration data is taken during 10 ~ 40 sec after the DT generator is fired to prevent electrical noise generated by the crane from contaminating the data. The number of \(^{16}\text{N}\) events at a single location in the SK tank is typically 30,000.

Fig.5.36 shows the observed energy spectrum at a typical position. The data are compared with the MC simulation, in which the reflection of Cherenkov photon from the DT generator housing and \(^{16}\text{N}\) decay simulation are included. The energy distributions of data and MC are fit with a Gaussian function between 5.5 MeV and 9.0 MeV. The energy at the peak of each distribution is treated as the energy scale.

Fig.5.37 shows the position dependences of the absolute energy scale. The left figure shows the variation of the absolute energy scale difference between data and MC as a function of radius. At each R-position, the results from different z-position are combined to make a position weighted result. Within the fiducial volume, the energy scale dependence on the radial position of the DT generator agree MC within \( \pm 0.5\% \). This is consistent with the systematic errors for the energy scale obtained by LINAC. The z-position variation is shown in the right figure. The absolute energy scale variation as a function of z-position is within \( \pm 1.0\% \). This is also consistent with the systematic errors for the energy scale obtained with LINAC.

The angular dependence of the absolute energy scale as a function azimuthal angle and zenith angle is presented in Fig.5.38. Results are obtained by a position weighted average
Contamination from natural background & < 0.01% \\ Modeling of $^{16}$N decay in MC & ±0.1% \\ Unmodeled decay lines & < 0.01% \\ Shadowing of Cherenkov photons & ±0.1% \\ Data selection & ±0.1% \\ Radioactive background from the DT generator & ±0.05%  

Table 5.6: Summary of systematic errors for DT calibration

at each point where data were taken. The variation of the energy scale in direction within the fiducial volume is within ±0.5%

Tab.5.6 summarizes the systematic errors for DT calibration of the absolute energy scale. The systematic error in the modeling of $^{16}$N decay in MC is due to gamma-rays from the decay of small amounts of background isotopes like $^{24}$Na, $^{62}$Co and $^{28}$Al created by the interaction between the DT neutron and the DT generator materials. It is conservatively estimated to be ±0.1%. The systematic error for the shadowing of Cherenkov photon by DT generator housing is determined from the fraction of photons which could be absorbed. This systematic error of ±0.1% is conservatively estimated.

5.5 Trigger efficiency

For the solar neutrino analysis, we use two different triggers, as described in Chapter 3: the Low Energy Trigger (LE trigger) and the Super Low Energy Trigger (SLE trigger). Hence, the trigger efficiency for both has to be measured.

The trigger efficiency is measured using the Ni-Cf system and the DT generator. The low energy trigger efficiency is defined as follows.

$$\epsilon_{LE\text{trigger}} = \frac{N_{LE\text{trigger}}}{N_{\text{special}}}$$  (5.5.1)

Here, $\epsilon_{\text{trigger}}$ is the trigger efficiency. $N_{LE\text{trigger}}$ is the number of events issuing both the LE trigger and a special low energy trigger with a threshold of -150 mV. $N_{\text{special}}$ is the number of events issuing the special low energy trigger. In measuring the SLE trigger efficiency, the number of event issuing both the SLE trigger and the special low energy trigger is used instead of $N_{LE\text{trigger}}$.
Figure 5.37: The position dependence of the absolute energy scale of DT calibration events.

Figure 5.38: The azimuthal angle and zenith angle dependence of the absolute energy scale of DT calibration events.
Figure 5.39: The LE trigger efficiency as a function of energy. The DT generator is situated near the center of the ID \((x, y, z) = (35.3, -70.7, 41.0)\).

Fig.5.39 shows the LE trigger efficiency as a function of energy for a typical data run. The efficiency is 99.8\% for events with energy of 6.5 \(\sim\) 7.0 MeV. Above 7.0 MeV, the LE trigger efficiency is 100\%.

The trigger efficiency is measured at various positions in the ID and at various water transparencies to study the possible dependence on the vertex position, the event direction, and water transparency. In order to consider these dependences in the MC simulation, a trigger simulator is used. In the trigger simulator, the trigger threshold is defined by the number of effective hits. The threshold level is set to reproduce the energy and position dependence of the trigger efficiency seen in data. The open circles in Fig.5.39 show the simulated trigger efficiency. The maximum deviation between the data and MC is \(+0.4\%\) in the energy region from 6.5 MeV to 7.0 MeV.

The SLE trigger was installed in May 1997 and the threshold has been lowered 3 times. At first, the threshold was set to \(-260\) mV, and it was changed to \(-250\) mV, \(-222\) mV, and \(-212\) mV. The changes in the SLE trigger threshold are summarized in Sec.3.6.3.

Fig.5.40 shows the energy dependence of the SLE trigger efficiency for each trigger threshold. Filled circles show data and open circles show the results from the SLE trigger simulation. The line shows the analysis threshold for this thesis (5.0 MeV). The deviation between the data and MC is obtained to be \(\pm 3.2\%\) in the energy region from 5.0 MeV to 5.5 MeV, and \(\pm 0.9\%\) in the energy region from 5.5 MeV to 6.0 MeV. To obtain these value, the deviations are combined with weights based on the run times for each SLE threshold. Above 6.0 MeV, the SLE trigger efficiency is 100\% for all positions.
Figure 5.40: The SLE trigger efficiency as a function of energy.
Chapter 6

Data Reduction

The data set used in this analysis covers the period from May 31, 1996 to April 24, 2000 and represents 1117 days of detector live time (roughly \( \sim 90\% \) of actual elapsed time). The data set selection criteria are described in Sec.6.1.

Background sources to the solar neutrino data sample are:

- electronic noise and flashing PMTs,
- cosmic ray muons,
- electrons from the decay of stopping muons,
- muon-induced spallation products,
- radioactivity in the detector materials and surrounding rock,

To reduce these backgrounds, various cuts are applied. The cuts are applied to the data in four stages: the first reduction, spallation cut, the second reduction and the \( \gamma \)-ray cut. These cuts are explained in Sec.6.2, 6.3, 6.4 and 6.5.

6.1 The data set

The basic unit of the dataset is a run. Each run is at most 24 hours long. Each run is divided into subruns, which are about 2 \( \sim 10 \) minutes long. The length of each subrun depends on the trigger rate.

The criteria used to select bad runs (subruns) are as follows:

- The length of a run is less than 5 minutes.
- The length of a subrun is less than 30 seconds.
Figure 6.1: The total charge distribution from a typical data sample. The hatched region shows events remaining after the total charge cut.

- The subrun has a higher than allowed rate of noise events.
- There are hardware and/or software problems in the run.
- Calibration devices are in the ID during the run.

Even after this bad run rejection, there is too much background to extract the solar neutrino signal. To reduce the background level, various cuts are applied. They are explained in the following sections. The efficiency of each cut is summarized in the last section.

6.2 First data reduction

6.2.1 Total charge cut

The total charge of an event is required to be less than or equal to 1000 p.e. to remove cosmic ray muons. The total p.e. distribution for a typical data sample is shown in Fig.6.1. 1000 p.e. corresponds to a recoil electron energy above 100 MeV, so the efficiency of this cut for solar neutrinos is 100%.

6.2.2 Fiducial volume cut

To reduce events caused by γ-rays coming from the surrounding rock and the materials comprising the ID wall, events that have vertex position within 2 m of the ID wall are
Figure 6.2: Fiducial volume cut applied to a typical data sample. The left figure shows the vertex distribution as a function of $z$-position. The right figure shows the vertex distribution as a function of $r$-position. The hatched histograms show the distributions after the fiducial volume cut.

eliminated. The fiducial volume of SK for the solar neutrino analysis is defined by this cut to be 22.5 kt, or 28.9 m in diameter and 32.2 m in height. Fig. 6.2 shows the vertex distribution from a typical data sample. The hatched histogram shows the vertex distribution after the fiducial volume cut.

### 6.2.3 Time difference cut

The time from the previous event is required to be greater than 50 $\mu$sec to remove electrons from the decay of cosmic ray muons. Fig. 6.3 shows the distribution of the time from the previous event from a typical data sample. The fractional dead time caused by this cut is $1.8 \times 10^{-4}$. This deadtime is taken into account in the MC simulation.

### 6.2.4 OD trigger cut

Events with an OD trigger (corresponds to $> 19$ OD-PMT-hits in a 200 nsec window) are rejected to remove cosmic ray muons with total p.e. less than 1000 p.e.
Figure 6.3: Time from the previous event ($\Delta T$). If $\Delta T < 50\mu$sec, the event is eliminated.

Figure 6.4: The left figure shows the $R_{\text{noise}}$ distribution from a typical data sample. If $R_{\text{noise}} > 0.4$, the event is eliminated. The right figure shows the ratio of the number of hit PMTs to the number of channels in an ATM. If more than 95% of the channels in one module have hits, the event is rejected.
Figure 6.5: Maximum charge vs the number of PMTs around the maximum charge PMT. Fig.(a) shows a typical distribution from a sample containing a flasher. Fig.(b) shows a distribution from a normal sample. The line in (a) shows the cut condition.

6.2.5 Noise event cut

In order to remove events due to electronic noise, the fraction of noise hits \( R_{\text{noise}} \) in an event is calculated. \( R_{\text{noise}} \) is defined as follows:

\[
R_{\text{noise}} = \frac{N_{\text{noise}}}{N_{\text{total}}},
\]

(6.2.1)

where \( N_{\text{noise}} \) is the number of hit PMTs with charge less than 0.5 p.e. and \( N_{\text{total}} \) is the total number of hit PMTs. If \( R_{\text{noise}} \) is larger than 0.4, the event is rejected.

There is another cut which targets electronic noise events. Noise hits are often clustered in one ATM board. If over 95% of channels in an ATM have hit PMTs, then the event is rejected.

The efficiency for the accidental reduction of solar neutrino events by these noise cuts is estimated by MC to be 0.01%.

6.2.6 Flasher event cut

A “Flasher” event is caused by a PMT that emits light due to arch discharge on dynodes. When a PMT flashes, it often has a large signal and the 24 (5 \times 5 - 1) surrounding PMTs also have signals. This characteristic is used to remove the flasher events.

Fig.6.5 is a scatter plot of the maximum charge PMT in each event and the number of hit PMTs which neighbor the maximum charge PMT.
Figure 6.6: Goodness distribution from a typical data sample. The events in the hatched area remain after this cut.

6.2.7 Goodness cut

The goodness of the reconstructed vertex (defined by Eq. 4.1.3) must be greater than 0.4. If the goodness value of an event is low, then the reconstructed vertex may be very inaccurate, so the event is rejected. Fig.6.6 shows the goodness distribution from a typical data sample.

6.2.8 Second flasher cut

To further eliminate flasher events, the uniformity of the hit PMT distribution about the reconstructed direction is used. In order to check the uniformity, the Kolmogorov-Smirnov test is applied. Examples are presented in Fig.6.7.

The upper figures illustrate a good event. The left figure shows the hit pattern in the event. The figure on the right shows the cumulative distribution function of the hit PMTs with respect to the azimuthal angle about the reconstructed direction. In the ideal case, the hit PMTs are on the broken line which represents an even distribution of hits around the reconstructed direction. Here, Dir$_{ks}$ is defined as the full width of the deviation of the hit PMT distribution from the ideal case, divided by 360. The lower figures are from a flasher event. As shown in the figure, Dir$_{ks}$ of flasher event is larger than that of the good event.

The condition to eliminate flasher events is as follows:

\[ \text{Dir}_{ks} \geq 0.25 \text{ and goodness} < 0.6 \]
Figure 6.7: An example of the $D_{irks}$ test. The upper figures illustrate a good event. The lower figures are from an external flasher event.
Figure 6.8: $\text{Dir}_{ka}$ distribution from a typical data sample. The histogram shows the distribution from a MC sample.

Figure 6.9: The distribution of the direction cosine of $z$ before (A) and after (B) the second flasher cut.
Fig. 6.9 shows the distribution of the direction cosine of \( z \) (\( \text{dir}_z \)) before and after this cut. After this cut, a small peak around \( \text{dir}_z = 0.7 \) disappears.

### 6.3 The spallation cut

Some cosmic ray muons which go through the detector produce radioactive elements by breaking up an oxygen nucleus.

\[
\mu + ^{16}O \rightarrow \mu + X + \ldots, \tag{6.3.1}
\]

where \( X \) represents radioactive nuclei. A summary of the possible radioactive nuclei is shown in Tab. 6.1 [47]. The nuclei decay by \( \gamma \) and \( \beta \) emission with a lifetime in the range of \( 0.001 \sim 14 \) sec. They are observed in SK and are called “Spallation events”. The energies of the spallation events are similar to those of recoil electrons from solar neutrinos, so they are one of the major backgrounds in the solar neutrino analysis.

To identify spallation events, a likelihood method is used [49]. The parameters for the likelihood function are:

- \( \Delta L \): Distance from the low energy event to the reconstructed track of the preceding muon event.
- \( \Delta T \): Time difference between the low energy event and the preceding muon event.
- \( Q_{\text{res}} \): Residual charge of the preceding muon event.

\[
Q_{\text{res}} = Q_{\text{total}} - Q_{\text{unit}} \times L_{\mu}, \tag{6.3.2}
\]

where \( Q_{\text{total}} \) is the total charge, \( Q_{\text{unit}} \) is the total charge per cm and \( L_{\mu} \) is reconstructed track length of the muon event.

If the muon track reconstruction fails, then only \( \Delta T \) and \( Q_{\text{total}} \) are used. For spallation events, \( \Delta L \) and \( \Delta T \) are shorter, and \( Q_{\text{res}} \) is larger, compared to solar neutrino events. The likelihood function is explained in Appendix A.

For the spallation cut, one calculates the likelihood values for muons in the previous 100 seconds and select a muon which gives the maximum likelihood value (\( L_{\text{max}} \)).

Fig. 6.10 shows the \( L_{\text{max}} \) distribution in data and in randomly sampled events. The conditions for eliminating the spallation events are given as follows: \( L_{\text{max}} > 0.98 \) for muon events with a reconstructed track and \( L_{\text{max}} > 0.92 \) for muon events in which track reconstruction failed. These thresholds are determined by maximizing the significance of the solar neutrino signal.

The dead time caused by the spallation cut is estimated to be 21.1%, which was obtained from randomly sampled events. The dead time depends upon the position in the detector.
<table>
<thead>
<tr>
<th>isotope</th>
<th>$\tau_{1/2}$ (sec)</th>
<th>decay mode</th>
<th>kinetic energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^8_2$He</td>
<td>0.122</td>
<td>$\beta^-$</td>
<td>10.66 + 0.99 ((\gamma)) (11%)</td>
</tr>
<tr>
<td>$^8_3$Li</td>
<td>0.84</td>
<td>$\beta^-$</td>
<td>12.5 ~ 13</td>
</tr>
<tr>
<td>$^9_5$B</td>
<td>0.77</td>
<td>$\beta^+$</td>
<td>13.73</td>
</tr>
<tr>
<td>$^9_3$Li</td>
<td>0.178</td>
<td>$\beta^-$</td>
<td>13.5 (75%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11.0 + 2.5 ((\gamma))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-$</td>
<td>$\sim$ 10 (35%)</td>
</tr>
<tr>
<td>$^9_6$C</td>
<td>0.127</td>
<td>$\beta^+\ p$</td>
<td>3 ~ 13</td>
</tr>
<tr>
<td>$^{11}_3$Li</td>
<td>0.0085</td>
<td>$\beta^-$</td>
<td>20.77 (31%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\ n$</td>
<td>$\sim$ 16 (61%)</td>
</tr>
<tr>
<td>$^{11}_4$Be</td>
<td>13.8</td>
<td>$\beta^-$</td>
<td>11.48 (61%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9.32 + 2.1 ((\gamma)) (29%)</td>
</tr>
<tr>
<td>$^{12}_4$Be</td>
<td>0.0114</td>
<td>$\beta^-$</td>
<td>11.66</td>
</tr>
<tr>
<td>$^{12}_6$Be</td>
<td>0.0204</td>
<td>$\beta^-$</td>
<td>13.37</td>
</tr>
<tr>
<td>$^{17}_7$N</td>
<td>0.0110</td>
<td>$\beta^-$</td>
<td>16.38</td>
</tr>
<tr>
<td>$^{13}_5$B</td>
<td>0.0173</td>
<td>$\beta^-$</td>
<td>13.42</td>
</tr>
<tr>
<td>$^{18}_8$O</td>
<td>0.0090</td>
<td>$\beta^-$</td>
<td>8 ~ 14</td>
</tr>
<tr>
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<td>0.0161</td>
<td>$\beta^-$</td>
<td>14.07 + 6.09 ((\gamma))</td>
</tr>
<tr>
<td>$^{16}_6$C</td>
<td>2.449</td>
<td>$\beta^-$</td>
<td>9.82 (32%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.51 + 5.31 ((\gamma))</td>
</tr>
<tr>
<td>$^{16}_7$N</td>
<td>0.7478</td>
<td>$\beta^-$</td>
<td>$\sim$ 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10.44 (26%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.27 + 6.13 ((\gamma)) (68%)</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of spallation products.
The left figure of Fig.6.11 shows the dead time distribution as a function of the distance from the side wall of ID, which corresponds to radial dependence. The z-position dependence of the dead time is shown in the right figure. This position dependence is taken into account in the MC simulation.

6.4 Second data reduction

6.4.1 Vertex test

Evaluation of the validity of the vertex reconstruction is carried out by two methods. One is a check of the stability of the *goodness* of the vertex reconstruction by examining the change in *goodness* around the reconstructed vertex (see Sec.6.4.1.1) and the other is a check of the stability of the vertex position by re-estimating the vertex position after noise hit rejection. (see Sec.6.4.1.2)

6.4.1.1 Goodness stability

Usually, if there are many noise hits in an event, the event is cut because of bad *goodness* from vertex reconstruction. However, some events have good *goodness* by chance. To estimate the validity of the *goodness*, differences between the *goodness* at the original reconstructed vertex and those at test vertices around the original one are examined. For an event with many noise hits, the timing distribution of PMT hits is broader. As a result, the change in the *goodness* is small.

The stability of the *goodness* of vertex reconstruction is checked as follows.

1. Make a two dimensional grid as shown in Fig.6.12 on the plane which contains the reconstructed vertex point and normal to the reconstructed direction.

2. The vertex is assumed to be at each grid point and the *goodness* defined by Eq. 4.1.3 is calculated at each grid point.

3. Calculate the difference between the *goodness* at the original vertex and at each grid point.

4. Count the number of grid points at which the deviation exceeds the threshold ($N_{bad}$). The threshold at each grid point is shown in the right figure in Fig.6.12.

5. Calculate the ratio of $N_{bad}$ to the total number of grid points ($R_{bad}$). If $R_{bad}$ is larger then 0.08, then the event is cut.

Fig.6.13 shows the $R_{bad}$ distribution from a typical data sample.
Figure 6.10: The distributions of spallation likelihood. The left figure shows the likelihood distribution for muon events with a reconstructed track. The right figure is for the case where muon reconstruction is failed. The open histogram shows a typical data sample and the hatched histogram shows the randomly sampled events.

Figure 6.11: Position dependence of the dead time caused by the spallation cut.
Figure 6.12: The left figure shows the definition of the grid used in the goodness stability cut. The original reconstructed vertex is situated at the center. The right figure shows the threshold of goodness difference as a function of the distance from the original vertex. This function is used to obtain $N_{\text{bad}}$.

Figure 6.13: $R_{\text{bad}}$ distribution from a typical data sample.
6.4.1.2 Rejection of noise hit

In order to eliminate the noise contribution to the hit PMTs, the distance and time difference from other hit PMTs in the event are used. The conditions for noise hit PMTs are given as follows:

- The minimum distance to any other hit PMT is larger than 1250 cm.
- The minimum time difference to any other hit PMT is larger than 35 nsec.

For further removal of noise hit PMTs, the selected hit PMTs are paired with each other, again. If the PMT signals are caused by Cherenkov light emitted from a point, then the time difference ($\Delta t$) and the distance ($\Delta r$) between a pair of hit PMTs satisfy the following relation as shown in Fig.6.14.

$$\Delta r/c > \Delta t \quad (6.4.1)$$

A hit PMT pair which satisfies the above relation is called a “related pair”. If the number of related pairs of a hit PMT is larger than 10, then the hit PMT is selected and used by the vertex fitter. If the distance from the new vertex to the nearest ID wall is smaller than 200 cm, the event is rejected. Fig.6.15 shows the distributions of the vertex reconstructed before and after rejecting noise hits.

6.4.2 Cherenkov ring image test

When there are additional $\gamma$-rays in an event, then the Cherenkov ring image is smeared and the direction reconstruction is not valid. To estimate the validity of the reconstructed Cherenkov ring, a likelihood function is made from a MC distribution of the angle between the reconstructed direction and the angle from reconstructed vertex to each hit PMT. Fig.6.16 shows the Cherenkov ring likelihood functions for several energy regions. The likelihood values are calculated for every hit PMTs which are used by the direction reconstruction. The product is the Cherenkov ring likelihood of the event. The likelihood distributions from a typical data sample are shown in Fig.6.17. The points with error bars are the likelihood distribution for events in data whose reconstructed directions are correlated with the solar direction (the inner products of those directions ($\cos\theta_{sun}$) is larger then 0.8). The histogram shows the likelihood distribution for events in data whose $\cos\theta_{sun}$ is smaller then 0.8. The cut threshold is defined to be -1.85.

6.5 $\gamma$-ray cut

One of the major backgrounds in the solar neutrino analysis is $\gamma$-rays from the PMT glass and the surrounding rock. To remove this background, the reconstructed direction of each
Figure 6.14: A schematic view of the condition used to reject noise hits.

Figure 6.15: The distribution of vertex reconstructed before and after rejecting noise hits. The dashed histograms show the original reconstructed vertex distribution.
Figure 6.16: The distribution of the likelihood functions for the ring image test.

Figure 6.17: The likelihood distribution of the Cherenkov ring image test from a typical data sample.
event is projected backward, and the distance from the reconstructed vertex to the detector wall \((d_{\text{eff}})\) is measured. The definition of \(d_{\text{eff}}\) is shown in Fig.6.18. If the distance is less than the threshold, the event is cut.

The \(\gamma\)-ray background is the largest source in the lower energy region. Therefore the threshold distance depends on energy. It is 8 m when the energy is less than 6.5 MeV and 4.5 m when the energy is greater than 6.5 MeV.

Fig.6.19 shows the vertex distributions from a typical data sample with energy between 6.5 to 20 MeV. The large peak near the ID wall disappears after this cut. The dead time introduced by this cut in the energy region 6.5 to 20 MeV is estimated to be 6.8%.

Fig.6.20 shows the vertex distributions from a typical data sample with energy between 5.0 to 6.5 MeV. Just as above, the large peak near the ID wall disappears after this cut. The peak at \(R^2 = 0\) in the \(R^2\) distribution of Fig.6.20 is caused by Rn contained in the input water. For the same reason, the peak in the Z-distribution appears. The dead time introduced by this cut in the energy region 5.0 to 6.5 MeV is estimated to be 20.9%.

### 6.6 Summary

The following table shows the number of events remaining after each reduction step for both data and MC. The number of events with energy between 5.0 MeV and 20 MeV after each reduction is summarized in this table. Fig.6.21 shows energy spectrum at each reduction step. The number of events after all cuts is 200,016 in data. Possible sources of the remaining background are electrons from the decay of \(^{222}\text{Rn}\) and \(\gamma\)-rays from the rock surrounding the SK tank and the materials which comprise the ID wall. Another possible source may be spallation products which have long lifetimes, such as \(^{16}\text{N}\).
Figure 6.19: The effect of the γ-ray cut in a typical data sample with $E = 6.5 \sim 20.0$ MeV. The upper figures are the vertex distributions and the lower ones are the direction cosines along the $x$ and $z$ directions. The hatched histograms show the distributions after the γ-ray cut.
Figure 6.20: The effect of the \( \gamma \)-ray cut in a typical data sample with \( E = 5.0 \sim 6.5 \) MeV. The upper figures are the vertex distributions and the lower ones are the direction cosines along the x and z directions. The hatched histograms show the distributions after the \( \gamma \)-ray cut.
<table>
<thead>
<tr>
<th>Reduction step</th>
<th>The number of remaining events</th>
<th>Data</th>
<th>MC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First reduction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total charge ≤ 1000 p.e.</td>
<td>114134159</td>
<td>4701645</td>
<td></td>
</tr>
<tr>
<td>Fiducial volume cut</td>
<td>21870014</td>
<td>3507003</td>
<td></td>
</tr>
<tr>
<td>Time to previous event &gt; 50 μsec</td>
<td>16541753</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OD trigger cut</td>
<td>15400842</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noise event cut</td>
<td>15353262</td>
<td>3506720</td>
<td></td>
</tr>
<tr>
<td>Flasher PMT cut</td>
<td>15201925</td>
<td>3472139</td>
<td></td>
</tr>
<tr>
<td>Goodness</td>
<td>14970860</td>
<td>3472134</td>
<td></td>
</tr>
<tr>
<td>Second flasher cut</td>
<td>10995187</td>
<td>3457795</td>
<td></td>
</tr>
<tr>
<td>Spallation cut</td>
<td>7624823</td>
<td>21.1% dead time</td>
<td></td>
</tr>
<tr>
<td><strong>Second reduction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Goodness stability</td>
<td>2006600</td>
<td>11.3% dead time</td>
<td></td>
</tr>
<tr>
<td>Noise hit rejection</td>
<td>1451519</td>
<td>2.0% dead time</td>
<td></td>
</tr>
<tr>
<td>Ring image cut</td>
<td>1089860</td>
<td>13.1% dead time</td>
<td></td>
</tr>
<tr>
<td>γ-ray cut</td>
<td>200016</td>
<td>11.2% dead time</td>
<td></td>
</tr>
<tr>
<td>Final sample (5.0 &lt; E &lt; 20 MeV)</td>
<td>200016</td>
<td>1805853</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6.21: The energy spectrum after each reduction step.
Chapter 7

Results

Solar neutrino events are detected by recoil electrons via $\nu$-e scattering. The strong angular correlation between the incident neutrino direction and the scattered electron direction is used to extract the solar neutrino flux.

Fig.7.1 shows the distribution of $\cos \theta_{\text{sun}}$ for all events passing the reduction cuts described in Chapter 7. $\theta_{\text{sun}}$ is defined as the angle between the reconstructed momentum and the radius vector from the sun. The energy range of the events is $5.0 \sim 20 \text{ MeV}$. The filled circles show data, and the histogram shows MC normalized to data. The dotted line shows the background shape which is used in extracting the solar neutrino signal.

A clear peak due to solar neutrinos is seen, as described above. The flat component in the $\cos \theta_{\text{sun}}$ distribution is due to radioactivity in the water, radioactive spallation products and $\gamma$-rays from the surrounding rock and materials comprising the ID wall.

In Sec.7.1, the method used to extract the solar neutrino signal is described. The flux measurement is described in Sec.7.2. The observed recoil electron energy spectrum is described in Sec.7.3.

7.1 Solar neutrino signal extraction

In order to obtain the measured flux relative to the SSM, a maximum likelihood method is used. The probability function for the likelihood consists of signal and background components:

$$P(E_e, \cos \theta_{\text{sun}}, x) = P_{\text{bg}} \times (1 - x) + P_{\text{sig}} \times x,$$ \hspace{1cm} (7.1.1)

where $E_e$ is the recoil electron energy and $x$ is the fraction of solar neutrino signals relative to the observed total number of events.

The probability function for the signal ($P_{\text{sig}}$) is obtained by MC simulation at various energy regions. Fig.7.2 shows distributions of $P_{\text{sig}}$ at 5, 7, 10, 15 MeV.
Figure 7.1: The $\cos \theta_{\text{sun}}$ distribution of all events ($E = 5.0 \sim 20$ MeV).

Figure 7.2: The probability density functions for signal, $P_{\text{sig}}$, at various energies.
Figure 7.3: The probability density function for background, $P_{bg}$, at various energies.

The background shape is almost flat. But, asymmetries in the $\gamma$-ray background from the surrounding rocks and PMT gain variations may slightly distort the background shape. Small deviations from a flat background shape are taken into account by simulating the background distribution. In order to obtain the background shape, the zenith angle ($\cos \theta_{sun}$) distribution of the events is fit with an 8-th degree polynomial. The background probability function ($P_{bg}$) is obtained by transforming this fit function into a function of $\cos \theta_{sun}$. Here, it is assumed that the background shape does not depend on the azimuth angle. Possible azimuthal dependence of the background shape will be discussed in Sec.7.4. Fig.7.3 presents distributions of $P_{bg}$ at 5, 7, 10, 15 MeV.

The fraction of signal $x$ differs between energy regions. In order to get this fraction at various energy regions separately, the probability function is defined as follows:

$$P(E_c, \cos \theta_{sun}, x_i(x)) = P_{bg}(E_c, \cos \theta_{sun}) \times (1 - x_i(x)) + P_{sig}(E_c, \cos \theta_{sun}) \times x_i(x),$$

(7.1.2)

where

$$x_i(x) = \frac{N_i^{SSM} \cdot N_{data}}{N_{all}^{SSM} \cdot N_{data}} \times x.$$  

(7.1.3)

$i$ is the index of the energy region. $N_{all}$ and $N_i$ are the total number of event and the number of event in each energy region, respectively.

From this probability function, the likelihood function is defined as follows.

$$L(x) = \prod_{i=1}^{N_{res}} \prod_{j=1}^{N_i} P_{ij}(E_c, \cos \theta_{sun}, x_i(x)),$$

(7.1.4)
where $N_{ene}$ is the number of the energy regions and $N_i$ is the number of events in i-th energy region. In this analysis, the data are divided into 19 energy regions. From 5.0 MeV to 14.0 MeV, data are divided into regions of 0.5 MeV width. The last region is from 14.0 MeV to 20.0 MeV.

The likelihood distribution as a function of $x$ is presented in Fig.7.4. From $x_{\text{max}}$ which gives the maximum likelihood value, the number of solar neutrino signals is calculated as:

$$N_{\text{signal}} = x_{\text{max}} \times N_{\text{data}}.$$  \hfill (7.1.5)

The statistical error of $N_{\text{signal}}$ is the difference between $x_{\text{max}}$ and $x$ which gives 22% of maximum likelihood value ($L(x_{\text{error}}) = e^{-0.5} \times L(x_{\text{max}})$). The systematic errors are explained in Sec.7.4.

### 7.2 Summary of the flux measurement

In this section, the flux measurement is summarized. For the flux measurement, the events with reconstructed recoil electron energy between 5.5 MeV to 20 MeV is used. For events collected in the first 280 days of livetime, events with energy above 6.5 MeV are considered. This is because no SLE trigger existed then, so that the trigger efficiency was 100% efficient only above 6.5 MeV.

#### 7.2.1 The $^8$B neutrino flux

The observed number of recoil electron events is
<table>
<thead>
<tr>
<th>Start date</th>
<th>$E_{thr}$ (MeV)</th>
<th>live time (day)</th>
<th>$N_{MC}$</th>
<th>$N_{red}$ ($E_{thr}-20\text{MeV}$)</th>
<th>expected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 May 1996</td>
<td>6.5</td>
<td>250.0</td>
<td>4172080</td>
<td>342171</td>
<td>6609.5</td>
</tr>
<tr>
<td>31 May 1997</td>
<td>5.5</td>
<td>545.2</td>
<td>8121750</td>
<td>865999</td>
<td>16731.7</td>
</tr>
<tr>
<td>14 May 1999</td>
<td>5.5</td>
<td>101.7</td>
<td>1498470</td>
<td>159322</td>
<td>3113.2</td>
</tr>
<tr>
<td>17 Sep 1999</td>
<td>5.5</td>
<td>78.57</td>
<td>1152790</td>
<td>117938</td>
<td>2401.9</td>
</tr>
<tr>
<td>22 Dec 1999</td>
<td>5.5</td>
<td>111.1</td>
<td>1628070</td>
<td>173323</td>
<td>3404.4</td>
</tr>
</tbody>
</table>

Table 7.1: The SSM(BP98) prediction of the number of the solar neutrino events in each run period.

$$N_{signal} = 15009^{+169}_{-157} \text{ (stat.)}^{+405}_{-435} \text{ (syst.)}.$$  

The expected number of solar neutrino events, $N_{expect}$, is obtained as follows:

$$N_{expect} = \sum_{i=1}^{N_{thr}} \frac{N_{red,i}}{N_{MC,i}} \times T_{live,i} \times N_{SSM}, \quad (7.2.1)$$

where $N_{thr}$ is the number of trigger thresholds. As described in Sec.3.6.3, there are 5 different trigger thresholds. $N_{MC,i}$ is the number of solar neutrino MC events in i-th run period. $N_{red,i}$ is the number of solar neutrino MC events after the reduction steps in i-th run period. $T_{live,i}$ is the live time of i-th run period. $N_{SSM}$ is the SSM prediction of the number of solar neutrino recoil electrons in SK, which is 287.8 events/day as described in Sec.3.1.1. $N_{expect}$ in each run period is summarized in Tab.7.1. The expected number of events for the flux analysis is:

$$N_{expect,SSM_{BP98}} = 32,261 \text{ events.}$$

Therefore, the ratio $Data/SSM_{BP98}$ is obtained as

$$\frac{Data}{SSM_{BP98}} = 0.465 \pm 0.005 \text{(stat.)}^{+0.015}_{-0.013} \text{(syst.)}. \quad (7.2.2)$$

The systematic error due to the uncertainty in the SSM is estimated to be $\pm 0.161$. The solar neutrino flux $\Phi_{\nu}$ is obtained by multiplying this ratio with the SSM$_{BP98}$ flux prediction ($5.15 \times 10^6 \text{cm}^{-2}\text{s}^{-1}$).

$$\Phi_{\nu} = 2.40 \pm 0.03 \text{(stat.)}^{+0.08}_{-0.07} \text{(syst.)} \times 10^6 / \text{cm}^2 / \text{sec}. \quad (7.2.3)$$

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7.2.2 Day and night flux difference

SK can measure the $^{8}$B neutrino flux in real time, so one can compare the daytime flux with the nighttime flux. For certain values of $\Delta m^2$ and $\sin^2 2\theta$, these fluxes are different because of regeneration in the Earth.

The data sample is divided into daytime and nighttime samples defined by the zenith angle of the direction of the sun as shown in Fig. 7.5.

The daytime flux is found to be:

$$\Phi_{\nu}^{day} = 2.35 \pm 0.04 (\text{stat.})^{+0.08}_{-0.07} (\text{syst.}) \times 10^6 / \text{cm}^2 / \text{sec}. \quad (7.2.4)$$

The ratio of the measured flux to the SSM prediction is:

$$\frac{\text{Data}_{day}}{\text{SSM}_{BP08}} = 0.456 \pm 0.007 (\text{stat.})^{+0.016}_{-0.014} (\text{syst.}) \quad (7.2.5)$$

The total runtime for daytime is 544.6 days. The mean distance between the sun and the Earth is the 1.001 AU for the daytime data. The measured flux is corrected for the eccentricity of the Earth’s orbit at 1 AU.

For nighttime, the measured flux is:

$$\Phi_{\nu}^{night} = 2.43 \pm 0.04 (\text{stat.})^{+0.08}_{-0.07} (\text{syst.}) \times 10^6 / \text{cm}^2 / \text{sec} \quad (7.2.6)$$

The ratio of the measured flux to the SSM prediction is:

$$\frac{\text{Data}_{night}}{\text{SSM}_{BP08}} = 0.472 \pm 0.007 (\text{stat.})^{+0.016}_{-0.014} (\text{syst.}) \quad (7.2.7)$$

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Figure 7.6: The time variation of the ratio of the measured flux to the SSM prediction. Each time bin is defined in Fig.7.5.

<table>
<thead>
<tr>
<th>time bin</th>
<th>$\cos \theta_z$</th>
<th>observed flux ($\times 10^6$cm$^{-2}$s$^{-1}$)</th>
<th>Data/SSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>0.0 ~ −0.2</td>
<td>2.53$^{+0.09}_{-0.09}$</td>
<td>0.49$^{+0.02}_{-0.02}$</td>
</tr>
<tr>
<td>N2</td>
<td>−0.2 ~ −0.4</td>
<td>2.41$^{+0.08}_{-0.08}$</td>
<td>0.47$^{+0.02}_{-0.02}$</td>
</tr>
<tr>
<td>N3</td>
<td>−0.4 ~ −0.6</td>
<td>2.45$^{+0.07}_{-0.08}$</td>
<td>0.48$^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>N4</td>
<td>−0.6 ~ −0.8</td>
<td>2.41$^{+0.08}_{-0.08}$</td>
<td>0.47$^{+0.02}_{-0.02}$</td>
</tr>
<tr>
<td>N5</td>
<td>−0.8 ~ −1.0</td>
<td>2.34$^{+0.08}_{-0.08}$</td>
<td>0.45$^{+0.02}_{-0.02}$</td>
</tr>
</tbody>
</table>

Table 7.2: Measured fluxes and Data/SSM in each nighttime bin. Only statistics errors are shown here.
The total runtime for nighttime is 572.1 days. The mean distance between the sun and the Earth is 0.998 AU for the nighttime data.

The difference between the daytime flux and nighttime flux is calculated as follows:

$$\frac{R_{day} - R_{night}}{(R_{day} + R_{night})/2} = -0.034 \pm 0.022\text{(stat.)}^{\pm0.013}\text{(syst.)}, \quad (7.2.8)$$

where R is Data/SSM. The difference is at the 1.5 $\sigma$ level. Fig.7.6 shows the fluxes for daytime and nighttime. The nighttime period is divided into 5 bins. Numerical results for each nighttime bin are presented in Tab.7.2.

### 7.3 Recoil electron energy spectrum

Fig.7.7 and 7.8 presents the $\cos \theta_{sun}$ distributions in various energy bins. The filled circles show the data, the histograms show the result of the likelihood fit, and the dashed lines show the background shape used to extract the solar neutrino signals.

The width of each energy bin is as follows:

- $5.0 \sim 14.0$ : 0.5 MeV for each bin
- $14.0 \sim 20.0$ : combined into one bin

As described in Sec.2.1, $^8$B neutrino energy spectrum endpoint is about 15 MeV. The hep flux is predicted to be 0.04% of the expected $^8$B neutrino flux, although the end point energy is 18.77 MeV. Therefore, the number of solar neutrino events with recoil electron energy above 14 MeV is expected to be much smaller than that below 14 MeV. That is the reason for the large width of the last bin.

The observed energy spectrum of recoil electrons between 5.0 and 20 MeV is presented in Fig.7.9. In this figure, the detection efficiency for solar neutrino events is taken into account. The errors in this figure show the statistical and systematic error, combined in quadrature. Fig.7.10 shows the ratio of the measured recoil electron spectrum to the expected spectrum. The results are summarized numerically in Tab.7.3.

The recoil electron energy spectra for daytime and nighttime are presented in Fig.7.11. The width of each energy bin is the same as above. The results are summarized numerically in Tab.7.4. The definition of daytime and nighttime is presented in Fig.7.5.

The systematic errors are summarized in the next section.
Figure 7.7: The $\cos \theta_{\text{sun}}$ distribution in each energy bin from 5.0 to 10.0 MeV.
Figure 7.8: The $\cos \theta_{\text{sun}}$ distribution for each energy bin from 10.0 to 20.0 MeV. For reference, the $\cos \theta_{\text{sun}}$ distribution of data with energy of 20.0 $\sim$ 30.0 MeV is also shown. Above 20 MeV, there is no peak in the $\cos \theta_{\text{sun}}$ distribution.
<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Data/SSM</th>
<th>Statistical error</th>
<th>Number of signal events</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0 ~ 5.5</td>
<td>0.447</td>
<td>±0.004 ±0.050</td>
<td>1281.8 ±135.3</td>
</tr>
<tr>
<td>5.5 ~ 6.0</td>
<td>0.461</td>
<td>±0.027 ±0.027</td>
<td>1374.3 ±184.9</td>
</tr>
<tr>
<td>6.0 ~ 6.5</td>
<td>0.436</td>
<td>±0.020 ±0.020</td>
<td>1248.3 ±154.9</td>
</tr>
<tr>
<td>6.5 ~ 7.0</td>
<td>0.453</td>
<td>±0.016 ±0.016</td>
<td>1939.2 ±154.9</td>
</tr>
<tr>
<td>7.0 ~ 7.5</td>
<td>0.473</td>
<td>±0.016 ±0.016</td>
<td>1835.2 ±163.0</td>
</tr>
<tr>
<td>7.5 ~ 8.0</td>
<td>0.489</td>
<td>±0.017 ±0.017</td>
<td>1692.3 ±278.0</td>
</tr>
<tr>
<td>8.0 ~ 8.5</td>
<td>0.477</td>
<td>±0.017 ±0.017</td>
<td>1445.0 ±209.6</td>
</tr>
<tr>
<td>8.5 ~ 9.0</td>
<td>0.453</td>
<td>±0.018 ±0.018</td>
<td>1163.5 ±174.1</td>
</tr>
<tr>
<td>9.0 ~ 9.5</td>
<td>0.464</td>
<td>±0.019 ±0.019</td>
<td>993.3 ±144.1</td>
</tr>
<tr>
<td>9.5 ~ 10.0</td>
<td>0.471</td>
<td>±0.024 ±0.020</td>
<td>828.5 ±30.1</td>
</tr>
<tr>
<td>10.0 ~ 10.5</td>
<td>0.459</td>
<td>±0.021 ±0.021</td>
<td>639.6 ±29.8</td>
</tr>
<tr>
<td>10.5 ~ 11.0</td>
<td>0.451</td>
<td>±0.023 ±0.022</td>
<td>492.5 ±24.1</td>
</tr>
<tr>
<td>11.0 ~ 11.5</td>
<td>0.473</td>
<td>±0.025 ±0.026</td>
<td>391.1 ±21.1</td>
</tr>
<tr>
<td>11.5 ~ 12.0</td>
<td>0.455</td>
<td>±0.028 ±0.029</td>
<td>280.3 ±17.3</td>
</tr>
<tr>
<td>12.0 ~ 12.5</td>
<td>0.429</td>
<td>±0.032 ±0.031</td>
<td>190.2 ±13.8</td>
</tr>
<tr>
<td>12.5 ~ 13.0</td>
<td>0.488</td>
<td>±0.038 ±0.037</td>
<td>151.3 ±11.6</td>
</tr>
<tr>
<td>13.0 ~ 13.5</td>
<td>0.493</td>
<td>±0.040 ±0.038</td>
<td>104.1 ±9.6</td>
</tr>
<tr>
<td>13.5 ~ 14.0</td>
<td>0.585</td>
<td>±0.044 ±0.037</td>
<td>80.6 ±6.2</td>
</tr>
<tr>
<td>14.0 ~ 20.0</td>
<td>0.506</td>
<td>±0.049 ±0.048</td>
<td>109.6 ±10.4</td>
</tr>
</tbody>
</table>

Table 7.3: A numerical summary of the results of the energy spectrum analysis
Figure 7.9: The measured solar neutrino recoil electron energy spectrum. The expected spectrum from SSM is also presented. The thick error bar shows the statistical error. The total error that includes the systematic error is represented by the thin error bar.
Figure 7.10: The ratio of the measured recoil electron energy spectrum to the predicted spectrum. The definition of the error bars is the same as in Fig. 7.9.
Figure 7.11: The daytime and nighttime recoil electron energy spectra. The error bars represent the total error.
<table>
<thead>
<tr>
<th>energy (MeV)</th>
<th>daytime</th>
<th>nighttime</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0 ~ 5.5</td>
<td>0.478±0.064</td>
<td>0.417±0.064</td>
</tr>
<tr>
<td>5.5 ~ 6.0</td>
<td>0.456±0.037</td>
<td>0.466±0.036</td>
</tr>
<tr>
<td>6.0 ~ 6.5</td>
<td>0.415±0.028</td>
<td>0.455±0.028</td>
</tr>
<tr>
<td>6.5 ~ 7.0</td>
<td>0.448±0.022</td>
<td>0.457±0.022</td>
</tr>
<tr>
<td>7.0 ~ 7.5</td>
<td>0.476±0.023</td>
<td>0.468±0.023</td>
</tr>
<tr>
<td>7.5 ~ 8.0</td>
<td>0.500±0.023</td>
<td>0.479±0.023</td>
</tr>
<tr>
<td>8.0 ~ 8.5</td>
<td>0.460±0.024</td>
<td>0.491±0.024</td>
</tr>
<tr>
<td>8.5 ~ 9.0</td>
<td>0.409±0.026</td>
<td>0.491±0.025</td>
</tr>
<tr>
<td>9.0 ~ 9.5</td>
<td>0.470±0.027</td>
<td>0.455±0.025</td>
</tr>
<tr>
<td>9.5 ~ 10.0</td>
<td>0.464±0.028</td>
<td>0.476±0.025</td>
</tr>
<tr>
<td>10.0 ~ 10.5</td>
<td>0.433±0.031</td>
<td>0.479±0.030</td>
</tr>
<tr>
<td>10.5 ~ 11.0</td>
<td>0.408±0.030</td>
<td>0.487±0.032</td>
</tr>
<tr>
<td>11.0 ~ 11.5</td>
<td>0.449±0.036</td>
<td>0.494±0.036</td>
</tr>
<tr>
<td>11.5 ~ 12.0</td>
<td>0.498±0.041</td>
<td>0.408±0.037</td>
</tr>
<tr>
<td>12.0 ~ 12.5</td>
<td>0.406±0.045</td>
<td>0.447±0.043</td>
</tr>
<tr>
<td>12.5 ~ 13.0</td>
<td>0.510±0.056</td>
<td>0.450±0.049</td>
</tr>
<tr>
<td>13.0 ~ 13.5</td>
<td>0.571±0.070</td>
<td>0.421±0.056</td>
</tr>
<tr>
<td>13.5 ~ 14.0</td>
<td>0.541±0.066</td>
<td>0.579±0.063</td>
</tr>
<tr>
<td>14.0 ~ 20.0</td>
<td>0.436±0.066</td>
<td>0.573±0.067</td>
</tr>
</tbody>
</table>

Table 7.4: A numerical summary of the daytime and nighttime recoil electron energy spectra. The error is statistical only.
7.4 Systematic errors

Systematic errors in the energy spectrum measurement are classified into two categories. The first are errors that are correlated with energy, and they are called correlated errors from now on. The other are errors that are not correlated with energy, and they are called uncorrelated errors. In this section, the correlated errors and the uncorrelated errors are described separately.

7.4.1 Correlated errors

Uncertainties in the absolute energy scale and energy resolution directly affect the shape of the observed energy spectrum. Moreover, the effect in each energy bin is interdependent. So the correlation between each energy bin should be considered in evaluating the systematic error of each bin.

The uncertainty in the energy scale and energy resolution are described in Sec.5.4. They are summarized here:

- Position dependence of the energy scale deviation, (DATA-MC)/MC, is measured to be within ±0.5% by LINAC calibration. For the energy resolution, it is within ±2.0%.

- Energy dependence of the energy scale deviation is also studied using LINAC calibration data and it is obtained to be within ±0.5%. That of energy resolution is obtained to be within ±2.0%.

- The systematic uncertainty due to the LINAC system itself is 0.33% for electron energy of 8.9 MeV. This error is summarized in Tab.5.4

- Directional dependence of the energy scale deviation measured using spallation data is found to be within ±0.5%

- The uncertainty due to the water transparency measurement is ±0.22%.

The effects of above uncertainties are evaluated as follows. In order to simulate the observed energy spectrum \( f(E_{\text{obs}}) \) given the original spectrum \( f_0(E_e) \), the following function is defined:

\[
f(E_{\text{obs}}) = \int_0^\infty f_0(E_e) R(E_{\text{obs}}, E_e) P(E_e) dE_e, \tag{7.4.1}
\]

where \( R(E_{\text{obs}}, E_e) \) is a function to simulate the energy resolution, and \( P(E_e) \) is a function for the detection efficiency.

By varying \( R(E_{\text{obs}}, E_e) \) and \( P(E_e) \), one can simulate the expected results that include the uncertainties. Then the effects of the uncertainties can be estimated by comparing the
simulated results with the observed results. The effect of the theoretical uncertainty in the
$^8$B neutrino energy spectrum is estimated in the same way. The estimated systematic error
in each energy bin is summarized in Tab.7.5.

The uncertainty in the energy scale, energy resolution and $^8$B neutrino energy spectrum
are also considered in the flux measurement. For the flux measurement, the systematic
error is estimated to be $^{+1.8\%}_{-1.7\%}$. For the Day/Night flux difference, the systematic error is
estimated to be $^{+1.2\%}_{-1.1\%}$.

7.4.2 Uncorrelated errors

Trigger efficiency The trigger efficiency depends on the vertex position and water trans-
parency. These dependences are considered in the MC simulation. The systematic error
due to the trigger efficiency is estimated by comparing the measured trigger efficiency with
the MC trigger simulation.

For the LE trigger, the efficiency is 100% above 7.0 MeV. In the energy region from the
6.5 MeV to 7.0 MeV, the maximum deviation between the data and MC is obtained to be
$^{+0.4\%}_{-1.7\%}$.

For the SLE trigger, the trigger efficiency varies with the hardware threshold change.
The trigger efficiency is measured for every SLE trigger threshold. The deviation between
the data and MC is obtained by taking the livetime-weighted average of the deviations in
each period. The deviations in each energy bin are obtained as follows : $5.0 \sim 5.5$ MeV,
$^{+3.2\%}_{-1.8\%}$, $5.5 \sim 6.0$ MeV, $^{+0.9\%}_{-0.5\%}$. The trigger efficiency is 100% above 6.0 MeV in both of data
and MC, so the systematic error is zero.

The systematic error due to the IT is studied using the Ni-Cf source. The deviation
between the reduction efficiency of the IT in data and that for Ni-Cf MC simulation is
calculated at various positions in the ID. The volume average of those deviations in each
energy bin is obtained as follows : $5.0 \sim 5.5$ MeV, $\pm 2.8\%$, $5.5 \sim 6.0$ MeV, $\pm 0.7\%$, and
$6.0 \sim 6.5$ MeV, $\pm 0.3\%$. Events with energy above 6.5 MeV, the LE trigger efficiency is
almost 100%, and the IT is applied to data issuing only SLE trigger, which mostly have
energy below 6.5 MeV. Therefore, the systematic error due to IT is zero above 6.5 MeV.

Reduction The systematic error due to reduction comes from the reduction efficiency
difference between data and MC simulation. Cuts which provide relatively large systematic
errors are presented as follows :

- The reduction efficiency difference of the flasher cut is obtained by comparing the
  reduction efficiency in spallation event data with that in $^8$B MC. The deviation,
  $(\text{DATA-MC})/\text{MC}$, is less than $\pm 1\%$.

- The vertex test reduction efficiency difference is obtained using Ni-Cf calibration data
  at various position and Ni-Cf MC at each corresponding position. In order to obtain
the energy dependence of the difference, a position average is calculated. Differences are less than ±1% in all energy regions.

- The systematic error from the vertex test by noise hit reduction is estimated using Ni-Cf calibration data at various positions. The reduction efficiency difference between data and the Ni-Cf simulation is obtained to be 0.7%.

- The systematic error caused by the Cherenkov ring image test is estimated using spallation products with short lifetime (≈ 30 msec). The data set is used because some of long-lifetime spallation products emit additional γ-rays as shown in Tab.6.1. The reduction efficiency for the spallation events is compared with that in $^8$B MC simulation. Deviations, (MC - DATA)/MC, between 5.5 MeV and 20.0 MeV are within ±1.0%.

The total systematic error of all reduction steps is $\pm 2.1\%$ for the flux measurement and ±0.85% for the energy spectrum measurement.

**Spallation dead time** The position dependence of the spallation deadtime is presented in Fig.6.11, and it is used to simulate the spallation cut in MC simulation. The systematic error due to this position dependence of the dead time is estimated to be 0.05% by comparing the deadtime calculated in MC with the deadtime calculated from data. For the Day/Night flux difference, the systematic error is estimated to be ±0.1%.

The time variation of spallation deadtime is found to be less than 0.8%.

**γ-ray cut** The γ-ray cut uses the reconstructed vertex and direction in events. So the differences of vertex and angular resolution between data and MC simulation can introduce systematic errors. To estimate the influence of these differences, the following method is adopted.

1. Shift the reconstructed vertices and direction of events within the differences in the vertex and angular resolution between data and MC.

2. Apply the γ-ray cut to the modified data and compare the reduction efficiency with the result of original data.

The differences in the vertex and angular resolution are measured with LINAC as described in Sec.5.4.1.4.

The systematic error for the flux measurement is ±0.05% and that for the energy spectrum measurement is ±0.1%.
**Vertex shift**  Systematic shifts in the reconstructed vertex relative to the original vertex is studied with Ni-Cf calibration at various positions. The estimated systematic error is 1.3% for the flux measurement and ±0.2% for the energy spectrum measurement.

**Non-flat BG**  In order to obtain the background shape used in the solar neutrino signal extraction, it is assumed that the background is independent of azimuth angle $\phi$. But, the $\phi$ asymmetry in the energy scale and background sources with possible asymmetric $\phi$ distribution may introduce deviations from flatness in the background $\cos \theta_{\text{sun}}$ distribution. So the difference between the originally measured signal events and that obtained using the background shape considering energy scale $\phi$ asymmetry is treated as the systematic error. The systematic error is ±0.1% for the flux measurement. The background shape depends on energy, so the systematic error also depends on energy. The systematic error in each energy bin is as follows: 5.0 ~ 5.5 MeV, ±1.2%, 5.5 ~ 6.0 MeV, ±0.4%, and above 6.0 MeV, ±0.15%. For the Day/Night flux difference, the systematic error is estimated to be ±0.4%.

**Angular resolution**  The measured angular resolution in MC simulation is slightly worse than in data as shown in Fig.5.28. In solar neutrino signal extraction, the MC angular resolution is corrected so that it agrees with data. The systematic error due to angular resolution is the difference between the number of signal events obtained using the MC angular resolution and that obtained using the corrected angular resolution. The estimated systematic error is ±0.6% for the flux measurement and ±1.0% for the spectrum measurement.

**Livetime**  The systematic error due to livetime calculation is evaluated from the difference between calculations using several different data samples (raw data, muon data or low energy triggered data). The systematic error for flux measurement is evaluated to be ±0.1%. For the Day/Night flux difference, the systematic error is estimated to be ±0.1%.
<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Uncorrelated error (%)</th>
<th>Correlated error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0 ~ 5.5</td>
<td>+4.6 - 3.7</td>
<td>+0.3 - 0.1</td>
</tr>
<tr>
<td>5.5 ~ 6.0</td>
<td>+1.8 - 1.6</td>
<td>+0.3 - 0.2</td>
</tr>
<tr>
<td>6.0 ~ 6.5</td>
<td>+1.4 - 1.4</td>
<td>+0.4 - 0.3</td>
</tr>
<tr>
<td>6.5 ~ 7.0</td>
<td>+1.4 - 1.4</td>
<td>+0.6 - 0.5</td>
</tr>
<tr>
<td>7.0 ~ 7.5</td>
<td>+1.4 - 1.4</td>
<td>+0.9 - 0.8</td>
</tr>
<tr>
<td>7.5 ~ 8.0</td>
<td>+1.4 - 1.4</td>
<td>+1.2 - 1.1</td>
</tr>
<tr>
<td>8.0 ~ 8.5</td>
<td>+1.4 - 1.4</td>
<td>+1.5 - 1.4</td>
</tr>
<tr>
<td>8.5 ~ 9.0</td>
<td>+1.4 - 1.4</td>
<td>+1.9 - 1.8</td>
</tr>
<tr>
<td>9.0 ~ 9.5</td>
<td>+1.4 - 1.4</td>
<td>+2.3 - 2.1</td>
</tr>
<tr>
<td>9.5 ~ 10.0</td>
<td>+1.4 - 1.4</td>
<td>+2.7 - 2.6</td>
</tr>
<tr>
<td>10.0 ~ 10.5</td>
<td>+1.4 - 1.4</td>
<td>+3.2 - 3.0</td>
</tr>
<tr>
<td>10.5 ~ 11.0</td>
<td>+1.4 - 1.4</td>
<td>+3.7 - 3.5</td>
</tr>
<tr>
<td>11.0 ~ 11.5</td>
<td>+1.4 - 1.4</td>
<td>+4.2 - 4.0</td>
</tr>
<tr>
<td>11.5 ~ 12.0</td>
<td>+1.4 - 1.4</td>
<td>+4.8 - 4.5</td>
</tr>
<tr>
<td>12.0 ~ 12.5</td>
<td>+1.4 - 1.4</td>
<td>+5.4 - 5.1</td>
</tr>
<tr>
<td>12.5 ~ 13.0</td>
<td>+1.4 - 1.4</td>
<td>+6.2 - 5.8</td>
</tr>
<tr>
<td>13.0 ~ 13.5</td>
<td>+1.4 - 1.4</td>
<td>+7.1 - 6.5</td>
</tr>
<tr>
<td>13.5 ~ 14.0</td>
<td>+1.4 - 1.4</td>
<td>+8.3 - 7.2</td>
</tr>
<tr>
<td>14.0 ~ 20.0</td>
<td>+1.4 - 1.4</td>
<td>+12.3 - 9.6</td>
</tr>
</tbody>
</table>

Table 7.5: Summary of the systematic errors in each energy bin.
Chapter 8

Discussion

In this chapter, constraints on neutrino oscillation parameters based on the results presented in Chapter 7 are discussed. First, the total solar neutrino rates as measured by the Chlorine experiment, Gallium experiments, and SK are used to obtain the allowed region for the neutrino mass difference and the mixing angle. After that, constraints on solar neutrino oscillation parameters obtained from the day and night recoil electron energy spectra are discussed.

8.1 Constraints on neutrino oscillations from the flux measurements

The observed flux in SK is 46.5% of the SSM prediction, as described in Chapter 7. The Chlorine experiment and Gallium experiments, which are sensitive to lower energy neutrino, show flux deficits as well (See Chapter2). As described in the same chapter, the most likely reason for the deficit is neutrino oscillations. Throughout this chapter, two-flavor neutrino oscillations are considered. In this section, the constraints on the neutrino oscillation parameters using flux measurements are discussed.

As described in Sec.7.2, the ratio of the observed solar neutrino flux in SK to the prediction by the SSM(BP98) is as follows.

\[ 0.465^{+0.005}_{-0.005} \text{ (stat.)}^{+0.015}_{-0.013} \text{ (syst.)} \]  \hspace{1cm} (8.1.1)

In Chlorine experiment (Homestake), the observed solar neutrino capture rate is

\[ 2.56 \pm 0.16 \text{ (stat.)} \pm 0.16 \text{ (syst.)} \text{ SNU}; \] \hspace{1cm} (8.1.2)

whereas the expected rate is 7.7 SNU.
Figure 8.1: The 95% C.L. allowed regions which satisfy all three flux measurements: Chlorine, Gallium and SK.
<table>
<thead>
<tr>
<th>Region</th>
<th>Parameter range</th>
<th>Number of divisions</th>
</tr>
</thead>
</table>
| MSW region        | $\Delta m^2 = 10^{-8} \sim 10^{-3}$ eV$^2$  
                      $\sin^2 2\theta = 10^{-1} \sim 1$ | 250                 |
| Vacuum region     | $\Delta m^2 = 10^{-11} \sim 10^{-9}$ eV$^2$  
                      $\sin^2 2\theta = 0 \sim 1$ | 790                 |

Table 8.1: The definition of regions in which neutrino oscillations is studied. $\chi^2$ is calculated only at hundreds of grid points. The product of the number of divisions of $\Delta m^2$ and $\sin^2 2\theta$ corresponds to the number of grid point.

In Gallium experiments (GALLEX and SAGE), the weighted average of observed solar neutrino capture rate is

$$72.1^{+5.4}_{-5.0} \text{ SNU}, \quad (8.1.3)$$

where the expected rate is 129 SNU.

Oscillation analyses are carried out in two regions. One is $\Delta m^2 = 10^{-8} \sim 10^{-3}$ eV$^2$ and $\sin^2 2\theta = 10^{-1} \sim 1$, which is called the “MSW region”. The other is $\Delta m^2 = 10^{-11} \sim 10^{-9}$ eV$^2$ and $\sin^2 2\theta = 0 \sim 1$, which is called the “Vacuum region”. Information on these regions is summarized in Tab.8.1.

In order to fit the observed flux from the three experiments, a standard $\chi^2$ method is adopted. The definition of $\chi^2$ is as follows :

$$\chi^2 = \sum_{i,j=1}^{N_{exp}} (R_{i}^{\text{data}} - R_{i}^{\text{osc}}) V_{ij}^{-1} (R_{j}^{\text{data}} - R_{j}^{\text{osc}}) \quad (8.1.4)$$

$N_{exp}$ is the number of flux measurements, which is 3 in this analysis. $R_{i}^{\text{data}}$ is the ratio of the measured solar neutrino flux to the SSM predicted flux for each experiment. $R_{i}^{\text{osc}}$ is the ratio of the solar neutrino flux calculated assuming neutrino oscillations to the SSM prediction. $V_{ij}$ is the error matrix which includes errors from the theoretical uncertainty in the solar neutrino flux. The definition in $V_{ij}$ is as follows :

$$V_{ij} = \sigma_{i}^{\text{exp}}^2 + \sigma_{i}^{\text{ssm}}^2 + \sigma_{ij}^{\text{exp}}^2, \quad (8.1.5)$$

where $\sigma_{ij}^{\text{exp}}$ is an error matrix which includes the uncertainty in the neutrino cross sections, $\sigma_{ij}^{\text{ssm}}$ is an error matrix for the uncertainty in the SSM. The uncertainties in the SSM are as follows [2] :

- Uncertainty in the interaction cross section for reactions in the PP-chain and the CNO-cycle.
• Uncertainty in the solar luminosity.
• Uncertainty in the abundances of heavy elements.
• Uncertainty in the radiative opacity of the sun.
• Uncertainty in the element diffusion in the sun.

$\sigma_{ij}^{\exp}$ is an error matrix for each experiment.

The global minimum of $\chi^2$ is 0.413 at $(\sin^2 2\theta = 0.99, \Delta m^2 = 1.02 \times 10^{-10} \text{ eV}^2)$, which is in the Vacuum region. Fig.8.1 shows the iso-$\chi^2$ contours of $\chi^2 - \chi^2_{\text{global min}} = 5.991$ corresponding to an allowed region at 95% C.L. The local minimum in the MSW region is $\chi^2 = 0.825$ at $(\sin^2 2\theta = 6.31 \times 10^{-3}, \Delta m^2 = 5.01 \times 10^{-6} \text{ eV}^2)$. As shown in Fig.8.1, there are several allowed regions in both regions. The notation for these allowed regions are given in Chapter 2.

8.2 Constraints from the day and night spectra

SK is a real-time detector, so we can measure the solar neutrino energy spectrum in daytime and nighttime separately. The separate measurement of daytime and nighttime energy spectra has the following merits. First, using possible distortion in the recoil electron energy spectrum, one can perform a solar model independent analysis of the neutrino oscillation parameters. Second, using possible differences between the daytime and nighttime recoil electron energy spectra, one can examine neutrino regeneration by matter-enhanced neutrino oscillations in the Earth.

Fig.8.2 shows the recoil electron energy spectrum expected for each allowed region described above. For the SMA solution, $(\sin^2 2\theta = 6.3 \times 10^{-3}, \Delta m^2 = 5.0 \times 10^{-6} \text{ eV}^2)$, the energy spectrum normalized to SSM increases with energy. For the LMA solution, $(\sin^2 2\theta = 0.71, \Delta m^2 = 4.0 \times 10^{-5} \text{ eV}^2)$, and the LOW solution, $(\sin^2 2\theta = 0.79, \Delta m^2 = 1.0 \times 10^{-7} \text{ eV}^2)$, the expected energy spectrum is almost flat. However, for the LMA solution, the overall normalization changes.

In the analysis using the daytime and nighttime recoil electron energy spectra, $\chi^2$ is defined as follows.

$$\chi^2 = \sum_{i=1}^{N_{\text{bin}}} \sum_{j=1}^{N_{\text{ene}}} \left( \frac{R_{ij}^{\text{data}} - \alpha \times R_{ij}^{\text{osc}} / \bar{f}_{j}^{\text{corr}}(\epsilon_{s}, \epsilon_{r}, \epsilon_{b})}{\sigma_{i}} \right)^2 + \epsilon_{s}^2 + \epsilon_{r}^2 + \epsilon_{b}^2 \quad (8.2.1)$$

where $N_{\text{bin}}$ is the number of day and night bins, which is 2 in this analysis. $N_{\text{ene}}$ is the number of energy bins, which is 19 in this analysis. $R_{ij}^{\text{data}}$ is the ratio of the observed flux in
Figure 8.2: Daytime and nighttime recoil electron energy spectrum with the expected energy spectrum for each allowed region.

bin \((i, j)\) to the SSM prediction. \(R_{ij}^{\text{osc}}\) is the ratio of the flux calculated assuming neutrino oscillations to the SSM prediction. \(\alpha\) is a free parameter that serves as the absolute flux normalization. \(f_{i}^{\text{corr}}\) is the response function for correlated errors in the \(i\)-th energy bin. As described in Sec.7.4, the correlated errors include that from the uncertainty in the energy scale, the energy resolution and the \(^8\text{B}\) spectrum. The definition of \(f_{i}^{\text{corr}}\) is as follows:

\[
f_i = (1 + s_i \epsilon_s)(1 + r_i \epsilon_r)(1 + b_i \epsilon_b),
\]

(8.2.2)

Here, \(s_i, r_i,\) and \(b_i\) are 1\(\sigma\) errors of the absolute energy scale, the energy resolution, and the expected solar neutrino spectrum shape. \(\epsilon_s, \epsilon_r, \epsilon_b\) are free parameters used in constraining the variation of the correlated systematic errors. \(\sigma_i\) is the quadrature sum of uncorrelated errors and statistical error of the \(i\)-th energy bin.

Fig.8.3 shows the excluded region at 95\%C.L. in the \(\Delta m^2\) and \(\sin^2 2\theta\) plane using the daytime and nighttime recoil electron energy spectra. The minimum \(\chi^2\) is 32.1 at \((\sin^2 2\theta = 0.47, \Delta m^2 = 4.47 \times 10^{-10} \text{ eV}^2)\), \(\alpha\) is \(0.582^{+0.08}_{-0.12}\) at the minimum of \(\chi^2\). The \(\chi^2\)-minimum point is shown in Fig.8.3 as a black star.

From this analysis, the VAC solution of the global analysis is excluded at 95\% C.L. and most of the SMA region is also excluded. For the LMA solution, the lower half of the allowed region is excluded. The LOW solution, the upper half of the LMA solution and a small part of the SMA solution are still allowed.

Since the SSM prediction of the hep neutrino flux has large uncertainty, as described in Chapter 2, the hep contribution to the recoil electron spectrum can be treated as an
Figure 8.3: The 95% C.L. excluded regions of $\Delta m^2$ and $\sin^2 2\theta$ obtained by the analysis using the daytime and nighttime recoil electron energy spectra. The black star shows the minimum $\chi^2$ point.
additional free parameter. For the free Hep analysis, Eq. (8.2) is changed as follows.

\[ \alpha \times R_{ij}^{osc} \rightarrow \alpha_{BS} \times R_{ij}^{osc}(8B) + \alpha_{Hep} \times R_{ij}^{osc}(Hep) \quad (8.2.3) \]

Here, \( \alpha_{Hep} \) is a free parameter.

From this analysis, the exclude regions are obtained as shown in Fig. 8.4. The minimum of \( \chi^2 \) is 31.6, \( (\sin^2 2\theta = 1.0, \Delta m^2 = 5.01 \times 10^{-5} \text{ eV}^2) \). It is shown in Fig. 8.4 as a black star. At the \( \chi^2 \) minimum point, \( \alpha_{BS} = 0.781, \alpha_{Hep} = 10.18 \). This result is almost the same as the standard Hep case, although the remaining allowed region of SMA solution becomes slightly smaller.

### 8.3 Flux-constrained oscillation analysis

In the previous section, the absolute flux normalization factor is treated as a free parameter. Here, an oscillation analysis is performed in which the SSM flux is constrained. The uncertainty in \( \alpha \) comes from the uncertainty in the 8B neutrino flux \( +19\% \) as described in Chapter 2. In this analysis, \( \chi^2 \) is defined as follows.

\[
\chi^2 = \sum_{i=1}^{N_{\text{data}}} \sum_{j=1}^{N_{\text{exp}}} \left( \frac{R_{ij}^{\text{data}} - \alpha \times R_{ij}^{osc}/f_i^{\text{corr}}(\epsilon_{\nu}, \epsilon_{\bar{\nu}})}{\sigma_i} \right)^2 \quad (8.3.1)
\]

where \( \epsilon_{\alpha} \) is the error due to the uncertainty in the 8B neutrino flux. Fig. 8.5 shows the the allowed region at 95\%C.L. on the \( \Delta m^2 \) and \( \sin^2 2\theta \) plane using the daytime and nighttime recoil electron energy spectra. The minimum \( \chi^2 \) is 33.1 at \( (\sin^2 2\theta = 0.89, \Delta m^2 = 1.1 \times 10^{-7} \text{ eV}^2) \). As shown in the figure, all allowed regions have large mixing angles \( (\sin^2 2\theta > 0.65) \). This is one of the most important results obtained in this thesis because it is obtained purely from SK results. The allowed regions cover the upper half of the LMA solution and the LOW solution. Although an allowed region exists in the Vacuum region, it does not overlap with the allowed regions of the flux analysis.

### 8.4 Combined contour

Using the measured flux from the Chlorine experiment, Gallium experiments, and SK and the daytime and nighttime recoil electron energy spectra, the allowed regions are obtained. The definition of \( \chi^2 \) is given as follows,

\[
\chi^2 = \sum_{i,j=1}^{N_{\text{exp}}} (R_{ij}^{\text{data}} - R_{ij}^{osc}) V_{ij} (R_{ij}^{\text{data}} - R_{ij}^{osc}) \quad (8.4.1)
\]
Figure 8.4: The 95% C.L. excluded regions of $\Delta m^2$ and $\sin^2 2\theta$ obtained by the analysis using the daytime and nighttime recoil electron energy spectra with free Hep flux. The black star shows the minimum $\chi^2$ point.
Figure 8.5: The 95% C.L. allowed regions of $\Delta m^2$ and $\sin^2 2\theta$ obtained in the analysis using the daytime and nighttime recoil electron energy spectra with constrained $^8$B neutrino flux. The black star shows the minimum $\chi^2$ point.
\[
\sum_{i=1}^{N_{db}} \sum_{j=1}^{N_{ext}} \left( \frac{R_{ij}^{\text{data}} - \alpha \times R_{ij}^{\text{osc}} / f_{ij}^{\text{corr}}(\epsilon_a, \epsilon_r, \epsilon_b)}{\sigma_i} \right)^2 + \epsilon_a^2 + \epsilon_r^2 + \epsilon_b^2. \tag{8.4.2}
\]

The minimum of $\chi^2$ is 37.8 at $(\sin^2 2\theta = 0.79, \Delta m^2 = 3.9 \times 10^{-5} \text{ eV}^2)$. It is shown in Fig.8.6 as a black star. Fig.8.6 shows the iso-$\chi^2$ contours at 68%, 90%, 95%, and 99% C.L. As seen in the figure, the SMA solution and VAC solutions remain only above $\sim 90\%$ C.L.

### 8.5 Summary

In this chapter, constraints on neutrino oscillation parameters were obtained using the daytime and nighttime recoil electron energy spectra.

First, a neutrino oscillation analysis using flux measurements was introduced. This analysis gives several allowed regions, which are called the SMA solution, the LMA solution, the LOW solution and the VAC solution.

After that, analyses using the daytime and nighttime recoil electron energy spectra are performed. An analysis in which the absolute normalization of the solar neutrino flux is treated as a free parameter provides several excluded regions. From this analysis, the VAC solution is excluded at 95% C.L., and most of the SMA solution and the lower half of the LMA solution are also excluded.

When the Hep flux, which has large uncertainty, is treated as a free parameter, the result is almost same as the result of the analysis using the standard Hep neutrino flux.

By constraining the absolute flux using the SSM prediction, allowed regions are obtained using only SK results. These allowed regions overlap with the LMA solution and the LOW solution.

From the analysis taking into account results of the flux measurements and the SK daytime and nighttime recoil electron energy spectra, allowed regions are obtained as shown in Fig.8.6. The LMA and LOW solutions are allowed at high confidence levels, while the SMA solution is disfavored at $90 \sim 95\%$ C.L.
Figure 8.6: The 68, 90, 95, and 99% C.L. allowed regions of $\Delta m^2$ and $\sin^2 2\theta$ obtained by the analysis using the results of four flux measurements (Cl, Ga, SK) and daytime and nighttime recoil electron energy spectra. The black star shows the point where the $\chi^2$ is minimum.
Chapter 9

Conclusion

SK has measured the $^8$B solar neutrino flux and energy spectrum based on 1117 days of data collected from May 31 1996 to April 24 2000.

The $^8$B solar neutrino flux is observed to be:

$$2.40 \pm 0.03 \text{ (stat.)} \pm_{0.07}^{0.08} \text{ (syst.)}(\times 10^6 \text{ cm}^{-2}\text{sec}^{-1}).$$

The ratio to the SSM prediction is:

$$0.465 \pm 0.005 \text{ (stat.)} \pm_{0.013}^{0.015} \text{ (syst.)}$$

The daytime and nighttime fluxes are measured to be:

- Daytime : $2.35 \pm 0.04 \text{ (stat.)} \pm_{0.07}^{0.08} \text{ (syst.)}(\times 10^6 \text{ cm}^{-2}\text{sec}^{-1})$.
- Nighttime : $2.43 \pm 0.04 \text{ (stat.)} \pm_{0.07}^{0.08} \text{ (syst.)}(\times 10^6 \text{ cm}^{-2}\text{sec}^{-1})$.

The difference between the daytime flux and the nighttime flux is calculated as follows:

$$\frac{\text{Day} - \text{Night}}{(\text{Day} + \text{Night})/2} : -0.034 \pm 0.022 \text{ (stat.)} \pm_{0.012}^{0.013} \text{ (syst.)}.$$

The measured recoil electron energy spectrum from 5.0 MeV to 20 MeV is presented in Fig.7,10. The numerical results are summarized in Table.7,3.

The neutrino oscillation parameters are constrained by using the daytime and nighttime recoil electron energy spectra from 5.0 MeV to 20 MeV, the result of which are presented in Fig.7,11. Here, neutrino oscillations between two active neutrinos, $\nu_e \rightarrow \nu_x$ (x = $\mu$ or $\tau$), are assumed.

There are four possible solutions (SMA, LMA, LOW, and VAC) which are obtained using the results of the flux measurements from several experiments, as summarized in Sec.8.1.

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When the absolute normalization of the flux is treated as a free parameter, the VAC solution is excluded at 95% C.L. Most of the SMA solution and the lower half of the LMA solution are also excluded.

When the Hep flux is treated as an additional free parameter, the result is almost the same as that from standard Hep analysis.

By constraining the absolute flux using the SSM prediction, allowed regions are obtained using only SK results. Allowed regions appear in the large mixing angle region ($\sin^2 2\theta > 0.65$), and overlap the LMA solution and the LOW solution.

Finally, the analysis considering the results of the flux measurements is performed. Four allowed regions are obtained as shown in Fig.8.6. The LMA solution and the LOW solution are allowed at high C.L., while the SMA solution and the VAC solution are disfavored at $90 \sim 95\%$ C.L.
Appendix A

Spallation Likelihood

As described in Sec. 6.3, spallation events are distinguished from solar neutrino events by the following 3 parameters.

\[
\Delta L = \text{Distance from the low energy event to the reconstructed track of the preceding muon.}
\]

\[
\Delta T = \text{Time difference between the low energy event and the preceding muon.}
\]

\[
Q_{\text{res}} = \text{Residual charge of the preceding muon. Definition is given Eq. (6.3.2).}
\]

The spallation likelihood functions are defined as follows:

For muon events with reconstructed track,

\[
L_{spa}(\Delta L, \Delta T, Q_{\text{res}}) = L_{spa}^{\Delta L}(\Delta L, Q_{\text{res}}) \times L_{spa}^{\Delta T}(\Delta T) \times L_{spa}^{Q_{\text{res}}}(Q_{\text{res}}) \quad (\text{A.0.1})
\]

For muon events with no reconstructed track,

\[
L_{spa}(\Delta T, Q_{\text{total}}) = L_{spa}^{\Delta T}(\Delta T) \times L_{spa}^{Q_{\text{res}}}(Q_{\text{total}}) \quad (\text{A.0.2})
\]

where \(L_{spa}^{\Delta L}(\Delta L, Q_{\text{res}}), L_{spa}^{\Delta T}(\Delta T)\), and \(L_{spa}^{Q_{\text{res}}}(Q_{\text{res}})\) (\(L_{spa}^{Q_{\text{res}}}(Q_{\text{total}})\)) are likelihood functions for \(\Delta L, \Delta T, \) and \(Q_{\text{res}}, \) respectively.

### A.1 Likelihood for \(\Delta L\) \((L_{spa}^{\Delta L}(\Delta L, Q_{\text{res}}))\)

Fig. A.1 shows the \(\Delta L\) distribution for 6 \(Q_{\text{res}}\) ranges from a spallation candidate event sample. The conditions to select the spallation candidate are: \(\Delta T < 0.1\) sec and \(N_{\text{eff}} \geq 50.\) Here, these distributions are represented as \(L_{spa, \text{canal}}^{i}(\Delta L), i = 1, 6.\) Each \(Q_{\text{res}}\) region is defined as follows.
Figure A.1: $\Delta L$ distribution from the spallation candidate event sample.

Figure A.2: $\Delta L$ distribution after subtraction.
1. \( Q_{res} < 5 \times 10^4 \) pC,
2. \( 1 \times 10^5 \geq Q_{res} > 5 \times 10^4 \) pC,
3. \( 2 \times 10^5 \geq Q_{res} > 1 \times 10^5 \) pC,
4. \( 1 \times 10^6 \geq Q_{res} > 2 \times 10^5 \) pC,
5. \( 2 \times 10^6 \geq Q_{res} > 1 \times 10^6 \) pC,
6. \( Q_{res} \geq 2 \times 10^6 \) pC.

The peak around the \( 0 \sim 100 \) cm is caused by spallation events and that around \( 1500 \) cm is caused by non-spallation events, whose shape is determined by phase space.

In order to obtain the \( \Delta L \) distribution of non-spallation events, the same distributions are made from a fake low energy sample whose events are generated uniformly in the fiducial volume \( \langle L_{non-spa}^i(\Delta L) \rangle \), \( i = 1, 6 \). The \( \Delta L \) distribution of spallation events is obtained by subtracting the non-spallation event distribution from the spallation candidate event distribution. The spallation probability function \( P_{spa}^i(\Delta L) \) and the non-spallation probability function \( P_{non-spa}^i(\Delta L) \) are calculated as follows:

\[
P_{spa}^i(\Delta L) = I_{spa-cand}^i(\Delta L) - I_{non-spa}^i(\Delta L) \quad (A.1.1)
\]

\[
P_{non-spa}^i(\Delta L) = I_{non-spa}^i(\Delta L) \quad (A.1.2)
\]

The prototype of the spallation likelihood, \( L_{spa}^{\Delta L}(\Delta L, Q_{res}) \), is defined as follows:

\[
L_{spa}^{\Delta L}(\Delta L, Q_{res}) = \frac{P_{spa}^i(\Delta L)}{P_{non-spa}(\Delta L)} \quad \text{for the } i\text{-th } Q_{res} \text{ region} \quad (A.1.3)
\]

The distribution of \( L_{spa}^{\Delta L}(\Delta L, Q_{res}) \) is shown in Fig. A.2.

The \( L_{spa}^{\Delta L}(\Delta L, Q_{res}) \) is obtained by fitting an assumed function:

\[
L_{spa}^{\Delta L, fit}(\Delta L, Q_{res}) = \frac{\exp(B_i - C_i \Delta L)}{A_i} \quad (A.1.4)
\]

The coefficients \( A_i, B_i \), and \( C_i \) are summarized in Tab. A.1. Then the likelihood function for \( \Delta L \) is obtained as follows:

\[
L_{spa}^{\Delta L}(\Delta L, Q_{res}) = L_{spa}^{\Delta L, fit}(\Delta L, Q_{res}) \quad \text{for the } i\text{-th } Q_{res} \text{ region} \quad (A.1.5)
\]
<table>
<thead>
<tr>
<th>$i$</th>
<th>$Q_{\text{res}}(pC)$</th>
<th>$A_i$</th>
<th>$B_i$</th>
<th>$C_i \Delta L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$&lt; 5 \times 10^4$</td>
<td>0.353</td>
<td>2.107</td>
<td>5.594</td>
</tr>
<tr>
<td>2</td>
<td>$1 \times 10^5 \sim 5 \times 10^4$</td>
<td>1.850</td>
<td>2.081</td>
<td>7.176</td>
</tr>
<tr>
<td>3</td>
<td>$2 \times 10^5 \sim 1 \times 10^5$</td>
<td>3.697</td>
<td>1.850</td>
<td>7.608</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.836</td>
<td>5.071</td>
<td>Δ$L &lt; 250.2$ cm</td>
</tr>
<tr>
<td>4</td>
<td>$1 \times 10^6 \sim 2 \times 10^5$</td>
<td>15.14</td>
<td>1.301</td>
<td>8.175</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.622</td>
<td>5.706</td>
<td>Δ$L \geq 250.2$ cm</td>
</tr>
<tr>
<td>5</td>
<td>$2 \times 10^6 \sim 1 \times 10^6$</td>
<td>92.98</td>
<td>0.9020</td>
<td>9.187</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5369</td>
<td>7.288</td>
<td>Δ$L &lt; 363.57$ cm</td>
</tr>
<tr>
<td>6</td>
<td>$\geq 2 \times 10^6$</td>
<td>352.88</td>
<td>0.7128</td>
<td>9.843</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.3417</td>
<td>7.813</td>
<td>Δ$L \geq 547.02$ cm</td>
</tr>
</tbody>
</table>

Table A.1: The parameters for the likelihood $L^\Delta L_{\text{spa}}(\Delta L, Q_{\text{res}})$ for $Q_{\text{res}}$ regions $i = 1 \sim 6$

Figure A.3: $\Delta T$ distributions from the spallation candidate event sample in 6 different time bins.

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\[ i \quad \text{radioactivity} \quad \tau_{1/2}^i \quad A_i \]

\begin{tabular}{|c|c|c|}
\hline
1 & $^{12}\text{B}$ & 0.02023 & 120100 \\
2 & $^{12}\text{N}$ & 0.01100 & 33900 \\
3 & $^{9}\text{Li}$ & 0.178 & 338.6 \\
4 & $^{8}\text{Li}$ & 0.84 & 1254 \\
5 & $^{15}\text{C}$ & 2.449 & 134.7 \\
6 & $^{7}\text{N}$ & 7.134 & 676.1 \\
7 & $^{11}\text{Be}$ & 13.83 & 7.91 \\
\hline
\end{tabular}

Table A.2: The parameters of the likelihood \( L_{\text{spa}}^{\Delta T}(\Delta T) \)

### A.2 Likelihood for \( \Delta T \) (\( L_{\text{spa}}^{\Delta T}(\Delta T) \))

Fig. A.3 shows the \( \Delta T \) distributions from the spallation candidate event sample in 6 different time bins. These figures are made from low energy events whose \( \Delta L < 300 \text{cm} \), and \( N_{\text{eff}} \geq 50 \) and muons with the \( Q_{\text{res}} < 10^6 \).

These distributions are fitted with an assumed function:

\[
L_{\text{spa}}^{\Delta T, \text{fit}}(\Delta T) = \sum_{i=1}^{7} A_i \left( \frac{1}{2} \right)^{-\frac{\Delta T}{\tau_{1/2}^i}} \tag{A.2.1}
\]

where \( \tau_i \) is the half-life of 7 radioactive nuclei produced by spallation. The results of the fit are summarized in Tab. A.2.

Then the likelihood function for \( \Delta T \) is obtained as follows.

\[
L_{\text{spa}}^{\Delta T}(\Delta T) = L_{\text{spa}}^{\Delta T, \text{fit}}(\Delta T) \tag{A.2.2}
\]

### A.3 Likelihood for \( Q_{\text{res}} \) (\( L_{\text{spa}}^{Q_{\text{res}}}(Q_{\text{res}}) \))

Fig. A.4 shows the \( Q_{\text{res}} \) distribution. The upper figure (\( Q_{\text{t-cor}}(Q_{\text{res}}) \)) is made from muon events whose timing is correlated (\( \Delta T < 0.1 \)) with low energy events (\( N_{\text{eff}} \geq 50 \)). The lower figure (\( Q_{\text{un-cor}}(Q_{\text{res}}) \)) is made from muon events uncorrelated with low energy events. The prototype likelihood function (\( L_{\text{spa}}^{Q_{\text{res}}}(Q_{\text{res}}) \)) is obtained as follows:

\[
L_{\text{spa}}^{Q_{\text{res}}}(Q_{\text{res}}) = \frac{Q_{\text{t-cor}}(Q_{\text{res}}) - Q_{\text{un-cor}}(Q_{\text{res}})}{Q_{\text{un-cor}}(Q_{\text{res}})} \tag{A.3.1}
\]

The distribution of \( L_{\text{spa}}^{Q_{\text{res}}}(Q_{\text{res}}) \) is shown in Fig. A.5.
Figure A.4: $Q_{res}$ distribution from the spallation candidate event sample (top), and from normal cosmic ray muon events (bottom).

Figure A.5: $Q_{res}$ distribution after subtraction.
The $L_{spa}^{Q_{res}} (Q_{res})$ is obtained by fitting a polynomial to the subtracted distribution:

For $Q_{res} \leq 5.0 \times 10^5$,

$$L_{spa}^{Q_{res}, fit} (Q_{res}) = \sum_{i=0}^{4} A_i (Q_{res})^i$$  \hspace{1cm} (A.3.2)

For $Q_{res} > 5.0 \times 10^5$,

$$L_{spa}^{Q_{res}, fit} (Q_{res}) = \sum_{i=0}^{2} A_i (Q_{res})^i$$  \hspace{1cm} (A.3.3)

The likelihood function for $\Delta T$ is obtained as follows:

For $Q_{res} < 0$ pC,

$$L_{spa}^{Q_{res}} (Q_{res}) = 1.5071 \times 10^{-4}$$  \hspace{1cm} (A.3.4)

For $0 < Q_{res} \leq 5.0 \times 10^5$ pC,

$$L_{spa}^{Q_{res}} (Q_{res}) = 1.5071 \times 10^{-4} + 7.138 \times 10^{-9} Q_{res} + 9.987 \times 10^{-14} Q_{res}^2$$

$$-1.307 \times 10^{-19} Q_{res}^3 + 6.407 \times 10^{-26} Q_{res}^4$$  \hspace{1cm} (A.3.5)

For $Q_{res} > 5.0 \times 10^5$ pC,

$$L_{spa}^{Q_{res}} (Q_{res}) = -2.644 \times 10^{-2} + 7.086 \times 10^{-08} Q_{res} - 3.661 \times 10^{-15} Q_{res}^2$$  \hspace{1cm} (A.3.6)
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