Search for Proton Decay \( p \rightarrow \pi K^+ \)
in Super-Kamiokande

スーパーカミオカンデによる
陽子崩壊 \( p \rightarrow \pi K^+ \) の探索

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Abstract

Proton decay via $p \rightarrow \pi K^+$, which is favored by supersymmetric grand unified theories, was searched for using the 84.1 kton-year exposure in Super-Kamiokande. No evidence was found in three different searches. Lower limit of partial proton life time $\tau / B_{p \rightarrow \pi K^+}$ was set to $2.2 \times 10^{33}$ years at 90% confidence level. The result depends on the model of residual nucleus.
Acknowledgment

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1 introduction

1.1 theoretical motivation

The standard model of particle physics is built on the gauge group $SU(3)\times SU(2)\times U(1)$ in which electroweak and strong interaction are mediated by gauge bosons, gluons, weak bosons and photon. It explains almost all the experimental data. In spite of the success of the standard model, there are several incomprehensible questions. Why are there $\sim 20$ parameters in the standard model? Why do the electrical charges of particles quantize? To answer these questions Grand Unified Theories (GUTs) were introduced. GUTs are to unify the electro-weak and strong interactions into the same Gauge group. Georgi and Glashow succeeded in the unification in 1974[1]. The unification scale is necessarily at extremely high energy ($\geq 10^{15}$ GeV). There are several attractive features in GUTs. One of them is that GUTs can explain the charge quantization. However no evidence for GUTs has been found. It is difficult to test the GUTs using accelerators, because the energy scale of GUTs is $\geq 10^{15}$ GeV. On the other hand GUTs have a unique prediction, nucleon instability, which directly leads to nucleon decay.

minimal SU(5) GUT

$SU(5)$ is the minimal gauge group containing $SU(3)\times SU(2)\times U(1)$ on which the standard model is based. The model using $SU(5)$ is called the minimal SU(5) which was first suggested by Georgi and Glashow[1]. In the minimal SU(5), the proton lifetime is expressed as

$$\tau_p \sim \frac{1}{\alpha_{\text{GUT}}} \frac{m_X^4}{m_p^3}$$

where $\alpha_{\text{GUT}}$ is the coupling constant at the GUT energy scale, $m_X$ is the mass of the new gauge boson and $m_p$ is the proton mass. The partial proton lifetime of the dominant mode $p\to e^+\nu^0\pi^0$ is predicted to $\sim 10^{30}$ years. However it has already been excluded by several experiments(section 1.2).

supersymmetric GUT

GUTs have two extremely different energy scales, $\sim 10^2$ GeV and $\sim 10^{15}$ GeV. Though the Higgs mass is $(m_H)^2 = \sim 250$ GeV$^2$, the radiative correction induces an enormous correction $\sim 10^{30}$ GeV$^2$. Fine tuning of the radiative correction is obviously needed. This is called the “hierarchy problem”. The supersymmetry(SUSY) is introduced in order to resolve the “hierarchy problem”. In the SUSY model, each particle of the standard model has a supersymmetric partner. The supersymmetric partner must have a spin which is $1/2$ unit different from the normal particle. The supersymmetric partner prevents the radiative correction from divergence. The standard model particles and their supersymmetric partners are summarized in Table 1.

<table>
<thead>
<tr>
<th>name</th>
<th>spin</th>
<th>super partner</th>
<th>name</th>
<th>spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>quark</td>
<td>1/2</td>
<td>squark</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>lepton</td>
<td>1/2</td>
<td>slepton</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Higgs</td>
<td>0</td>
<td>Higgsino</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>gluon</td>
<td>1</td>
<td>gluon</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>photon</td>
<td>1</td>
<td>photon</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>Z$^0$</td>
<td>1</td>
<td>zino</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>W$^\pm$</td>
<td>1</td>
<td>wino</td>
<td>1/2</td>
<td></td>
</tr>
</tbody>
</table>

1.2 earlier experiment

Experimental searches for nucleon decay had been carried out before nucleon decay was predicted by Grand Unified Theories. After the introduction of GUTs, many experiments began. First, several experiments of the tracking calorimeter type started in the early 1980s. Two years later, experiments of the water Cherenkov type went on.
1.2.1 experiments before 1980

There are two types of nucleon decay searches before 1980. One is the indirect search. Isotopes which are daughters of nucleon decays were searched for. This method is independent of the decay mode. The first indirect search was carried out by Goldhaber in 1954[7]. Until 1977 the most sensitive lower limit of the nucleon lifetime was $2.2 \times 10^{36}$ years[9]. The other is the direct search. The first direct search was carried out by Reines, Cowan and Goldhaber in 1954[6]. Particles from a nucleon decay were searched for. Therefore this method depends on the decay mode. Until 1974 the most sensitive partial lower limit of the nucleon lifetime was $6 \times 10^{30}$ years[8].

1.2.2 iron calorimeter

The iron calorimeter consists of iron plates, which is a source of nucleons, and tracking devices, which track charged particles traversing the detector. It has a good tracking capability. The first generation detectors(1980s) were constructed with a sensitivity of $\sim 10^{31}$ years. Characteristics and the results of the calorimetric experiments are summarized in Table 2.

KGF (1980-1992)

The Kolar Gold Field(KGF) detector was located in the Kolar Gold mine in India at a depth of 6,800 m water equivalence(w.e.). The detector consisted of 34 layers of alternating 12 mm thick iron plates with 10 cm x 10 cm proportional counters. The detector mass was 0.14 kton. They reported an anomaly in their data which could be caused by nucleon decays[10]. However their claim was not confirmed by all the other experiments. If it were true, several hundreds of proton decays would have been observed in Super-Kamiokande.

Soudan (1981-1990)

The Soudan detector was located in the Soudan mine in Minnesota, USA at a depth of 2,000 m w.e.. The detector consisted of 432 concrete slabs with 8 proportional tubes each. Total mass was 0.031 kton.

NUSEX (1982-1988)

The NUdron Stability EXperiment(NUSEX) was located at Mont Blanc, France at a depth of 4,800 m w.e.. The detector was composed of 134 layers of alternating 1cm thick iron plates with 9 mm x 9 mm streamer tubes. The detector mass was 0.15 kton.

Frejus (1984-1988)

The Frejus detector was located in the Frejus highway tunnel in the Alps, Italy at a depth of 4,850 m w.e.. The 0.9 kton detector was a 912 layer sandwich of 3 mm thick iron plates and 6 m long plastic tubes of 5 mm x 5 mm cross section.

Soudan 2 (1988-present)

The Soudan 2 detector is located at the same site of the Soudan. The 0.97 kton detector consisted of 224 modules alternating 1.6 mm thick corrugated iron sheets. Each module stacks in hexagonal structure, which has two million plastic drift tubes (1.0 m long by 15 mm diameter). The Soudan 2 detector can detect $K^+$ from $p \rightarrow \pi K^+$ directly.

1.2.3 water Cherenkov

The water Cherenkov detector is to detect Cherenkov radiations produced by charged particles traversing faster than light in a medium. Section 2.1 will explain it in more detail. A source of nucleons is water and photo multiplier tubes(PMTs) detect Cherenkov radiations. The water Cherenkov detector can have a large mass, because water is cheap. Characteristics and the results of the water Cherenkov experiments are summarized in Table 2.

IMB/IMB3 (1982-1991)

The Irvine-Michigan-Brookhaven(IMB) detector was located in the Fairport salt mine near Ohio, USA. The IMB was a cubical tank with the total mass of 8 kton.

The Kamiokande detector was located 200 m away from the Super-Kamiokande. The Kamiokande was a cylindrical tank with the total mass of 3 kton. 948 50 cm PMTs were installed. This large photo-coverage made Cherenkov rings clear.

**HPW (1983-1984)**

The Harvard-Purdue-Wisconsin (HPW) detector was located in a mine near Utah, USA at a depth of 1500 m w.e.. The detector was a cylindrical tank with the total mass of 0.7 kton. 700 PMTs were mounted on vertical strings of hoops and mirrors were on the wall to increase light detection efficiency. However, Cherenkov images were not resolved. Information of Michel electron was used to eliminate atmospheric neutrino backgrounds. Hence no results on \( p \rightarrow e^+\pi^0 \) were obtained.

### 1.3 present work

In this thesis, proton decay via \( p \rightarrow \pi K^+ \), which is favored by SUSY GUTs, is searched for in Super-Kamiokande. The Super-Kamiokande detector is introduced in section 2. Proton decay and the background simulation are described in section 3. Data selection in Super-Kamiokande is described in section 4. Event reconstruction algorithms are explained in section 5. Various calibrations are studied in section 6. Proton decay analysis is described in section 7. Finally this thesis concludes in section 8.
<table>
<thead>
<tr>
<th>detector</th>
<th>detector type</th>
<th>total detector mass (kton)</th>
<th>depth (m)</th>
<th>sensitivity (years)</th>
<th>limit for $p \to e^+\pi^0$ (years)</th>
<th>limit for $p \to \pi^+\kappa^+$ (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KGF</td>
<td>iron calorimeter</td>
<td>0.14</td>
<td>6,800</td>
<td>$10^{30-31}$</td>
<td>[10]</td>
<td>[10]</td>
</tr>
<tr>
<td>Soudan</td>
<td>iron calorimeter</td>
<td>0.031</td>
<td>2,000</td>
<td>$10^{29}$</td>
<td>$1.3 \times 10^{30}$[11]</td>
<td>$2.7 \times 10^{29}$[11]</td>
</tr>
<tr>
<td>NUSEX</td>
<td>iron calorimeter</td>
<td>0.15</td>
<td>5,200</td>
<td>$10^{30-31}$</td>
<td>$1.5 \times 10^{31}$[12]</td>
<td>$2. \times 10^{30}$[12]</td>
</tr>
<tr>
<td>Frejus</td>
<td>iron calorimeter</td>
<td>0.9</td>
<td>4,800</td>
<td>$10^{31-32}$</td>
<td>$1.5 \times 10^{31}$[13]</td>
<td>$1.5 \times 10^{31}$[13]</td>
</tr>
<tr>
<td>Soudan2</td>
<td>iron calorimeter</td>
<td>0.97</td>
<td>2,000</td>
<td>$10^{31-32}$</td>
<td>[10]</td>
<td>$4.3 \times 10^{31}$[14]</td>
</tr>
<tr>
<td>HPW</td>
<td>water Cherenkov</td>
<td>0.7</td>
<td>1,500</td>
<td>$10^{30}$</td>
<td>[10]</td>
<td>$5.0 \times 10^{30}$[15]</td>
</tr>
<tr>
<td>IMB/IMB3</td>
<td>water Cherenkov</td>
<td>8</td>
<td>1,600</td>
<td>$10^{31-32}$</td>
<td>$5.4 \times 10^{32}$[16]</td>
<td>$1.8 \times 10^{32}$[16]</td>
</tr>
<tr>
<td>Kamiokande</td>
<td>water Cherenkov</td>
<td>3</td>
<td>2,700</td>
<td>$10^{31-32}$</td>
<td>$2.6 \times 10^{32}$[17]</td>
<td>$1.0 \times 10^{32}$[17]</td>
</tr>
</tbody>
</table>
2 detector

Nucleon decay search was carried out using the Super-Kamiokande detector shown in Figure 2. In this chapter the Super-Kamiokande detector is introduced.

2.1 principle of measurement

The Super-Kamiokande detector is of the Cherenkov ring imaging type. Events are observed by detecting Cherenkov photons. Cherenkov light was originally discovered by Cherenkov in 1934. This light was explained theoretically and quantitatively by Tamm and Frank [18]. Cherenkov light is an electromagnetic shock wave which is emitted along a charged particle that traverses medium faster than light \((v \geq c/n, \ v; \ \text{velocity of the charged particle,} \ n; \ \text{refractive index of the medium,} \ c; \ \text{the light velocity in vacuum})\). The light is emitted in the forward direction of the traveling particle with a half opening angle \(\theta\).

\[
\cos \theta = \frac{1}{n \beta}, \quad (\beta = v/c) \tag{2}
\]

The Cherenkov light spectrum is

\[
\frac{d^2 N}{dxd\lambda} = 2\pi \alpha (1 - \frac{1}{(n\beta)^2}) \frac{1}{\lambda^2} \tag{3}
\]

\(\alpha\) : the fine structure constant  
\(N\) : the number of the emitted photons  
\(\lambda\) : wavelength  
\(x\) : track length

In water, about 3/40 photons are emitted for the wavelength of 300 nm to 600 nm every 1 cm and Cherenkov photons are emitted along the cone of 42° half opening angle. Various physics quantities are reconstructed using Cherenkov light detected in the Super-Kamiokande detector.

2.2 Super-Kamiokande detector

The Super-Kamiokande detector is located at the Kamioka Observatory of Institute for Cosmic Ray Research in the Kamioka mine, Gifu Prefecture, Japan. The latitude and longitude are 36°25’N and 137°18’E, respectively. Since it lies 1,000m below the top of Mt. Ikenoyama (i.e. 2,700m w.e. underground), cosmic ray muons are reduced down to \(10^{-5}\) and their event rate is only 2.2 Hz. The tank is made of stainless steel and is filled with 50,000 tons of ultra-pure water. To keep the water clean, a water purification system is instrumented. The size of the tank is 39.3 m in diameter and 41.4 m in height. The tank is optically separated into two cylindrical sections, the inner detector(ID) and the outer detector(OD). The ID is fully covered with the OD. The OD is the veto counter to reject cosmic ray muons. Data is processed in the control room near the Super-Kamiokande detector. In the following sections each component is explained briefly.

2.2.1 inner detector (ID)

The size of the ID is 33.8 m in diameter and 36.2m in height. In total 1,146 of 50 cm \(\phi\) PMTs are uniformly mounted on the wall of the ID. The photocathode coverage is 40%. To reduce reflection light and to observe Cherenkov rings clearly, the other region is covered with black sheets. Figure 3 shows the schematic view of the PMTs, the support structures and the black sheets.

![Figure 3: The schematic view of the frame which support the PMTs in the ID and OD.](image)
Figure 2: The Super-Kamiokande detector
2.2.2 outer detector (OD)

The OD completely surrounds the ID. The thickness of the OD is 1.95 ~ 2.20 m. 1.885 of 20 cm φ PMTs are mounted on the wall. Each PMT is equipped with a 60 cm square and 1.3 cm thick wavelength shifter plate which absorbs ultra violet light and radiates blue light. DuPont Tyvek, a white reflective material covers all the surface of the other region. Thus the collection efficiency of Cherenkov photons increases. Moreover 2 m thickness of water shields against gamma-rays and neutrons from the rock. The top view of Figure 3 illustrates the OD structure.

2.2.3 photomultiplier tube

The PMTs we use in the ID are 50 cm(20 inch) in diameter. They, R3600-5s, are much larger than the normal type as shown in Figure 4. They were developed by Hamamatsu Photonics K.K. and the Kamiokande collaboration[19]. The characteristics of the PMT is summarized in Table 3. The PMT has a uniform response provided that the geomagnetic field is less than 100 mG. This condition is fulfilled with 26 sets of Helmholtz coils winding the detector. The OD PMTs are Hamamatsu R1408 with 20 cm(8 inch) in diameter.

Table 3: The characteristics of the 20 inch PMT.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>photocathode area</td>
<td>50 cm (20 inch) in diameter</td>
</tr>
<tr>
<td>shape</td>
<td>hemispherical</td>
</tr>
<tr>
<td>window material</td>
<td>Pyrex glass (4 ~ 5mm)</td>
</tr>
<tr>
<td>photocathode material</td>
<td>Bialkali</td>
</tr>
<tr>
<td>dynodes</td>
<td>Venetian blind type, 11 stages</td>
</tr>
<tr>
<td>pressure tolerance</td>
<td>6 kg/cm² water proof</td>
</tr>
<tr>
<td>quantum efficiency gain</td>
<td>20% at λ = 400 nm</td>
</tr>
<tr>
<td>dark current</td>
<td>10⁷ at ~ 2000 V</td>
</tr>
<tr>
<td>dark pulse rate</td>
<td>~ 3 kHz at gain = 10⁷</td>
</tr>
<tr>
<td>cathode non-uniformity</td>
<td>&lt; 10%</td>
</tr>
<tr>
<td>anode non-uniformity</td>
<td>&lt; 40%</td>
</tr>
<tr>
<td>transit time</td>
<td>90 nsec typical at gain = 10⁷</td>
</tr>
<tr>
<td>transit time spread</td>
<td>2.8 nsec RMS at 1 p.e. level</td>
</tr>
</tbody>
</table>

water flowing near the detector site. The water purification system processes and circulates at the rate of about 30 ton/hour to keep it ultra-pure. Figure 5 shows the water purification system, which consists of eight components as follows:

1. **μm filter** Comparatively large dusts are rejected.
2. **heat exchanger** The water temperature is kept about 13 °C to suppress bacteria growth.
3. **ion exchanger** Metal ions such as Ca, Ra and Th in the water are removed.
4. **ultra-violet sterilizer** Bacteria in the water are killed.
5. **vacuum degasifier** Gases in the water, such as air and radon, are removed. About 99% of Oxygen and 96% of Radon are removed.
6. **cartridge polisher** This is a high performance ion exchanger. Remaining metal ions are removed.
7. **ultra filter** Small dusts whose size is on the order of 10 nanometers are removed.
8. **reverse osmosis** Organisms of the order of 10² molecular weights are removed.

2.2.4 water purification system

50,000 ton of ultra-pure water is needed for the Super-Kamiokande detector. It was produced from natural

2.2.5 data acquisition system (DAQ)

**Inner Detector DAQ**

The data acquisition system of the ID is shown in Figure 6. Signals from each PMT are sent to a front end
module called ATM (Analog Timing Module). In the ATM module, 12 PMTs signals are processed. 934 ATM modules are used in total. Each channel has its own amplifier, discriminator, TAC(Timing to Analog Converter) and QAC(Charge to Analog Converter). If the signal exceeds the threshold value(about 0.32 P.E.s), a rectangular signal(200 ns in width and 11 mV in height) is generated to be used for the global trigger. These signals are summed up in each ATM module. It is called HITSUM signal. If the global trigger is issued, timing and charge of the signal are held and digitized by TAC and QAC. The dynamic range of ATM is \( \approx 450 \text{ pC} \) with a resolution of 0.2 pC and \( \approx 1300 \text{ ns} \) with a resolution of 0.4 ns. There are two TACs and QACs in each ATM module. Therefore we can take without dead time two successive events like a muon and the following Michel electron. We call the latter events “sub-event”. The data are read out by 8 online computers(SUN classic) through 48 VME memory modules(Super Memory Partner, SMP). They are collected to the online host computer(SUN sparc10) and merged to make complete events.

**Outer Detector DAQ**

The data acquisition system of the OD is shown in Figure 7. PMT signals are fed into QTC(Charge to Timing Converter) modules which generate a rectangle signal whose width is proportional to the input charge. It is digitized by TDC(Time to Digital Converter). Timing of the signal window is set to \( \approx 10^{-6} \) \( \mu \text{s} \) from the global trigger. The TDC is controlled by

**Figure 5:** The water purification system. Solid line shows the main circulation of water and dotted line shows the supplementary circulation line.

**Figure 6:** The schematic view of data acquisition system of the ID

**Figure 7:** The schematic view of data acquisition system of the OD

**Trigger**

The HITSUM signal from each ATM is summed up to generate the grand HITSUM signal of the ID. Moreover the HITSUM signal from each QTC is summed up to generate the grand HITSUM signal of the OD. From these two grand HITSUM signals, four global trigger signals are generated; high energy trigger(HE), low energy trigger(LE), super low energy trigger(SLE) and outer detector trigger(OD). Table 4 summarizes all of the triggers. The trigger module
records the trigger type and run, subrun and event number. Each run has about 24 hour data. “Subrun” is subsidiary unit of the run. Each subrun has 1~3 minutes data. The trigger information is appended in the PMT data. Data used for nucleon decay and at-

Table 4: the kind of trigger type.

<table>
<thead>
<tr>
<th>trigger type</th>
<th>using PMTs</th>
<th>threshold (#of hit PMTs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE</td>
<td>ID</td>
<td>31</td>
</tr>
<tr>
<td>LE</td>
<td>ID</td>
<td>29</td>
</tr>
<tr>
<td>SLE</td>
<td>ID</td>
<td>19</td>
</tr>
<tr>
<td>OD</td>
<td>OD</td>
<td></td>
</tr>
</tbody>
</table>

mospheric neutrino analyses are triggered by HE. HE requires a coincidence of at least 31 HITSUM signals within a 200 nsec time window. The trigger rate is 5~6 Hz. Most of the data are caused by radioisotopes and cosmic-ray muons.

data flow

Data taken by the HE trigger are transmitted to an offline computer outside the mine through optical fibers. Each data is tagged by the run mode. Data for the proton decay analysis corresponds to the normal mode. Other modes are calibration, test mode etc. Data of only the normal mode are flown to the reduction process.
3 simulation

Proton decay analysis is carried out in full use of a Monte Carlo(M.C.) simulation. The detection efficiency is estimated by proton decay M.C.. The background is estimated by the atmospheric neutrino interaction M.C.. Details of each simulation are explained in the following sections.

3.1 proton decay simulation

The source of proton decay in the Super-Kamiokande detector is water(H₂O). H₂O has 10 protons. 2/10 of protons are free, 8/10 of protons are bound in an Oxygen nucleus. Bound protons have the Fermi momentum and nuclear binding energy. After the proton decays, meson-nucleon interactions in the Oxygen nucleus and a nuclear gamma-ray emission from the proton hole should be taken into account.

3.1.1 Fermi momentum and nuclear binding energy

In our simulation, we use the Fermi momentum measured by electron-[¹²C] scattering[20]. Figure 8 shows the Fermi momentum for the s-state and p-state.

![Figure 8: Fermi momentum distribution. Left figure shows p-state and right figure shows s-state. Full circles show data measured by electron scattering on [¹²C][20]. Solid lines show theoretical calculation, which are used in our M.C. simulation.](image)

In case that a proton decays in an Oxygen nucleus, the proton mass is modified by the nuclear binding energy. The modified proton mass mₚ' is calculated by mₚ' = mₚ - Eₚₚ, where mₚ is the proton rest mass and Eₚₚ is the nuclear binding energy. Figure 9 shows the modified proton mass distribution in [¹⁶O].

![Figure 9: The modified proton mass distribution in [¹⁶O]. Each state is expressed by Gaussian function \( G(\text{mean}(\text{MeV}/c^2),\text{RMS}(\text{MeV}/c^2)) \). s-state is \( (938 - G(39,10.2)) \text{ MeV}/c^2 \) and p-state is \( (938 - G(15.5,3.82)) \text{ MeV}/c^2 \).](image)

3.1.2 meson-nucleon interaction in nucleus

The position where a proton decays in [¹⁶O] is determined by the following Wood-Saxon nuclear density model \( \rho_p(r) \), which is expressed as

\[
\rho_p(r) = \frac{Z}{A} \rho_0 \frac{1}{1 + e^{(r - a) / c}}
\]

\( r \): the distance from the center of the nucleus
\( \rho_0 = 0.48m_e^2 \) (for [¹⁶O])
\( a = 0.41 \text{ fm} \) (for [¹⁶O])
\( c = 2.69 \text{ fm} \) (for [¹⁶O])

The K⁺ produced by \( p \to \pi K^+ \) can interact with nucleons in the residual [¹⁵N] nucleus before escaping. The elastic scattering(K⁺N→K⁺N), inelastic scattering(K⁺N→K⁺N(resonance)) and charge exchange scattering(K⁺N→K⁰N¹⁺) are considered in our M.C. simulation. We use the cross sections provided by the particle data group[21], which are shown in Figure[23][22]. The K⁺ interaction is mostly elastic because the momentum of the K⁺ produced by \( p \to \pi K^+ \) is between 150 and 500 MeV/c.

In our simulation of the K⁺ scattering in the [¹⁶O] nucleus, the Pauli blocking is taken into account. Namely the momentum of the recoil nucleon in the final state should be more than the Fermi surface momentum \( p_F(r) \).

\[
p_F(r) = \left( \frac{3}{2} \pi^2 \rho_p(r) \right)^{\frac{1}{3}}
\]

Since the K⁺ from a proton decay is below the threshold of the Cherenkov light emission and only its de-

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3.1.3 nuclear gamma-ray from nucleon hole

When a proton in $^{16}\text{O}$ decays, the remaining $^{15}\text{N}$ nucleus is left in either the ground state or excited states. From the latter case a prompt gammaray is emitted. We use this prompt gamma-ray to tag the proton decay. The emission probability is estimated by Ejiri[25]. Figure 13 shows the level scheme of the proton hole states in the $^{15}\text{N}^*$. Table 5 shows the emission probability from the $^{15}\text{N}^*$ cited from [25].
We use these values in our proton decay and atmospheric neutrino simulation. Most probable gamma-ray is 6.32 MeV from the p3/2 state. 41% of the proton holes in the \(^{16}\text{O}\) are estimated to emit 6.32 MeV gamma-rays. Table 6 shows the emission probability from the \(^{15}\text{O}\)\(^{*}\). These values are used in the atmospheric neutrino simulation. In case of \(\pi^0\) absorption, the simulation assumes that 1~4 gamma-rays, that have 5~20 MeV in total, emitted.

![Energy & deexcitation scheme of \((\ell_p)^{-1}\)](image)

**Table 6: Gamma-ray emission probability from neutron hole in \(^{15}\text{O}\)\(^{*}\)[25]**

<table>
<thead>
<tr>
<th>hole state</th>
<th>energy (MeV)</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1/2</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>p3/2</td>
<td>6.18</td>
<td>0.44</td>
</tr>
<tr>
<td>s1/2</td>
<td>7.03</td>
<td>0.02</td>
</tr>
<tr>
<td>other</td>
<td>3.0 - 4.0</td>
<td>0.22</td>
</tr>
</tbody>
</table>

### 3.1.4 detector simulation

After the proton decay or atmospheric neutrino interaction simulations, the secondary particles are applied to the detector simulation. It simulates particle interactions in the water, PMTs responses and electronics. The simulation of particles in the water is based on the GEANT package[26]. It can simulate the Cherenkov radiation, ionization loss, bremsstrau radiation, delta-ray creation, multiple scattering of leptons.

Figure 14 shows the photon attenuation coefficient of the water in the Super-Kamiokande detector. We consider the Rayleigh scattering, Mie scattering and absorption. The Rayleigh scattering is dominant for short wavelengths because the Rayleigh scattering length is proportional to \(\lambda^{-4}\). The attenuation length in 350~450 nm is more than 80 m.

Moreover the hadron interactions in the water are simulated. We use the CALOR package[27]. However for pions of \(p_\pi \leq 500\text{ MeV/c}\), we use our own simulation program[28].

### 3.2 background simulation

Almost all of cosmic-ray muons are rejected by the data selection processes (see section 4). Therefore the cosmic-ray muons are ignored in our background simulation. The severe background is caused by the atmospheric neutrino interactions.

#### 3.2.1 atmospheric neutrino

Various cosmic-ray particles enter the earth atmosphere continuously. Figure 15 shows the chemical composition of the primary cosmic rays as a function of energy. The chemical composition of the cosmic rays is \(\sim 95\%\) protons, \(\sim 4.5\%\) heliums and \(\sim 0.3\%\) CNO nuclei for the energy above 2 GeV/nucleus. An experimental uncertainty of the primary cosmic ray
fluxes is $\sim 20\%$[29]. Atmospheric neutrinos are the decay products from pions, kaons and muons which are produced by the primary cosmic-ray - nucleon interactions in the atmosphere. Figure 16 shows a schematic view of the atmospheric neutrino production processes.

Various calculations of the atmospheric neutrino fluxes are shown in Figure 17. We use the flux calculation by Honda et al.[30] in our simulation. The uncertainty of the atmospheric neutrino flux is $\pm 20\%$[34]. Neutrino interactions are categorized to

- quasi-elastic and elastic scattering
- single meson production
- deep inelastic scattering

### 3.2.2 quasi-elastic and elastic scattering

We considered six modes shown in Table 7 for the quasi-elastic and elastic scattering.
The quasi-elastic and elastic scattering cross sections are calculated using the following terms of the hadronic current.

\( J_{\nu}^{\text{hadron}} = \cos \theta_c \bar{u}(N) \left( \gamma_\nu F_V^V(q^2) \right. \)

\[ + \frac{i\sigma_\nu q^\mu (\mu_p - \mu_n) F_P^V(q^2)}{2M_N} + \gamma_\nu \gamma_5 F_A(q^2) \left. \right) u(N) \]

\( F_V^V(q^2), F_P^V(q^2) \) : vector form factors

\( F_A(q^2) \) : axial vector form factor

\( \theta_c \) : Cabibbo angle

\( M_N \) : nucleon mass

\( \mu_p, \mu_n \) : anomalous magnetic moments of the proton and neutron, respectively.

These form factor parameters are determined by experiments[36]. The charged current cross sections are calculated using this equation. Figure 18 shows the comparison of the calculated cross section with the experimental data. The neutral current cross sections are estimated using the following relations[37].

\( \sigma(\nu p \rightarrow \nu p) = 0.153 \times \sigma(\nu n \rightarrow e^- p) \) \hspace{1cm} (7)

\( \sigma(\bar{\nu} p \rightarrow \bar{\nu} p) = 0.218 \times \sigma(\bar{\nu} p \rightarrow e^+ n) \) \hspace{1cm} (8)

\( \sigma(\nu n \rightarrow \nu n) = 1.5 \times \sigma(\nu p \rightarrow \nu p) \) \hspace{1cm} (9)

\( \sigma(\bar{\nu} n \rightarrow \bar{\nu} n) = 1.0 \times \sigma(\bar{\nu} p \rightarrow \bar{\nu} p) \) \hspace{1cm} (10)

Calculated cross sections for free protons are shown in Figure 19.

### 3.2.3 Single meson production

We considered two different single meson production processes. One is the single meson production mediated by the baryon resonances. \( \pi, K \) and \( \eta \) are considered. The other is the coherent \( \pi \) production by the neutrino-nucleus interaction.

Single meson productions mediated by the baryon resonances are simulated by the Rein and Sehgal method[42]. Figure 20 shows a schematic view of the Rein and Sehgal method. The interactions consist of two parts. First is \( \nu N \rightarrow N^* \) (\( N \) is a nucleon, \( N^* \) is the baryon resonance). This is calculated using the FKR model(Feynman-Kiskinger-Raynal). Second is \( N^* \rightarrow mN \) (\( m \) is a meson, \( N^* \) is a scattered nucleon). This process is calculated using the experimental data.
Figure 19: Calculated cross sections for quasi-elastic and elastic scattering.
We considered 14 modes for the single pion production through the baryon resonances shown in Table 8. In each interaction, 18 resonances below 2 GeV are considered. The calculated cross section for $\nu_\mu p \rightarrow \mu^- p n^+$ is compared with the experimental data in Figure 21. Figure 22 shows the calculated cross section for each single pion production mode through the baryon resonances.

Table 8: Single $\pi$ resonance production mode considered in our M.C. simulation.

<table>
<thead>
<tr>
<th>Interaction mode</th>
<th>Interaction mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.C. $\nu p \rightarrow l^- p n^+$</td>
<td>N.C. $\nu n \rightarrow \nu n^0$</td>
</tr>
<tr>
<td>C.C. $\nu n \rightarrow \Gamma p n^0$</td>
<td>N.C. $\nu p \rightarrow \nu p n^0$</td>
</tr>
<tr>
<td>C.C. $\nu n \rightarrow \Gamma n^- n^+$</td>
<td>N.C. $\nu n \rightarrow \nu n^- n^+$</td>
</tr>
<tr>
<td>C.C. $\pi p \rightarrow l^+ p n^-$</td>
<td>N.C. $\nu p \rightarrow \nu p n^-$</td>
</tr>
<tr>
<td>C.C. $\pi p \rightarrow l^+ n^0 n^+$</td>
<td>N.C. $\nu p \rightarrow \nu p n^+$</td>
</tr>
<tr>
<td>C.C. $\pi n \rightarrow l^+ n^0 n^+$</td>
<td>N.C. $\pi n \rightarrow \pi n^0 n^+$</td>
</tr>
<tr>
<td>C.C. $\pi n \rightarrow l^+ n^-$</td>
<td>N.C. $\pi n \rightarrow \pi n^-$</td>
</tr>
<tr>
<td>C.C. $\pi p \rightarrow l^+ n^+$</td>
<td>N.C. $\pi p \rightarrow \pi n^+$</td>
</tr>
</tbody>
</table>

Figure 18: Comparison between calculated cross section for quasi-elastic scattering and experimental data (ANL[38], GGM[39], Serpukhov[40], BNL[41]). Upper figure (a) shows $\nu_\mu n \rightarrow \mu^- p$ and lower figure (b) shows $\pi_\mu p \rightarrow \mu^+ n$.

Figure 20: A schematic view of the Rein and Sehgal method.

Figure 21: Comparison between calculated cross section for single $\pi$ production through resonance and experimental data (ANL[43], GGM[44], Serpukhov[40], BNL[41]).
Figure 22: Calculated cross sections for single pion production through baryon resonances.
Moreover single K and $\eta$ productions through the baryon resonances are considered. Table 9 shows the modes considered in our M.C. simulation. There are six modes in K and six modes in $\eta$ production. It is important to estimate the K production for the $p \to \pi K^+$ search, because this process can make the same signal. Table 10 shows the resonances which are considered in the K production. The cross section for the K production is calculated by the Rein and Seghal method. Figure 23 shows the calculated cross section of the single K$^+$ production.

Table 9: Single K and $\eta$ production mode considered in our M.C. simulation.

<table>
<thead>
<tr>
<th>resonance</th>
<th>$\text{interaction mode}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.C.</td>
<td>$\nu n \to \Lambda K^+$</td>
</tr>
<tr>
<td>C.C.</td>
<td>$\pi n \to \Lambda K^-$</td>
</tr>
<tr>
<td>N.C.</td>
<td>$\nu n \to \Lambda K^0$</td>
</tr>
<tr>
<td>N.C.</td>
<td>$\pi n \to \Lambda K^0$</td>
</tr>
<tr>
<td>N.C.</td>
<td>$\pi p \to \Lambda K^0$</td>
</tr>
<tr>
<td>N.C.</td>
<td>$\pi p \to \Lambda K^+$</td>
</tr>
</tbody>
</table>

Table 10: The resonances through $N^* \to AK$ process. These are considered in our M.C. simulation.

<table>
<thead>
<tr>
<th>$I=\frac{1}{2}$</th>
<th>$\Gamma_{\text{total}}$(MeV)</th>
<th>branching ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>S$_{11}(1535)$</td>
<td>150</td>
</tr>
<tr>
<td>D$_{15}(1675)$</td>
<td>D$_{13}(1700)$</td>
<td>100</td>
</tr>
<tr>
<td>P$_{11}(1710)$</td>
<td>P$_{13}(1720)$</td>
<td>150</td>
</tr>
<tr>
<td>P$_{11}(1710)$</td>
<td>150</td>
<td>5-25%</td>
</tr>
<tr>
<td>P$_{13}(1720)$</td>
<td>200</td>
<td>1-15%</td>
</tr>
</tbody>
</table>

Table 11: Coherent $\pi$ production mode considered in our M.C. simulation.

<table>
<thead>
<tr>
<th>interaction mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.C. $\nu^{16}\text{O} \to \nu^{16}\text{O}\pi^+$</td>
</tr>
<tr>
<td>C.C. $\pi^{16}\text{O} \to \pi^{16}\text{O}\pi^-$</td>
</tr>
<tr>
<td>N.C. $\nu^{16}\text{O} \to \nu^{16}\text{O}\pi^0$</td>
</tr>
<tr>
<td>N.C. $\pi^{16}\text{O} \to \pi^{16}\text{O}\pi^0$</td>
</tr>
</tbody>
</table>

The last process of the single meson production is a coherent pion production. It is the interaction between a neutrino and a $^{16}\text{O}$ nuclei as shown in Table 11. In this process the outgoing lepton and pion directions are strongly correlated with the neutrino direction, because the momentum transfer is
very small. The differential cross section of the coherent pion production is expressed as the following equation and is shown in Figure 24.

\[
\frac{d^2 \sigma}{dQdydt} = \frac{G^2 M_N}{2\pi^2} f_\pi \Delta^2 E_\nu (1-y) \frac{1}{16\pi} \left( \frac{-m_A^2}{m_A^2 + Q^2} \right)^2 e^{-t\eta} |F_{abs}|^2
\]

\[
Q^2 = (1+y)(\frac{m_A^2}{m_A^2 + Q^2}) e^{-t\eta} |F_{abs}|^2
\]

(11)

\[
\frac{d^2 \sigma}{dxdy} = \frac{c_2 M_N E_\nu}{\pi} ((1-y + \frac{1}{2} y^2 + c_1) F_2(x, q^2)
\]

\[
\pm y(1-\frac{1}{2} y + c_2) [xF_3(x, q^2)]
\]

(13)

\[
C_1 = \frac{y M_f^2}{4 M_N E_\nu x} - \frac{x y M_N}{2 E_\nu} - \frac{M_f^2}{2 M_N E_\nu}
\]

\[
C_2 = \frac{M_f^2}{4 M_N E_\nu x}
\]

(14)

where \(x = -\frac{Q^2}{2M(E_\nu - E_\nu)}\) is the Bjorken scaling variable and \(y = \frac{E_\nu}{E_\nu + E_\nu}\) is the fractional energy transferred to the hadron system. The cross section for the N.C. interaction is estimated from the C.C. cross section using the following relations. The relations are estimated from the experimental data[47].

\[
\frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\bar{\nu} N \rightarrow \bar{\nu} X)} = \begin{cases} 
0.26, & (E_\nu \leq 3 \text{ GeV}) \\
0.26 + 0.04 \times \frac{E_\nu - 3}{\text{ GeV}}, & (3 < E_\nu \leq 6 \text{ GeV}) \\
0.30, & (6 \text{ GeV} < E_\nu)
\end{cases}
\]

(15)

The calculated cross sections for the deep inelastic scattering are shown in Figure 25.

There are two methods in the calculation of the outgoing hadron system. In \(W < 2.0 \text{ GeV}/c^2\), we use the original particle production program and in \(W > 2.0 \text{ GeV}/c^2\), we use the PYTHIA package[48]. \(W\) is the invariant mass of the hadron system. In \(W < 2.0 \text{ GeV}/c^2\), we considered only multiple pion production processes. The contribution from the one pion production is subtracted in order to keep the consistency with the single pion production processes described in section 3.2.3. The mean multiplicity of pions is calculated by the following equation, which is estimated by the experimental data[45].

\[
\langle n_\pi \rangle = 0.09 + 1.83 \ln W^2
\]

assuming \(\langle n_\pi \rangle = \frac{1}{3} \langle n_{\pi^+} \rangle = \frac{1}{3} \langle n_{\pi^-} \rangle = \frac{1}{3} \langle n_0 \rangle\)

The pion multiplicity of each event is determined by the KNO(Koba-Nielsen-Olesen) scaling[46]. We use
the following relation, which is estimated by the experimental data, in the forward-backward asymmetry of the pions in the hadronic center of mass system[49].

\[
\frac{\pi^{\text{forward}}}{\pi^{\text{backward}}} = \frac{0.35 + 0.41 \ln W^2}{0.5 + 0.001 \ln W^2}
\] (17)
4 data selection

The trigger rate (HE; see section 2.2.5) is about 6 Hz. Data whose run mode = normal are all applied to the reduction process. The reduction flow is shown in Figure 26. Most of the collected data are the backgrounds caused by cosmic muons and radioisotopes. Events, in which event vertices and whole Cherenkov photons are not contained in the ID are rejected by the subsequent reduction processes. After applying all the reduction programs, the events induced by atmospheric neutrinos and possible nucleon decays remain. Moreover the livetime calculation is explained in this chapter.

<table>
<thead>
<tr>
<th>event rate ( /day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>raw data</td>
</tr>
<tr>
<td>1st &amp; 2nd reduction</td>
</tr>
<tr>
<td>3rd reduction</td>
</tr>
<tr>
<td>bad run/subrun rejection</td>
</tr>
<tr>
<td>4th reduction</td>
</tr>
<tr>
<td>5th reduction</td>
</tr>
<tr>
<td>final rejection</td>
</tr>
<tr>
<td>final sample</td>
</tr>
</tbody>
</table>

Figure 26: reduction flow

4.1 1st and 2nd reduction

First, the simple and effective algorithms are applied in the 1st and 2nd reduction steps because the data size is large. Selection criteria in the 1st and 2nd reduction steps are

1st reduction step:
- (1-a) \( PE_{300} \geq 200 \) P.E.s
- (1-b) \( N_{hit} \leq 50 \)
- (1-c) \( t_{diff} \geq 10 \) \( \mu \)sec

2nd reduction step:
- (2-a) \( PE_{max}/PE_{300} \leq 0.5 \)
- (2-b) \( N_{hit}(800ns) \leq 25 \)

\( PE_{300} \) is the maximum number of P.E.s within the 300 nsec time sliding window in the ID. Figure 27 shows the \( PE_{300} \) distribution\(^1\). \( PE_{300} = 200 \) P.E.s corresponds to 22 MeV/c for electron and 190 MeV/c for muon. A peak around 60 P.E.s is caused by radioisotopes and a peak around 500 P.E.s is caused by Michel electrons from cosmic-ray muons. Criterion (1-a) eliminates radioisotope events. \( N_{hit}(800ns) \) is the number of hits within 800 nsec window in the OD. Criteria (1-b) and (2-b) eliminate cosmic-ray muons. Figure 28 shows the \( N_{hit}(800ns) \) distributions of (a) data and (b) Xe lamp events. The Xe lamp light is emitted at the center of the ID (see section 6.2.1) and more than 300,000 P.E. are observed in the ID. Figure 28 (b) shows that the number of accidental coincidences caused by the dark current and radioisotope hits are smaller than criterion (2-b). Moreover, it shows that the OD are optically well separated from the ID and the light leak is negligible. Figure 29 shows the \( t_{diff} \) distribution, where \( t_{diff} \) is the time difference from the previous event. A peak around \( 10^{-6} \) sec corresponds to the reflection of PMT signals and a peak around \( 10^{-5} \) sec corresponds to after pulses. Criterion (1-c) eliminates fake trigger events. Figure 30 shows the \( PE_{max} \) distributions, where \( PE_{max} \) is the maximum number of P.E.s detected by single PMTs. Criterion (2-a) eliminates electrical noises and events which occur near PMTs. The \( p \rightarrow \nu K^+ \) M.C. events are not rejected by the 1st and 2nd reduction.

4.2 3rd reduction

99.99% of the backgrounds are rejected by the 1st and 2nd reduction. But four types of the backgrounds still remain. They are cosmic-ray muons, flashing PMTs, accidental events and low energy events. In the 3rd reduction, the remaining backgrounds are rejected.

\(^1\)full window is 1.8 \( \mu \)s wide
Figure 27: \( \text{PE}_{\text{300}} \) distribution for raw data (histogram) and cosmic-ray muons (hatched region). The region pointed by arrow shows criterion (1-a).

Figure 28: \( N_{\text{hita}}(800\text{ns}) \) distribution. Each figure shows: (a) data and (b) Xe lamp events. The region pointed by arrow shows criteria (1-b)(2-b).

Figure 29: \( t_{\text{diff}} \) distribution. The region pointed by arrow shows criterion (1-c).

Figure 30: \( \text{PE}_{\text{max}}/\text{PE}_{\text{300}} \) distribution for events that survive 1st reduction step. Upper figure shows data (run#5501-5600) and lower figure shows \( p \rightarrow \pi K^+ \) M.C. The region pointed by arrow shows criterion (2-a).

4.2.1 cosmic-ray muon cut

There are three cuts for the cosmic-ray muons rejection: through-going muon cut, stopping muon cut and cable-hole muon cut.

through-going muon cut

A through-going muon event produces OD hit clusters at the entrance and exit points. Figure 31 shows a typical through-going muon event. The entrance and exit points are seen in the OD. Events satisfying the following criteria are rejected as through-going muons.

- \( \text{PE}_{\text{max}} > 231 \text{P.E.} \)
- \( N_{\text{hite}} \geq 1000 \)
- \( N_{\text{hita\_cluster (entrance)}} > 9 \) or \( N_{\text{hita\_cluster (exit)}} > 9 \)
- \( E_{\text{through}} > 0.75 \)

The data are fitted by a special reconstruction tool for through-going muons and the entrance and exit positions are reconstructed. \( N_{\text{hita\_cluster}} \) is the number
of hits within 8 m from the reconstructed entrance or exit positions. The goodness of fit is calculated, whose definition is

$$g_i = \frac{1}{\sum \frac{1}{\sigma_i^2}} \times \frac{\sum \frac{1}{\sigma_i^2} \exp \left[ \frac{(t_i - T)^2}{2(1.5 \times \sigma_i)^2} \right]}$$  \hspace{1cm} (18)$$

The only good fitted events as through-going muon are rejected.

**stopping muon cut**

A typical stopping muon event is shown in Figure 32. A hit cluster exists only at the entrance point in the OD. The selection criteria for stopping muon are

- \( N_{hit-cluster}(\text{entrance}) > 9 \)
- or
- \( N_{hit-cluster}(\text{entrance}) > 4 \) and \( g_{stop} \geq 0.5 \)

A special rejection is applied for rejecting stopping muons, too. \( N_{hit-cluster}(\text{entrance}) \) is the number of hits within 8 m from the reconstructed entrance position. \( g_{stop} \) is a goodness for the stopping \( \mu \) fitter. The definition is the same as that of the through-going muon fitter (Equation 18). In the \( 4 < N_{hit-cluster}(\text{entrance}) \leq 9 \) region, events satisfying \( g_{stop} \geq 0.5 \) are rejected. Figure 33 shows the \( N_{hit-cluster}(\text{entrance}) \) distributions. No \( p \rightarrow \pi K^+ \) M.C. events are rejected by these cuts.

Figure 31: An event display of a typical through-going muon.

Figure 32: An event display of a typical stopping muon.

Figure 33: \( N_{hit-cluster}(\text{entrance}) \) distribution for events after 1st and 2nd reduction. Upper figure shows data (run #5501-5600) and lower figure shows \( p \rightarrow \pi K^+ \) M.C.. The region pointed by solid line shows criterion \( N_{hit}(\text{entrance}) > 9 \). And the region pointed by dotted line shows criterion \( N_{hit}(\text{entrance}) > 4 \). The events satisfying \( 4 < N_{hit}(\text{entrance}) \leq 9 \) are rejected only if \( g_{stop} \geq 0.5 \).
cable-hole muon cut

PMT and high voltage cables are fed out of the tank through the cable-holes. There are 12 cable-holes and 4 of them are located at the top OD region. 4 cable-holes cause the inefficiency region to veto cosmic-ray muons. Therefore plastic scintillation counters, whose size is 2.0×2.5 m, were installed above each cable hole in April 1997. Events are removed if one of the veto counter is fired and the vertex positions are reconstructed within 4 m from the veto counter. Figure 34 shows the effect of the veto counters. One can see four clusters around the cable-holes in (a). Four clusters are eliminated in (b) after installing the veto counters.

![Figure 34](image)

Figure 34: X-Y vertex distribution for the FC events with Z>1760 cm. The outer circle shows the ID wall and the inner circle shows the fiducial volume line. Figure(a) shows data before installing veto counter. Figure(b) shows data after installing veto counter. Four boxes show location of veto counter.

4.2.2 flashing PMT cut

Flashing PMT events, so called “flasher”, are caused by an internal corona discharge from a PMT dynode. There are various kinds of “flasher”; long tail flasher, dark flasher and repeating flasher. For rejecting flashing PMT events, the following two cuts are applied in the 3rd reduction step.

long tail flasher cut

One of the flasher’s characteristics is the broad timing distribution. In this type, flashing continues in several hundreds nano seconds. The following criteria are applied to reject this type of flashers.

- \( N_{hit}(\text{min}) \geq 15 \)
- or \( N_{hit}(\text{min}) \geq 10 \) and \( N_{hit} \leq 800 \)

\( N_{hit}(\text{min}) \) is the minimum number of hit in 100ns timing window sliding from 300ns to 9000ns after the event trigger. In the low energy events \( (N_{hit} \leq 800) \) the cut is more tight. Figure 35 shows the \( N_{hit}(\text{min}) \) distribution. \( p \rightarrow \pi \kappa^{+} \) M.C. are not rejected by this cut.

![Figure 35](image)

Figure 35: \( N_{hit}(\text{min}) \) distribution for events that satisfy 1st and 2nd reduction. Upper figure shows data(runs 5501-5600) and lower figure shows \( p \rightarrow \pi^{+} K^{+} \) M.C.. The region pointed by solid arrow shows \( N_{hit}(\text{min}) \geq 15 \) and dotted arrow shows \( N_{hit}(\text{min}) \geq 10 \)

dark flasher cut

Dark flashers have the same characteristics as other types of flashers. However, since the number of hit PMTs is small, it is difficult to distinguish from the proton decay or atmospheric neutrino events. Therefore a special fitter is applied for those events satisfying \( N_{hit} \leq 500 \) and The goodness of the low energy fitter is calculated. The definition of the goodness is the same as that of the through-going muon fitter(Equation 18). The dark flasher event pattern is different from a typical Cherenkov ring pattern and the flashing time is not short, compared with the proton decay or atmospheric neutrino events. The goodness becomes low. Events satisfying the following criteria are rejected.

- \( N_{hit} \leq 500 \)
4.2.3 accidental coincidence event cut

There are events which contain two or more peaks in the timing window. These events consist of low energy particles and a cosmic-ray muon which happens in a single window. If these events remain in final sample, the correct analysis is not obtained especially for Michel electrons. The selection criteria to reject them are

- \(N_{hit}(off) > 19\)
- \(PE_{off}(off) > 5000\)

\(N_{hit}(off)\) is the number of hits between 400 and 900 ns after the event trigger in the OD. \(PE_{off}(off)\) is the total P.E. between 400 and 900 ns after the event trigger in the ID.

4.2.4 low energy event cut

The vertex is reconstructed for an event satisfying \(N_{hit} < 500\) using the special low energy fitter. Criteria for the low energy events is

- \(N_{hit} < 500\)
- \(N_{hit}(low) < 50\)

\(N_{hit}(low)\) is the maximum number of hits in a 50 ns window in the time residual. \(N_{hit}(low) = 50\) corresponds to about 10 MeV for electron. Figure 37 shows the \(N_{hit}(low)\) distribution.

Figure 36: \(g_{low}\) distribution for events that survive 1st and 2nd reduction and \(N_{hit} \leq 500\). Upper figure shows data(run#5501-5600) and lower figure shows \(p \rightarrow \nu + K^+\) M.C.. The region pointed by arrow shows \(g_{low} \leq 0.4\)

Figure 37: \(N_{hit}(low)\) distribution for events that satisfy 1st and 2nd reduction and \(N_{hit} < 500\). The region pointed by arrow shows rejected criterion \(N_{hit}(low) < 50\)

4.3 bad run/subrun rejection

The bad run/subruns are categorized to the noisy run/subrun and the run/subrun which has many bad PMTs. The noisy run/subrun means that the event rate is much higher than that of the usual level. The noisy run/subrun rejection is needed before the 4th reduction, because the 4th reduction algorithm doesn't work well if there are many noise events (see section 4.4). The noisy run/subrun is caused by the PMT flasher or data acquisition troubles. The noisy run/subruns are rejected by two criteria.
• event rate after 1st reduction step is more than 0.09 event/sec.
• event rate after 3rd reduction step is more than 50 event/hour.

The usual event rate after the 1st and 3rd reductions is 0.03 event/sec and 1.8 event/hour, respectively. The first criterion is applied in each subrun and the second criterion is applied in each run.

The bad PMT means that the PMT pedestal level or gain is highly changed. The following criteria are applied in order to reject the run/subruns which have many bad PMTs.

• the number of bad PMTs of the ID is by 60 more than usual level.
• the number of bad PMTs of the OD is by 20 more than usual level.

60 channels of the ID corresponds to 5 ATM modules. Moreover, the OD bad PMT cut is applied, because the reductions of cosmic-ray muons don’t work well if the number of the bad PMT of the OD increases. 20 channels of the OD corresponds to 1% of all the OD channel.

4.4 4th reduction

The 4th reduction is another flasher cut to reject the repeating type flasher which repeats the same PMT hit pattern. The detector wall is divided to 2 × 2 m patches which include ~8 PMTs. In total there are 1,350 regions. The estimator \( r_{\text{match}} \) is calculated between event A and B.

\[
 r_{\text{match}} = \frac{\sum (q_i^A - \langle q_i^A \rangle)(q_i^B - \langle q_i^B \rangle)}{N\sigma_q + \sigma_q^0} \tag{19}
\]

\( q_i^j \) : the number of the P.Es summing in the \( i \)-th patch where \( j=A,B \)
\( \langle q_i^j \rangle \) : the average of \( q_i^j \) in the \( i \)-th patch.
\( \sigma_q \) : the deviation of \( q_i^j \).

An event is examined whether it matches or not with 10,000 events just before and after the event. The event A and B is considered as “match” if they satisfy the following criterion.

\[
 r_{\text{match}} > 0.23 \times \log_{10} \frac{Q_{\text{tot}}^A + Q_{\text{tot}}^B}{2} - 0.029 \tag{20}
\]

\( Q_{\text{tot}}^j \) is the total number of P.Es of the event \( j \). We use three parameters to reject flashers. They are \( N_{\text{match}}, r_{\text{match, max}} \) and \( r_{\text{offset}} \). \( N_{\text{match}} \) is the number of “match” events and \( r_{\text{match, max}} \) is the maximum \( r_{\text{match}} \). \( r_{\text{offset}} \) is 0.15 if the PMT with the maximum P.Es is within 75 cm from that of the most “match” event. Otherwise \( r_{\text{offset}} \) is zero. Figure 38 shows the \( r_{\text{match, max}} + r_{\text{offset}} \) versus \( N_{\text{match}} \) distribution and the selection criteria. The region pointed by arrow is rejected by the 4th reduction. Less than 1% of the events categorized as atmospheric neutrino and nucleon decay events by eyescan are rejected by this reduction. The probability of rejecting proton decays is estimated to be less than 1%. It is estimated from Figure 38 (b), because the number of proton decay M.C. events is not so many.

![Figure 38](image)

4.5 5th reduction

This is the final reduction process, in which five criteria are applied to reject remaining cosmic-ray muons, flashers and electric noises.

• (5-a) \( N_{\text{hit}} > 9 \)
• (5-b) \( N_{\text{hit}}(\text{entrance}) \geq 4 \)
• (5-c) \( g_p < 4, N_{\text{hit}}(\text{mini}) \geq 6 \)
• (5-d) \( N_{hitac}(early) \geq 5 \), total P.Es < 1000  
  or  
  \( N_{hitac}(early) \geq 10 \)

• (5-e) \( N_{hit}(q < 1.) > 500 \), \( N_{hit}(q > 1.) < 200 \)

Events satisfying either of these criteria are eliminated. \( N_{hitac} \) is the number of OD hit PMTs in the cluster. Criterion (5-a) eliminates the remaining cosmic-ray muons and events whose vertices are out of the ID. Moreover, criterion (5-b) eliminates the remaining cosmic-ray muons. \( N_{hitac}(entrance) \) is the number of OD hit PMTs within 8 m from the reconstructed entrance point. The entrance point is determined by back-extrapolating the vertex reconstructed by “vertex fit” (see section 5.1). Criterion (5-c) is the long tail flasher cut that is basically the same; however, tighter than the 3rd reduction. \( g_p \) is the goodness defined in “vertex fit” (see section 5.1). \( N_{hit}(min) \) is the same definition as that of the 3rd reduction. \( N_{hitac}(early) \) is the maximum number of OD hits in a 200 ns timing window between ~8000 ~ 800 ns from the event trigger. Criterion (5-d) eliminates so called “invisible muon”. “invisible muon” is a Michel electron event. However, since the muon momentum is so close to or below the Cherenkov threshold, the event is not triggered by the muon. Muon Cherenkov light is searched for. Criterion \( N_{hitac}(early) \geq 5 \) is applied in case that total P.Es < 1000. \( N_{hit}(q < 1.) \) and \( N_{hit}(q > 1.) \) are the numbers of hit PMTs whose detected number of P.Es is less than 1. and more than 1. Electric noises are rejected by Criterion (5-e). They have a strange peak in the number of P.Es distribution. It is due to flashing of fluorescent lamp in a hut. A typical electric noise event is shown in Figure 39. The 5th reduction is applied to \( p \rightarrow \pi K^+ \) M.C.. No events are rejected.

4.6 final rejection
The bad run/subrun cuts tighter than section 4.3 are applied. It is the final bad run/subrun rejection. Criteria is

• the number of bad PMTs of the ID is by 20 more than usual level.

• the number of bad PMTs of the OD is by 7 more than usual level.

Moreover, runs the duration of which is shorter than 30 minutes are rejected. It is to reject junk runs, which are caused by the data acquisition troubles.

Figure 39: An event display of a typical electric noise event.

Finally we check all the runs from the run logs. The runs where someone works in the detector are rejected. Most of these types of runs are not “normal run” mode. However, several runs remain. Therefore these runs are rejected by hand. About 6% of time are categorized to the bad run/subrun by these criteria.

The events induced by the neutrino beam from Tsukuba are rejected. This neutrino beam is used in K2K experiment. These events can be tagged using GPS system. The probability of rejecting nucleon decay and atmospheric neutrino data from this cut is less than \( 10^{-3} \) event [50]. That is all of the data selection processes. The selected data are applied to the event reconstruction processes. Figure 40 shows the time variance of event rate of each reduction step after all of the bad run/subrun rejection. The event rate of raw data increases because PMTs gain increases (see section 6.4.2). Recently repeating type flashers increase drastically.

4.7 livetime calculation
The livetime of each subrun is calculated in the online computer. The total livetime is calculated by summing all the livetime of subruns after rejecting the bad subruns which is selected in section 4.3 and 4.6. The data using in this thesis are taken from May 27 1996 to March 17 2001. The total livetime is 1367 days which corresponds to 84 kt-year.
Figure 40: Time variance of event rate of each reduction step. "1st" means the event rate after 1st reduction. "final sample" means the data set after applying $E_{vis} > 30$ MeV, $d_{wall} > 200$ cm cuts (see section 7.1)
5 event reconstruction

The procedure of the event reconstruction is explained in this section. Physical quantities are reconstructed for the events selected by the data reduction processes (section 4). Figure 41 shows the reconstruction flow. There are seven steps in the event reconstruction.

vertex fit \downarrow
ring fit \downarrow
particle identification \downarrow
MS vertex fit \downarrow
momentum determination \downarrow
muon decay electron fit \downarrow
ring correction

Figure 41: The flow of the event reconstruction.

5.1 vertex fit

The vertex position where Cherenkov light originates is reconstructed. In the “vertex fit” routine there are the following three processes.

1. point fit
2. ring edge finding
3. precise vertex fit

5.1.1 point fit

First, to estimate the vertex position roughly, we assume that Cherenkov light is emitted from one point \((x, y, z)\). The timing of the \(i\)-th PMT after the time of flight subtraction is then expressed as

\[
t_{0,i} = t_i - \frac{n}{c} \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}
\]  

(21)

t_i \text{ is the time of the } i\text{-th PMT, } c \text{ is light velocity and } n \text{ is the refractive index of water. } (x_i, y_i, z_i) \text{ is the } i\text{-th PMT position. In the ideal case } t_{0,i} \text{ follows the delta function distribution. However, in the actual case PMTs have a finite timing resolution and Cherenkov light scatters in water. We define the goodness function } (g_p).

\[
g_p = \frac{1}{N_{hit}} \sum_i \exp\left(\frac{-(t_{0,i} - \langle t \rangle)^2}{2\sigma_p^2}\right)
\]  

(22)

\(N_{hit}\) is the number of hit PMTs. \(\sigma_p\) is typical timing resolution (3.5 ns). \(\langle t \rangle\) corresponds to the peak in the \(t_{0,i}\) distribution. \((x, y, z)\) is searched for so that \(g_p\) becomes maximum.

5.1.2 ring edge finding

In this step the particle direction(\(\vec{r}\)) and Cherenkov ring edge(\(\theta_{edge}\)) are determined. \(\theta_{edge}\) is defined such that the following conditions are satisfied.

- \(\frac{d^2PE(\theta, r)}{d\theta^2}|_{\theta=\theta_{edge}} = 0\)
- \(\theta_{edge}\) is more than the peak value of \(\theta\)

\(PE(\theta, r)\) is the corrected number of P.E.s as a function of the angle(\(\theta\)) with respect to the particle direction(\(\vec{r}\)). The particle direction is determined such that the following \(Q(\theta_{edge}, \vec{r})\) takes the maximum value.

\[
Q(\theta_{edge}, \vec{r}) = \frac{\int^{\theta_{edge}}_{\theta} PE(\theta, \vec{r}) d\theta}{\sin \theta_{edge}} \left(\frac{dPE(\theta, \vec{r})}{d\theta}|_{\theta=\theta_{edge}}\right)^2
\]

\[
\times \exp\left(-\frac{\left(\theta_{edge} - \theta_{exp}\right)^2}{2\sigma^2}\right)
\]  

(23)

\(\theta_{exp}\) is the expected Cherenkov angle from the P.E.s. The above two processes are iterated until the precise \(\theta_{exp}\) and the particle direction are obtained. Figure 42 shows the typical \(PE(\theta)\) and its second derivative distributions.
using $g_i$ and $g_o$.

$$g_k = \frac{g_i + g_o}{\sum_i \frac{1}{\sigma_i^2}}$$ (25)

The vertex position is determined where $g_k$ takes the maximum value. In multi-ring events the vertex is determined by only this “vertex fit”. The vertex resolution in $p \rightarrow \pi^0K^+,K^+ \rightarrow \pi^+\pi^0$ M.C. is shown in Figure 43. The resolution is 37 cm.

Figure 43: The vertex resolution in $p \rightarrow \pi^0K^+,K^+ \rightarrow \pi^+\pi^0$ M.C.. Filled area shows 68% of total events.

5.1.3 precise vertex fit

The vertex position is determined more precisely by considering the scattering light and particle track length. Two goodness functions, $g_i, g_o$, are defined as

$$g_i = \sum_i \frac{1}{\sigma_i^2} \exp\left(-\frac{(t_{0,i} - \langle t \rangle_i)^2}{2(1.5\sigma_i)^2}\right)$$ (24)

$$g_o = \sum_i \frac{1}{\sigma_i^2} \max\left[\exp\left(-\frac{(t_{0,i} - \langle t \rangle_i)^2}{2(1.5\sigma_i)^2}\right), 0.8 \exp\left(-\frac{t_{0,i} - \langle t \rangle_i}{t_{delay}}\right)\right]$$

$g_i$ is the goodness calculated using the PMTs inside the Cherenkov ring edge. $\langle t \rangle$ is the mean of $t_{0,i}$, $\sigma_i$ is the timing resolution as a function of the charge. The track length is calculated using the PMTs within the 70° half opening angle towards the particle direction and $t_i$ is recalculated by considering the track length.

$g_o$ is the goodness calculated by the PMTs outside the Cherenkov ring edge. The effect of the scattered light is considered. $t_{delay}$ is a typical timing delay of the scattered light and is set to be 20 ns from the M.C. study. The total goodness function is defined

5.2 ring fit

In “ring fit”, the number of rings and the direction and Cherenkov angle of each ring are reconstructed using the vertex reconstructed by “vertex fit”. The number of rings is counted from two. The method of finding the $i$-th ring is

1. search for the $i$-th ring candidate
2. determine whether the candidate is probable or not
3. If it is true, go back to step 1 and search for the $(i + 1)$-th ring candidate. Otherwise the number of ring is determined as $(i - 1)$. 

31
5.2.1 search for the next ring candidate

The next ring candidate is searched for using the Hough transformation [51] which is the technique for a pattern recognition. The scheme of the Hough transformation in the Cherenkov ring search is shown in Figure 44. First, P.E.s of each PMT is corrected for the PMT acceptance and attenuation length and contributions from the known rings are subtracted. From the given P.E.s, a special charge map $HS(\Theta, \Phi)$ is made where $\Theta, \Phi$ are polar and azimuthal angles at the reconstructed vertex, respectively. The P.E.s are mapped to the point for which the opening angle toward the PMT is $\Theta$ with a weight of the expected P.E.s $Q_e(\theta)$, which is the expected P.E. distribution for 500 MeV/c electron as a function of the opening angle toward the PMT($\theta$). The point where $HS(\Theta, \phi)$ is maximum is identified as the direction of the candidate ring.

![Figure 44: The scheme of Hough transformation. Solid small circles show hit PMTs and hatched area shows Cherenkov ring. These data transform to Dashed circles. Most probable direction is pointed by center.](image)

\[
L(N + 1) = - \sum_{i,j} \log \text{prob}(q_{ij}) \sum_{j} \alpha_j \cdot q_{ij}^{\text{exp}}
\]

(26)

The probability function $\text{prob}(q_{\text{obs}}, q_{\text{exp}})$ is expressed as Equation (44). PMTs inside the cone whose half opening angle is $1.2\theta_j$, which is the half opening angle of the $j$-th ring. Moreover the likelihood that $N$ rings exist is calculated as

\[
L(N) = - \sum_{i,j} \log \text{prob}(q_{ij}) \sum_{j} \alpha_j \cdot q_{ij}^{\text{exp}}
\]

(27)

If $L(N) > L(N + 1)$, the number of rings is determined as $N$. Moreover four evaluation functions, $F_1, F_2, F_3$ and $F_4$ are used.

$F_1$ $F_1$ is the corrected $(L(N + 1) - L(N))$ for the total P.E.s.

$F_2$ The P.E.s around the Cherenkov ring edge of the $(N + 1)$-th ring($Q_{\text{peak}}^N, Q_{\text{out}}^N$) are calculated. $Q_{\text{peak}}^N$ should be large if the candidate ring is probable.

$F_3$ The P.E.s around and outside the Cherenkov ring edge of $(N + 1)$-th ring($Q_{\text{peak}}^{N+1}, Q_{\text{out}}^{N+1}$) are calculated. The difference($Q_{\text{peak}}^{N+1}, Q_{\text{out}}^{N+1}$) should be large if the candidate ring is probable.

$F_4$ If the $N$ ring assumption is true, the residual P.E.s for each PMT is $q_{\text{res}}^i = q_i - q_i^{\text{exp}}$ is small. The absolute value of the summation of vector $\sum_i q_{\text{res}}^i \cdot OP$ is calculated where $OP$ is the vector from the vertex position to $i$-th PMT position. If $|\sum_i q_{\text{res}}^i \cdot OP|$ is large, the candidate ring is probable.

Using these evaluation functions, the total evaluation function is calculated.

\[
F = F_1 + F_2 + 6F_3 + 1.5F_4
\]

(28)

If $F < 0$, the number of rings is determined as $N$. Otherwise, the $(N + 1)$-th ring is probable and the $(N + 2)$-th ring is searched for back to step 1.

In $p \rightarrow K^+, K^+ \rightarrow \mu^+ \nu$ M.C., 94% are determined as 1 ring events. In $p \rightarrow K^+, K^+ \rightarrow \pi^+\pi^-\pi^0$ M.C., 57% are determined as 2 ring events.
5.3 particle identification

Particle identification is carried out using the Cherenkov ring pattern and Cherenkov angle.

5.3.1 particle identification with Cherenkov ring pattern

An Electron and gamma-ray make an electromagnetic shower. Secondary particles in the shower suffer multiple Coulomb scattering due to their low energies, and hence the edge of the Cherenkov ring is blurred and not sharp. We call this ring pattern e-like. On the other hand, a muon, charged pion and proton don’t make showers in the momentum region of 0~1,000 MeV/c for muon and charged pion and 0~2,000 MeV/c for proton. These particles make single Cherenkov rings. We call this type of the ring pattern μ-like. Typical e-like and μ-like events are displayed in Figure 45.

The particle identification using the Cherenkov ring pattern is performed by comparison between the detected P.E.s in the j-th PMT, $q_{i,j}$, and expected P.E.s in each PMT, $q_{i,j}^{exp}$. $q_{i,j}$ is the observed number of P.E.s in the i-th PMT for the j-th ring. $q_{i,j}^{exp}$ is calculated by the summation of P.E.s expected from direct light and scattered light as follows:

$$ q_{i,j}^{exp} = q_{i,j}^{dir} + q_{i,j}^{scat} $$

To estimate the expected P.E.s for electrons, electron M.C. events in non light attenuated water are generated and $q_{MC,j}(p_e, \theta_i)$ was made. $q_{MC,j}(p_e, \theta_i)$ is the averaged number of P.E.s arrived in a 50 cm diameter circle located on a sphere of 16.9 m in radius. $p_e$ is the electron momentum and $\theta_i$ is the half opening angle towards the electron direction.

Expected P.E.s for direct light, $q_{i,j}^{dir}(e)$, for electron is expressed using $q_{MC,j}(p_e, \theta_i)$.

$$ q_{i,j}^{dir}(e) = \alpha_e \cdot q_{MC,j}(p_e, \theta_i) \cdot \left( \frac{16.9}{l_i} \right)^{1.5} \cdot \exp \left( -\frac{l_i}{L} \right) \cdot f(\Theta_i) $$

$\alpha_e$: normalization constant for electrons.
$\theta_i$: half opening angle of the i-th PMT towards the j-th ring direction.
$l_i$: the distance from i-th PMT to the vertex.
$L$: light attenuation length in water.
$f(\Theta_i)$: relative PMT photo sensitive area shown in Figure 47.

The light attenuation is corrected for by $\exp(-l_i/L)$. For muons we need to consider the change of the Cherenkov angle due to the ionization loss. Figure 46 shows the relation between PMT and Cherenkov photons. The expected P.E.s from the direct light for muon is expressed as

$$ q_{i,j}^{dir}(\mu) = \left( \frac{\alpha_\mu \cdot \sin^2 \theta_{i,j}}{l_i (\sin \theta_{i,j} + l_i (\frac{leff}{l_i}))} + q_{i,j}^{kno}(\mu) \right) \cdot \exp \left( -\frac{l_i}{L} \right) \cdot f(\Theta_i) $$

$q_{i,j}^{kno}$ is the number of P.E.s from knock-on electrons as a function of $\Theta_i$, which is estimated by M.C..
Figure 45: Typical $\gamma$-like(left) and $\mu$-like(right) event displays. These are M.C. events. The momenta of electron and muon are 279 MeV/c and 362 MeV/c, respectively.

The observed photons are categorized to the direct and scattered photons using the timing information.

$$30\text{ns} < t_i - t_0 < 2\sigma_i + 5\text{ns} \rightarrow \text{direct photons}$$
$$2\sigma_i + 5\text{ns} < t_i - t_0 \rightarrow \text{scattered photons}$$

$t_0$ is the peak of the TOF subtracted timing distribution and $\sigma_i$ is the measured timing resolution. The probability function is defined as follows using $\text{prob}(q_{\text{obs}}, q_{\text{exp}})$ defined in Equation (44).

$$\text{Prob}(e/\mu) = \begin{cases} 
\text{prob}(q_i, q_{\text{exp}}(e/\mu) + \sum_{j \neq i} q_{\text{exp}}^j) & \text{for direct photons} \\
\text{prob}(q_{\text{dir}}(e/\mu) + \sum_{j \neq i} q_{\text{exp}}^j) \times \text{prob}(q_i, q_{\text{calc}}^i + q_i - q_{\text{dir}}^i) & \text{for scattered photons}
\end{cases}$$

The likelihoods for the electron and muon assumptions are calculated using $\text{Prob}(e/\mu)$.

$$L(e/\mu) = \prod_{0 < \theta_i < 1.5\theta_c} \text{Prob}(e/\mu) \quad (33)$$

The maximum likelihood is searched for by moving the particle direction and Cherenkov angle. The estimated Cherenkov angle is used in the particle identification (see section 5.3.2). The likelihood is transformed into $\chi^2$ in order to combine with the result using the Cherenkov angle.

$$\chi^2(e/\mu) = \frac{1}{\log_{10}e} \cdot [-\log_{10}L(e/\mu) - \text{constant}] \quad (34)$$

The probabilities from the Cherenkov ring pattern are calculated as

$$P_j^{\text{charge}}(e) = \exp \left( -\frac{1}{2} \left( \frac{\chi^2(e) - \min[\chi^2(e), \chi^2(\mu)]}{\sigma^2} \right)^2 \right)$$

$$\sigma^2 = \sqrt{2N_D}$$

where $N_D$ is the degree of freedom, namely, the number of PMTs used in Equation (33).

5.3.2 particle identification with Cherenkov angle

Moreover, the Cherenkov angle is used. The Cherenkov angle depends on the particle mass. Figure 48 shows the Cherenkov angle distribution of each particle.

The particle identification with the Cherenkov angle is carried out by comparison between the observed and expected Cherenkov angle. The observed Cherenkov angle ($\theta$) is estimated by the Cherenkov ring pattern fit (see section 5.3.1). The expected Cherenkov angle ($\theta_{\text{exp}}$) is estimated from the expected momentum. The probability is calculated as

$$P_j^{\text{angle}}(e/\mu) = F \cdot \exp \left( -\frac{1}{2} \left( \frac{\theta_{\text{exp}}(e/\mu) - \theta_j}{\Delta \theta_j} \right)^2 \right) \quad (36)$$

$\Delta \theta$ is an error of the reconstructed $\theta$. A typical $\Delta \theta$ is $2\degree$. The momentum of each particle is reconstructed by RTOT, which is corrected for P.E.s
defined as Equation (39). Therefore the expected Cherenkov angle is determined by $RTOT$. Figure 49 shows the Cherenkov angle distribution as a function of $RTOT$. It is effective to identify the particle using the Cherenkov angle in the low $RTOT$ region.

5.3.3 **total particle identification**

Particle identification is carried out using both Cherenkov ring pattern and Cherenkov angle. The total probability is calculated by the following equation.

$$P_{j}^{prob}(e/\mu) = P_{j}^{charge}(e/\mu) \times P_{j}^{angle}(e/\mu)$$  \hspace{1cm} (37)

The ring is identified whether it is $e$-like or $\mu$-like by the following equation.

$$P_{j}^{PID} = \sqrt{\log P_{j}(\mu)} - \sqrt{\log P_{j}(e)}$$  \hspace{1cm} (38)

$P_{j}^{PID} > 0$ corresponds to $\mu$-like, and vice versa. The $P_{j}^{PID}$ distribution of data and atmospheric neutrino M.C. are shown in Figure 50. In CC quasi-elastic events of the atmospheric neutrino M.C. (visible energy is 50~1000 MeV), the contamination of the misidentification probability is 0.6% for $\nu_e$ and 1.3% for $\nu_\mu$. Moreover, this method of particle identification was checked with a 1 kton water Cherenkov detector with beams of several hundreds MeV/c electrons and muons at KEK[52].

Figure 48: Cherenkov angle distribution of electron, muon, charged pion and proton as a function of momentum.

Figure 49: Cherenkov angle distribution of electron, muon and proton as a function of $RTOT$. $RTOT$ is corrected P.E.s defined as Equation (39). $RTOT=300$ corresponds to 60 MeV/c for electron, 233 MeV/c for muon and 1430 MeV/c for proton.

Figure 50: Performance of particle identification. Each figure shows $P_{j}^{PID}$ distribution of data (upper) and CC$\nu_e$, CC$\nu_\mu$ and NC interaction of atmospheric M.C. (lower). These data are selected by criterion that visible energy is between 50~1000 MeV.
5.4 MS vertex fit

The resolution of the vertex reconstructed by "vertex fit" is good enough to determine the number of rings and particle identification. However, in single-ring events, the vertex along the particle direction is not well determined, because "vertex fit" is performed using only the timing information. The precise vertex position is essential for the $^{16}\text{O} \rightarrow \pi K^{+15}N\gamma$, $K^+ \rightarrow \mu^+\nu$ analysis (see section 7.3). An improved method is applied as following steps.

- 1. vertex fit in the particle direction
- 2. vertex fit in the perpendicular direction

Since the particle type has already been identified in section (5.3), the ring pattern can be estimated. Step 1 of "MS vertex fit" searches for the position where the observed and expected ring pattern best match by moving the vertex along the particle direction. The pattern matching is carried out using the likelihood function defined in the particle identification with the Cherenkov ring pattern (see section (5.3.1)). In step 2, the vertex position with the best goodness defined by Equation (25) is searched for by moving the vertex perpendicular to the particle direction. These steps are iterated until the vertex and direction difference from the previous ones is less than 5 cm and 0.5°. The performance of the "MS vertex fit" is shown in Figure 51. The vertex resolution is better than "vertex fit", 34 cm for $e$-like and 25 cm for $\mu$-like events.

However, in some $^{16}\text{O} \rightarrow \pi K^{+15}N\gamma$, $K^+ \rightarrow \mu^+\nu$ events, the vertex is reconstructed poorly owing to the Michel electron. Therefore, after applying all the above procedures another vertex fit is applied to those events having $\geq 1$ in-gate type Michel electrons and with the number of total P.E.s less than 1,200. The method is the same as "MS vertex fit". However, the PMTs that have the timing residual between -50 and 100 ns are used. It is tighter than that of "MS vertex fit". The vertex resolution of $p \rightarrow \pi K^+, K^+ \rightarrow \mu^+\nu$ M.C. is shown in Figure 52. The vertex resolution is 47 cm.

Figure 51: The vertex resolution along the particle direction for "MS vertex fit" (upper) and "vertex fit" (lower) from M.C. study. Left figures show $e$-like events and right figures show $\mu$-like events.

Figure 52: The vertex resolution for $p \rightarrow \pi K^+, K^+ \rightarrow \mu^+\nu$ M.C. Filled area show 68% of total events.
5.5 momentum determination

The particle momentum is determined by $RTOT_j$. $RTOT_j$ is the number of P.E.s of the $j$-th ring corrected for light attenuation in water, PMT acceptance and coverage. $RTOT_j$ is calculated by the following equation:

$$RTOT_j \equiv A \times \frac{G}{G_{MC}} \times \sum_{\theta_{ij} < 70^0 \ \ \ \ \ \ \ \ \ \ |\theta_{ij}| < 260} q_{ij} \times \frac{1}{\exp(-t_{ij}/L)} \times \cos \Theta_{ij} \times f(\Theta_i)$$

(39)

$A : A = \exp(-16.9/55.0)$ normalization factor.
$G$: PMT gain parameter. It is calculated from through going cosmic muons (see section 6.4.2). In M.C. we use $G_{MC}$.
$q_{ij}$: the P.E.s detected by $i$-th PMT. It is calculated by ring separation (see section 5.8).
$t_{ij}$: the distance from $i$-th PMT to vertex.
$L$: light attenuation length in water.
$f(\Theta_{ij})$: Relative PMT photo sensitive area shown in Figure 47.
$\theta_{ij}$: opening angle towards the $j$-th ring direction.
$t_{ij}^{res}$: the time of $i$-th PMT from the peak of timing residual.

We use the PMTs selected by $\theta_{ij} < 70^0$ in order to eliminate the scattered light. Moreover, the timing cut $-50 < t_{ij}^{res} < 250$ ns is applied. It is for eliminating the effect of Michel electrons. The gain difference between data and M.C. is corrected for by $G/G_{MC}$. The calculated $RTOT_j$ is converted to the momentum. The conversion function made by M.C. is shown in Figure 53. Moreover, the resolution of the reconstructed momentum for single ring events is shown in Figure 53. Resolutions for electrons and muons are $(2.5/\sqrt{p(GeV)+0.5})\%$ and $\sim 3\%$, respectively.

5.6 Michel electron fit

Michel electrons are categorized to three of the following detection types:

- (a) in-gate type ($0.1 < t_{\mu e} < 0.8$ µsec)
- (b) split type ($0.8 < t_{\mu e} < 1.2$ µsec)
- (c) sub-event type ($1.2 < t_{\mu e} < 30$ µsec)

$t_{\mu e}$ is reconstructed lifetime of the muon. The signal of the type (a) electron is included in the primary event gate. The electrons of type (c) decay outside the gate of the primary event and triggered as “sub-event” (see section 2.2.5). In type (b), some Cherenkov photons are detected in the primary event gate and others are detected as “sub-event”. We do not count type (b) electrons in the proton decay analysis. To search for the type (a) electrons, another peak of the PMT timing distribution is searched for after the primary event peak. We required more than 20 hits in 30 ns window above the background level. To search for the type (c) electrons, we required the following criteria to “sub-event”:

- $t_{\mu e} < 30$ µsec
- $N_{hit} > 50$
- $g_{\mu e} > 0.5$
- $N_{hit}(50\text{ns}) > 30$
- total P.E.s < 2000

$N_{hit}$ is the number of hit PMTs in “sub-event”.$g_{\mu e}$ is the goodness function defined as Equation (18). $N_{hit}(50\text{ns})$ is the maximum number of hit PMTs in a 50 ns TOF subtracting timing window.

5.7 ring correction

“ring correction” checks whether each of the reconstructed rings is true or fake. If the momentum of the rings is very low, it is rejected as a fake ring. The selection criteria is as follows:

- (1a) $E_i < E_j$
- (1b) $\theta_{ij} < 30^0$
- (1c) $E_i \cos \theta_{ij} < 60$ MeV
- (2a) $E_i/\sum_j E_j < 0.05$
- (2b) $E_i < 40$ MeV

$E_i, E_j$ are visible energies of the $i$-th and $j$-th rings, respectively. $\theta_{ij}$ is the opening angle between the $i$-th and $j$-th ring directions. If the $i$-th ring satisfies (1a, 1b, 1c) and/or (2a, 2b), it is rejected.

5.8 ring separation

“ring separation” allocates P.E.s to each ring in multi-ring events. It is used in “ring fit”, “particle identification” and “momentum determination”.

37
Figure 53: Each figure shows: (left) $RTOT$ distribution for electron, muon and charged pion as a function of momentum, (right) reconstructed momentum resolution for electron, muon and charged pion as a function of momentum.

Expected P.E.s of the $i$-th PMT for the $j$-th ring ($q_{i,j}$) is expressed as

$$q_{i,j} = q_i \times \frac{q_{i,j}^{\text{exp}}}{q_i^{\text{exp}}} (q^{\text{exp}}_i = \sum_{j=1}^{n} q_{i,j}^{\text{exp}})$$  \hspace{1cm} (40)$$

$q_i$: the number of P.E.s of $i$-th PMT
$q_{i,j}$: the number of P.E.s of $i$-th PMT to $j$-th ring contribution
$q_{i,j}^{\text{exp}}$: the expected P.E.s of $i$-th PMT
$q^{\text{exp}}_i$: the expected P.E.s of $i$-th PMT from $j$-th ring contribution

The initial $q_{i,j}$ is calculated assuming that all particles are electrons. Using this $q_{i,j}$, corrected P.E.s $q'_{i,j}$ is calculated from $q_{i,j}$,

$$q'_{i,j} = q_{i,j} \times A \times \frac{1}{\exp(-l/L)} \times \frac{1}{f(\Theta_i)}$$  \hspace{1cm} (41)$$

$A$, $l_i$, $L$, $f(\Theta_i)$ is the same definition as Equation (39). Correction terms are explained in Equation (39).

From the $q'_{i,j}$, the expected P.E.s distribution as a function of the half opening angle towards the particle direction ($Q_j^{\text{exp}}(\theta)$) is calculated.

$$Q_j^{\text{exp}}(\theta_{i,j}) = \alpha_{i,j} \times q_{i,j}^{\text{exp}}$$  \hspace{1cm} (42)$$

To reestimate the ring separation of the observed P.E.s, the following likelihood function $L_j$ is defined.

$$L_j = - \sum_{\theta_{i,j} < \theta_0} \left( \log[\text{prob}(q_i, \sum_{j}^{n} (\alpha_{j}q_{i,j}^{\text{exp}}))] \times W_{i,j} \right)$$  \hspace{1cm} (43)$$

$\alpha_j$ is the normalization factor of the $j$-th ring, $W_{i,j}$ is the weight factor which enhances the contribution from around and inside the Cherenkov ring. $W_{i,j}$ is defined as $\sqrt{Q_j^{\text{exp}}(\theta_{i,j}) \min[1, \sqrt{q_{i,j}^{\text{exp}}} / \theta_0]}$ where $\theta_0$ is the reconstructed Cherenkov angle of the $j$-th ring. prob($q_{\text{obs}}, q_{\text{exp}}$) is the probability function of observing $q_{\text{obs}}$ with expected $q_{\text{exp}}$. The definition of prob($q_{\text{obs}}, q_{\text{exp}}$) is as follows:

$$\text{prob}(q_{\text{obs}}, q_{\text{exp}}) = \begin{cases} \text{conv} \{PE_{\text{one}}(q), P(q_{\text{obs}}, q_{\text{exp}}) \} & \text{for } q_{\text{exp}} < 20 \text{ P.E.s} \\ \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(q_{\text{obs}}-q_{\text{exp}})^2}{2\sigma^2}) & \text{for } q_{\text{exp}} > 20 \text{ P.E.s} \end{cases}$$  \hspace{1cm} (44)$$

It is calculated for $q_{\text{exp}} < 20$ P.E.s by convolving the measured one P.E. distribution and Poisson distribution $P(r | \mu) \equiv (\mu^r e^{-\mu}) / (r!)$, $\sigma$ is defined as $\sigma^2 = (1.2 \sqrt{Q_i^{\text{exp}}})^2 + (0.1q_{\text{exp}})^2$. The maximum $L_j$ is searched for by changing $\alpha_j$. Finally the separated P.E.s $q_{i,j}$ is calculated by given the best $\alpha_j$.

$$q_{i,j} = q_i \times \frac{\alpha_j \cdot q_{i,j}^{\text{exp}}}{\sum_{j=1}^{n} (\alpha_j \cdot q_{i,j}^{\text{exp}})}$$  \hspace{1cm} (45)$$
6 calibration

6.1 PMT timing calibration

It is important to calibrate the timing information of the PMT signals, because the precise timing is essential for the determination of the event vertex position. The timing response depends on:

- the transit time of a PMT
- the pulse height of a PMT (time walk)
- the cable length of a PMT

The timing response is measured using a calibration system shown in Figure 54. The light source is a N₂ laser which emits 337 nm in wavelength and a dye laser module converts it to 384 nm which is close to the Cherenkov light wavelength. The light intensity can be changed by an optical filter. The light goes to a diffuser ball located in the ID. The diffuser ball consist of a TiO₂ tip which is located at the center of the diffuser ball and LUDOX which is silica gel with 20 nm glass fragments. The light reflected by the tip is diffused by the LUDOX. The time jitter and the width of the laser light are small enough (±0.5 ns and ∼3 ns, respectively). The typical timing versus pulse height is shown in Figure 55. We call it “TQ-map”.

Each channel has its own TQ-map and the timing information is corrected for by the TQ-map in physics analyses. Figure 56 shows the timing resolution distribution. The timing resolution (RMS) at 1 P.E. is 2.8 ns.

Figure 54: A schematic view of timing calibration system.

Figure 55: Typical timing versus pulse height distribution (TQ-map).

Figure 56: Timing resolution distribution as a function of Q(P.E.).
6.2 PMT gain calibration

Before we started to take data in Super-Kamiokande, the PMT gain was calibrated by a light source and the high voltage values of PMTs were determined. The PMT gain was fine tuned after the experiment started. The PMT gain is calibrated by two methods. The absolute gain is calibrated by low energy gamma-rays and relative gain is calibrated by a Xe lamp.

6.2.1 relative gain calibration

The relative calibration system is shown in Figure 57. Light from the Xe lamp passes through the U.V. filter and is injected into a scintillator ball in the ID. The scintillator is an acrylic ball doped with BBOT scintillator and MgO powder. The BBOT scintillator absorbs U.V. light and emits light whose wavelength is close to that of Cherenkov light. The MgO powder is used to diffuse the light uniformly. The light intensity of the Xe lamp is monitored by two photo-diodes and a scintillator viewed with a PMT. This measurement is performed at several positions regularly. Observed P.E.s in each channel are corrected for light attenuation, PMT acceptance and anisotropy of the scintillator ball. The corrected P.E.s in each channel are compared with each other in Figure 58. The spread of the PMT gain is 7%.

![Figure 57: Relative gain calibration system](image)

Figure 58: The corrected charge distribution of all PMTs in the relative gain calibration. The charge is normalized by the mean value.

6.2.2 absolute gain calibration

Gamma-rays emitted from thermal neutron captured Ni(Ni(γ,Ni))N* are used for the absolute gain calibration. Neutrons are produced by the spontaneous fission of 252Cf. The expected number of P.E.s is one, because the energy of the gamma-rays are low (6~9 MeV). Therefore we can measure the one P.E. distribution. Figure 59 shows the observed one P.E. distribution. One P.E. is determined as 2.055 pC from this measurement. The absolute gain of the PMTs is 1×10^7.

![Figure 60: PMT noise measurement](image)

Figure 60: The time variance of the dark noise rate. The mean of the dark noise rate is 3.2 kHz. The time variance of the dark noise rate is very stable. We use this measured value in our simulation.

6.3 PMT dark noise measurement

The PMT dark noise rate is very important for 16O → 7K + 15Nγ,K → μ+ν search (section 7.3). The PMT dark noise rate is measured by events with more than 1,000 hit PMTs, corresponding to more than 100 MeV in visible energy, mostly cosmic-ray muons. The number of hits in the 162 ns window just before the trigger timing are counted and the mean of the dark noise rate is calculated. Figure 60 shows the time variance of the dark noise rate. The mean of the dark noise rate is 3.2 kHz. The time variance of the dark noise rate is very stable. We use this measured value in our simulation.
6.4 water transparency measurement

The light attenuation length in the detector water is measured by two methods. One is to use a laser and CCD camera and the other is by through-going cosmic muons.

6.4.1 measurement by a laser and CCD camera

The system with a laser and CCD camera is shown in Figure 61. The light is produced by a N₂ laser and is converted to the monochromatic light by a dye. The produced wavelength is 337, 365, 400, 420, 460, 500 and 580 nm. The light goes to a diffuser ball in the ID. The light intensity coming from several different points is measured by the CCD camera at the top of the ID. The light intensity is monitored by the PMT. The measured attenuation length are summarized in Figure 62. The comparison between this measurement and M.C. parameter are shown in section 3.1.4.

6.4.2 through-going cosmic muon measurement

Cosmic-ray muons enter the detector at 2.2 Hz. The water transparency can be measured using Cherenkov light emitted by the cosmic-ray muons. The measured attenuation length at monochromatic wavelengths cannot be measured. However, these data can be taken regularly. Therefore the stability of the water transparency is measured. Moreover the
PMT gain stability is checked by this measurement. To select vertical through-going muons, the following criteria are applied.

- (a) $5000 < \text{total P.E.} < 125000$.
- (b) $r_{\text{in}} = \sqrt{x_{\text{in}}^2 + y_{\text{in}}^2} < 15.9 \text{ m}$, $z_{\text{in}} > 18.1 \text{ m}$.
- (c) $r_{\text{out}} = \sqrt{x_{\text{out}}^2 + y_{\text{out}}^2} < 15.9 \text{ m}$, $z_{\text{out}} < 18.1 \text{ m}$.
- (d) $\sqrt{(x_{\text{in}} - x_{\text{out}})^2 + (y_{\text{in}} - y_{\text{out}})^2} < 5 \text{ m}$

Muons with an additional radiation or hadrons are rejected by criterion (a). Coordinates of the entrance and exit points are defined as $(x_{\text{in}}, y_{\text{in}}, z_{\text{in}})$ and $(x_{\text{out}}, y_{\text{out}}, z_{\text{out}})$, respectively. Vertically going muons which go through near the center of the ID are selected by criteria (b,c,d). For events selected by criteria (a~d), $q_{\text{corr}}$ is calculated by the following equation.

$$q_{\text{corr}} = q \times l \times \frac{1}{f(\Theta)}$$  \hspace{1cm} (46)

$q$ : the detected P.E.s at a PMT
$l$ : the travel length of Cherenkov photon
$f(\Theta)$ : PMT acceptance as shown in Figure 47

Figure 63 shows the typical distribution of corrected P.E.s ($q_{\text{corr}}$) as a function of the photon travel length ($l$). This distribution is fitted by the following equation.

$$q_{\text{corr}}(l) = G \times \exp\left(-\frac{l}{L}\right)$$  \hspace{1cm} (47)

$G$ is proportional to the PMT gain and $L$ is the attenuation length. In an event shown in Figure 63, $L$ is 101 m and $G$ is 405 P.E.s.

The time variance of the attenuation length and PMT gain are shown in Figure 64. The mean attenuation length is 110 m. The gain has increased by 10% for five years. In the momentum reconstruction, the corrected P.E.s from this measurement are used.

Figure 64: Time variance of the attenuation length (a) and PMT gain (b) distributions from through-going cosmic muon measurement. The regions pointed by arrows show data used in this analysis.

6.5 energy calibration

The absolute energy is calibrated by the following methods.

- Michel electron from cosmic muon
- $\pi^0$ produced by neutrino interactions
- cosmic ray stopping muon (Cherenkov angle)
- cosmic ray stopping muon (range)

6.5.1 Michel electron from cosmic muon

Michel electrons are a good calibration source at ~ 50 MeV. The positron energy spectrum (Michel spectrum) is calculated by V-A theory analytically:

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{12\pi^2} m_{\mu}^2 E_e^2 (3 - \frac{4E_e}{m_{\mu}}), \quad E_e \leq \frac{m_{\mu}}{2}$$  \hspace{1cm} (48)

$E_e$: positron energy  
$m_{\mu}$: muon mass  
$G_F$: Fermi coupling constant

However, the energy spectrum of $\mu^-$ is slightly different from that of $\mu^+$, because most of $\mu^-$ are bound in $^{10}\text{O}$. $\mu^+/\mu^-$ ratio used in M.C. are 1.37 that is measured by M. Yamada et al.[53]. The difference of the mean energy is less than 1%. We select Michel electrons satisfying the following selection criteria.

- (a) $100 < N_{\text{hit}} < 210$
- (b) $1.5 < t_{\mu e} < 8.0 \mu\text{sec}$
- (c) $d_{\text{wall}} > 200 \text{ cm}$

$N_{\text{hit}}$ is the number of hit PMTs in the ID. $t_{\mu e}$ is the timing interval between a stopping muon and the Michel electron. $d_{\text{wall}}$ is the distance from the wall of the ID to the reconstructed Michel electron vertex. In Figure 65, the distribution of the electron momentum is shown.

6.5.2 $\pi^0$ produced by neutrino interaction

$\pi^0$ is produced by atmospheric neutrino interactions. The invariant mass of $\pi^0$ is also a calibration source at a few hundreds MeV/c. The selection criteria are as follows:

- (a) $d_{\text{wall}} > 200 \text{ cm}, E_{\text{vis}} > 30 \text{ MeV}$
- (b) 2-ring
- (c) $e$-like, $\bar{e}$-like
- (d) no Michel electron

Figure 65: Reconstructed momentum distribution for Michel electrons from cosmic ray muon. Full circles show data and histogram shows M.C.

d_{\text{wall}}$ is the distance from the wall of the ID to the reconstructed vertex. $E_{\text{vis}}$ is the energy reconstructed as electron. Figure 66 shows the distribution of the reconstructed $\pi^0$ mass ($m_{\pi^0}$). The number of events that satisfy all the criteria and $85 < m_{\pi^0} < 185 \text{ MeV/c}^2$ are 436. The fitted peak value is 140.9 MeV/c$^2$ for data and 141.3 MeV/c$^2$ for M.C. The discrepancy of the fitted peak between data and M.C. is less than 1%.

6.5.3 cosmic-ray stopping muon (Cherenkov angle)

We use the Cherenkov angle of cosmic ray stopping muons for energy calibration. It is effective for the low momentum region ($p < 400 \text{ MeV/c}$), because $d\theta/dp$ is large where $\theta$ is the Cherenkov angle. The muon data are selected by the following criteria.

- (a) $200 < \text{total P.E.s} < 1500$
- (b) one cluster of hit PMTs in the OD
- (c) entrance point is on the top wall, $r < 14.9 \text{ m}$, $\cos\theta_z > 0.94$
- (d) 1 Michel electron, $t_{\mu e} > 0.8 \mu\text{sec}$
Figure 66: The distribution of reconstructed $\pi^0$ mass for events that satisfy criteria (a), (b), (c), (d). Full circles show data and hatched boxes show atmospheric neutrino M.C. normalized by livetime. The region pointed by arrows shows criterion $85 < m_{\pi^0} < 185$ MeV/c$^2$.

Criterion (a) corresponds to a region in which the muon momentum is roughly 190~500 MeV/c. $\cos \theta_z$ is the zenith angle of the muon. Vertical downward-going muons are selected by criteria (b), (c). $t_{\mu e}$ is a timing interval between the stopping muon and Michel electron. Criterion (d) is required in order not to be affected by the P.E.s of Michel electrons. The Cherenkov angle of the selected data is reconstructed using the method of the particle identification (see section 5.3). The reconstructed Cherenkov angle versus momentum distribution is shown in Figure 67. The comparison between data and M.C. are shown in Figure 68. They agree within 2.5% level.

6.5.4 cosmic-ray stopping muon (range)

Momentum estimation from the muon range can be another calibration method. We required the following criteria in order to select stopping muons.

- (a) 200 < total P.E.s
- (b) one cluster of hit PMTs in the OD

Figure 67: Reconstructed Cherenkov angle versus momentum distribution for cosmic ray stopping muons. Figure (a) and (b) show data and M.C., respectively.

Figure 68: Figure (a) shows the ratio of the reconstructed and expected momentum for cosmic ray stopping muons. The expected momentum is reconstructed by P.E.s. Full circles show data and open circles show M.C.. Figure (b) shows the ratio of the momentum of data and M.C.
• (c) entrance point is on the top wall, \( \cos \theta_z > 0.94 \)
• (d) 1 Michel electron, \( t_{\mu e} > 0.8 \mu\text{sec} \)
• (e) \( d_{\mu e} > 7 \text{ m} \)

\( \cos \theta_z \) and \( t_{\mu e} \) is the same definition as section 6.5.3. Vertical downgoing muons are selected by criteria (b,c). \( d_{\mu e} \) is the distance between reconstructed muon and electron vertex. The resolutions of the vertex reconstruction of muon and electron are less than 50 cm. The muon range versus (momentum/range) distributions are shown in Figure 69. The data within the flat distribution region are used by criterion (e). The momentum deposit per 1cm is \( \sim 2.5 \text{ MeV/c} \). Data and M.C. are compared in Figure 70. Data and M.C. agree within 2.5% level.

6.5.5 summary of energy calibration

The absolute energy calibrations are performed using five different methods. The measured difference between data and M.C., shown as a function of momentum in Figure 71, is within 5% level in the momentum range from \( \sim 200 \text{ MeV/c} \) to a few GeV/c.

Figure 71: Summary of absolute energy calibrations. The difference between data and M.C. as a function of momentum are shown.
7 \ p\rightarrow \bar{\nu}K^{+} \ mode \ analysis

Since \ \bar{\nu} \ cannot \ be \ detected, \ the \ main \ signal \ of \ proton \ decay \ is \ provided \ by \ K^{+}. \ The \ momentum \ of \ the \ K^{+} \ produced \ by \ p(\text{free \ proton})\rightarrow \bar{\nu}K^{+} \ is \ 340 \ MeV/c. \ Because \ the \ interaction \ cross \ section \ of \ the \ low \ momentum \ K^{+} \ is \ small, \ more \ than \ 90\% \ of \ K^{+} \ exit \ from \ the \ 16O \ nucleus \ without \ interaction. \ The \ decay \ ratio \ of \ K^{+} \ is \ as \ follows:

\[
\begin{align*}
    K^{+} & \rightarrow \mu^{+}\nu_{\mu} \quad (63.5\%) \\
    & \rightarrow \pi^{+}\pi^{0} \quad (21.5\%) \\
    & \rightarrow \pi^{+}\pi^{+}\pi^{-} \quad (5.6\%)
\end{align*}
\]

Therefore we searched for the two dominant modes, \ K^{+} \rightarrow \mu^{+}\nu_{\mu} \ and \ K^{+} \rightarrow \pi^{+}\pi^{0}. \ Two \ separate \ methods \ were \ used \ to \ search \ for \ K^{+} \rightarrow \mu^{+}\nu_{\mu}. \ First, \ the \ characteristics \ of \ the \ final \ data \ sample \ are \ shown.

7.1 \ characteristics \ of \ data \ sample

We obtained the data sample after applying the event reconstruction tools. \ E_{\text{vis}} >30 \ MeV, \ d_{\text{wall}} >200 \ cm \ cuts \ are \ applied \ at \ the \ end. \ E_{\text{vis}} \ is \ the \ electron-equivalent \ energy. \ d_{\text{wall}} \ is \ the \ distance \ from \ the \ nearest \ wall \ of \ the \ ID \ to \ the \ vertex. \ The \ event \ rate \ of \ the \ final \ sample \ is \ \sim 8 \ event/day. \ The \ time \ variance \ of \ the \ event \ rate \ is \ shown \ in \ Figure \ 72. \ Most \ data \ are \ atmospheric \ neutrino \ event. \ The \ event \ rate \ is \ stable \ except \ the \ last \ two \ points. \ This \ decrease \ is \ caused \ by \ the \ maximum \ solar \ activity. \ Figure \ 74,75 \ shows \ vertex \ distributions, \ which \ are \ almost \ uniform \ in \ the \ fiducial \ volume. \ The \ number \ of \ rings \ is \ shown \ in \ Figure \ 73. \ Neutrino \ oscillations[34] \ are \ not \ taken \ into \ account \ in \ the \ atmospheric \ neutrino \ M.C.. \ Therefore \ the \ number \ of \ observed \ events \ is \ smaller \ than \ that \ of \ M.C.. \ However \ the \ shape \ of \ distribution \ is \ similar.

Table 12: Number of events in data sample

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>atmospheric M.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ring</td>
<td>7532</td>
<td>8954.7</td>
</tr>
<tr>
<td>2ring</td>
<td>2095</td>
<td>2514.0</td>
</tr>
<tr>
<td>\geq3ring</td>
<td>1566</td>
<td>1973.9</td>
</tr>
<tr>
<td>total</td>
<td>11193</td>
<td>13442.6</td>
</tr>
</tbody>
</table>

Figure 73: Number of ring distribution. Points show data and histogram shows atmospheric neutrino M.C.. M.C. is normalized by livetime.

Figure 72: Time variance of event rate of final sample. The mean of event rate is 8.2 event/day.
Figure 74: $r^2$ distributions for events that satisfy $|z| \leq 16.1$ m. Each figure shows: (left) Full circles show data and histogram shows atmospheric neutrino M.C. (right) $p \rightarrow K^+M.C$. The region pointed by arrow shows the fiducial volume.

Figure 75: $z$ distributions for events that satisfy $|r| \leq 14.9$ m. Each figure shows: (left) Full circles show data and histogram shows atmospheric neutrino M.C. (right) $p \rightarrow K^+M.C$. The region pointed by arrow shows the fiducial volume.
The momentum distributions of e-like and μ-like events are shown in Figure 76. Though e-like data agree with M.C. well, μ-like data show the strong deficit due to neutrino oscillation[34]. Evidence for neutrino oscillation $\nu_\mu \rightarrow \nu_\tau$ was reported by Super-Kamiokande collaboration in 1998[35] using the atmospheric neutrino data. Not only other atmospheric neutrino experiments, MACRO and Soudan2, but also an accelerator-based experiment, K2K[50], give support to the result of Super-Kamiokande. Neutrino oscillation should be taken into account in background estimation. Figure 77 shows the ratio of single-ring data to M.C. as a function of the reconstructed momentum, from which the number of expected backgrounds is normalized by multiplying 0.68 for C.C. $\nu_\mu$ events and 1.07 for N.C. and C.C. $\nu_\tau$ events. These numbers are calculated using μ-like and e-like single ring event ratio shown in Table 13.

Table 13: The number of e-like and μ-like events in data sample. Only single ring events are used. M.C. is normalized by livetime.

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>atmospheric M.C.</th>
<th>(data)/(M.C.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e-like</td>
<td>3990</td>
<td>3721.3</td>
<td>1.07</td>
</tr>
<tr>
<td>μ-like</td>
<td>3542</td>
<td>5225.2</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Figure 76: Momentum distributions. Points show data and histograms show atmospheric neutrino M.C.. Each figure shows: (upper) e-like and (lower) μ-like single ring events. M.C. is normalized by livetime.

Figure 77: The ratio of data to atmospheric neutrino M.C. as a function of momentum. Full circles show μ-like and open circles show atmospheric neutrino M.C.
7.2 \( p \rightarrow \bar{p} K^+, K^+ \rightarrow \mu^+ \nu \) search

Figure 78 shows a schematic view of \( p \rightarrow \bar{p} K^+, K^+ \rightarrow \mu^+ \nu \). Selection criteria for this analysis is expressed as (A1~A5)

- (A1) 1 ring
- (A2) \( \mu \)-like
- (A3) \( 215 < p_\mu < 260 \text{ MeV/c} \)
- (A4) 1 Michel electron
- (A5) not (B1~B8)

Since 90\% of the \( K^+ \) stop before decaying to \( \mu^+ \), the \( \mu^+ \) momentum is mono-energetic at 236 MeV/c. To tag this muon, we apply the criteria (A1~A4). Figure 79 displays a typical event of \( p \rightarrow \bar{p} K^+, K^+ \rightarrow \mu^+ \nu \) M.C..

\[ \chi^2(a_{at}, a_{bd}) = \sum_{i=1}^{3} \frac{(N_{\text{data}}(i) - (a_{at} \cdot N_{at}(i) + a_{bd} \cdot N_{bd}(i)))^2}{N_{\text{data}}(i)} \]  

(i=1,2,3 correspond to 200~215 MeV/c, 215~260 MeV/c and 260~300 MeV/c, respectively.)

- \( a_{at} \): fitting parameter for backgrounds
- \( a_{bd} \): fitting parameter for the signal
- \( N_{\text{data}}(i) \): the number of events in data
- \( N_{at}(i) \): the number of events in atmospheric neutrino M.C.
- \( N_{bd}(i) \): the number of events in proton decay M.C.

The result is shown in Table 14. The number of events at the best fit is 209.6 for backgrounds and 8.3 for signals. There is no excess around 236 MeV/c. The upper limit of the excess from proton decay is estimated. Since minimal \( \chi^2 = 0.26 \) is located in the unphysical region (\( a_{bd} < 0 \)), \( \chi^2 \) at the 90\% C.L upper limit is calculated by the following equation.

\[ \frac{\int_{-\infty}^{0}(x-a_{min})^2 \exp(-x^2/2)dx}{\int_{-\infty}^{0} \exp(-x^2/2)dx} = 0.9 \]
\( a_{pd}^{\text{min}} (\approx -0.031) \) is minimal \( a_{pd} \), \( a_{pd}^{(0)} \) is minimal \( a_{pd} \) in the physical region and \( a_{pd}^{(0.013)} \) is at the 90% C.L. upper limit. The upper limit of \( a_{pd} \) at 90% C.L. \( (a_{pd}^{(0.013)}) \) is determined as the maximum value by changing the value \( a_{at} \) for the fixed \( \chi^2_{00} \). The results are shown in Table 14. The upper limit number of events for signal is 16.0.

Table 14: The results of \( \chi^2 \) fit of muon spectrum

<table>
<thead>
<tr>
<th></th>
<th>( \chi^2 )</th>
<th>( a_{at} )</th>
<th># of B.G.</th>
<th>( a_{pd} )</th>
<th># of signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimal</td>
<td>0.26</td>
<td>-0.054</td>
<td>209.6</td>
<td>-0.031</td>
<td>~380</td>
</tr>
<tr>
<td>90% C.L.</td>
<td>8.21</td>
<td>0.046</td>
<td>178.6</td>
<td>0.013</td>
<td>16.0</td>
</tr>
</tbody>
</table>

Figure 81 shows the fitting results. Lower limit of proton lifetime from this search \( (\tau/B(p \rightarrow \nu K^+)) \) is calculated by the following equation.

\[
\tau/B(p \rightarrow \nu K^+) = \frac{1}{\chi_{\text{limit}}} \epsilon BT \tag{51}
\]

\( \chi_{\text{limit}} \) : the upper limit of the number of signal.
\( \epsilon B \) : the detection efficiency of this mode.
\( T \) : the detector exposure

\( \chi_{\text{limit}} = 16.0, T = 84.05(\text{kt-year}) \times 3.33 \times 10^{32}(\text{kt}) = 2.8 \times 10^{31} \) and the detection efficiency is 35%. Therefore the limit of this search is \( 6.1 \times 10^{32} \) years (90% C.L.).
Figure 80: Reconstructed muon momentum distribution for events that satisfy criteria (A1,A2). Each figure shows: (left) Full circles shows data and histogram shows atmospheric neutrino M.C. normalized by livetime and neutrino oscillation. (right) $p \rightarrow K^+\nu M.C.$ The region pointed by arrows shows criterion (A3).

Figure 81: Comparison between data and fitting results of muon momentum distribution for events that satisfy criteria (A1,A2,A4,A5). Full circles show data with statistical errors. Solid line shows atmospheric neutrino M.C. plus $p \rightarrow K^+\nu M.C.$ 90\% C.L. lower limit. Dashed line shows atmospheric neutrino M.C. contribution from solid line. Left figure shows 3 bin plots used in the spectrum fit and right figure shows smaller binning plots.
7.3 $^{16}\text{O} \rightarrow \pi^0 K^{+15}\gamma, K^+ \rightarrow \mu^+\nu$ search

The other method is applied for $K^+ \rightarrow \mu^+\nu$ search, because the former method (section 7.2) is dominated by backgrounds. When a proton in $^{16}\text{O}$ decays, the remaining $^{15}\text{N}$ nucleus is left in either the ground state or excited states. From the latter case a prompt gamma-ray is emitted. The level scheme of $^{15}\text{N}$ is shown in Figure 13 and their estimated gamma-ray emission probability is shown in Table 5 [25]. The most significant branch is the 6.32 MeV gamma-ray from the p3/2 hole state. This emission probability is 41% [25]. Figure 82 shows the scheme of $p \rightarrow \pi K^+$ in $^{16}\text{O}$.

- (B5) $g > 0.6$
- (B6) $d_{\mu e} < 150$ cm
- (B7) $t_{\mu-e} < 100$ ns
- (B8) $7 < N_{hit\gamma} < 60$

The selection criteria (B1–B4) are the same as the former method. Criteria (B5–B6) are applied for rejecting backgrounds caused by the poor vertex reconstruction. Most of this type of backgrounds are recoil protons produced by neutral current interactions. The Cherenkov threshold of protons is 1.1 GeV/c. Because the Cherenkov angle of protons is much smaller than that of muons, the vertex is mis-reconstructed by the “MS vertex fit” routine (section 5.4). Since the muon is assumed in the fit, a fake peak, which mimics gamma-rays, is sometimes produced in TOF distribution at the wrong vertex position. To reject these events, a goodness cut $g \leq 0.6$ is applied in (B5). $g$ at the position reconstructed by “MS vertex fit” is calculated by Equation (25). This goodness is used in “vertex fit” routine (see section 5.1). Figure 83 shows the goodness distribution. The hatched region shows the recoil protons. 98% of $p \rightarrow \pi K^+$, $K^+ \rightarrow \mu^+\nu$ M.C. survive by the criterion (B5). However, criterion (B5) cannot reject all of the recoil protons. Therefore we apply the other criterion (B6) which uses the distance between the reconstructed muon and the Michel electron vertex. A muon from the $K^+$ decay is expected to travel 55 cm and decays to an Michel electron as shown in Figure 82. To estimate the vertex resolution, $d_{\mu e}$ is defined as

$$d_{\mu e}(cm) = |\vec{\mu e} - 55\times \vec{d}_{\mu}^r|$$  \hspace{1cm} (52)

$\vec{\mu e}$ is a vector from the reconstructed muon to the Michel electron vertex. $\vec{d}_{\mu}^r$ is a unit vector of the reconstructed muon direction. $d_{\mu e}$ should be close to zero. Only events which have sub-event type Michel electrons are applied, because vertex resolution is not good in in-gate type Michel electrons. Figure 84 shows the $d_{\mu e}$ distribution. 95% of $p \rightarrow \pi K^+$, $K^+ \rightarrow \mu^+\nu$ M.C., which have sub-event type Michel electrons, survive criterion (B6).

Figure 82: A schematic view of $^{16}\text{O} \rightarrow \pi K^{+15}\gamma$, $K^+ \rightarrow \mu^+\nu$. Particles with a ring emit Cherenkov light and can be detected as Cherenkov rings. Particles with dotted ring emits Cherenkov light, however a Cherenkov ring is not reconstructed due to low momentum.

Since the $K^+$ is below the Cherenkov threshold and the $K^+$ lifetime is 12.6 ns, we can separate the gamma-ray signal from the $\mu^+$ signal. By tagging the prompt gamma-ray as well as the $\mu^+$ and $e^+$, most backgrounds are eliminated. The selection criteria for this analysis are

- (B1) 1 ring
- (B2) $\mu$-like
- (B3) 215 < $p_\mu$ < 260 MeV/c
- (B4) 1 Michel electron
Figure 83: Goodness distributions for events that satisfy criteria (B1~B4). Each figure shows: (left) Full circles shows data and histogram shows atmospheric neutrino M.C. normalized by livetime and neutrino oscillation. Filled area shows proton-recoil events (protons have more than 1,300 MeV/c). (right) $p\rightarrow\bar{v}K^+\text{M.C.}$ The region pointed by arrows shows criterion (B5).

Figure 84: $d_{\mu\text{e}}$ distribution for events that satisfy criteria (B1~B4). Only sub-event type Michel electrons are used. Each figure shows: (left) Full circles shows data and histogram shows atmospheric neutrino M.C. normalized by livetime and neutrino oscillation. Filled area shows proton-recoil events (protons have more than 1,300 MeV/c). (right) $p\rightarrow\bar{v}K^+\text{M.C.}$ The region pointed by arrows shows criterion (B6).
Criteria (B7,B8) are required for the nuclear gamma-ray tagging. The three quantities, \( t_{\mu} \), \( t_{\gamma} \) and \( N_{hit}\gamma \) are calculated. \( t_{\mu} \) is the decay time of \( K^+ \rightarrow \mu^+\nu \) and \( t_{\gamma} \) is the decay time of \( ^{16}\text{O} \rightarrow \pi K^+\gamma \). \( N_{hit}\gamma \) is the number of hit PMTs caused by the prompt gamma-ray. Figure 85 shows a schematic view of the prompt gamma-ray search. \( dN(t)/dt \) is calculated after TOF subtraction, where \( N(t) \) is the number of hit PMTs as a function of time. \( t_{\mu} \) is defined as the time when \( dN(t)/dt \) is maximum, \( t_{\gamma} \) and \( N_{hit}\gamma \) is determined so that the number of hits in a 12 ns window takes maximum. \( N_{hit}\gamma \) is the maximum number of hits at \( t_{\gamma} \) in the 12 ns window. However, PMTs outside the 50° opening angle with respect to the muon is used in the \( t_{\gamma} \) determination in order to reject hit PMTs from the muon. \( t_{\mu} \) is searched for in the region of \( t \leq t_{\mu0} \). \( t_{\mu0} \) is where \( dN_{hit}\mu/dt = 0 \).

![Figure 85: A schematic view of gamma-ray search.](image)

\( t_{\mu} - t_{\gamma} \) corresponds to the decay time of \( K^+ \rightarrow \mu^+\nu \). In Figure 86 \( t_{\mu} - t_{\gamma} \) is compared with the real decay time of \( K^+ \rightarrow \mu^+\nu \). The resolution is within 10 ns. Events with \( t_{\mu} - t_{\gamma} > 100 \) ns are rejected by criterion (B7). The dark noise rate of the PMTs is 3 kHz/PMT. The expected hit in a 12 ns window from the dark noise is 0.4. Therefore this effect is negligible. Figure 87 shows the \( N_{hit}\gamma \) versus \( t_{\mu} - t_{\gamma} \) distribution. Five events remain in 70 year atmospheric neutrino M.C.. The expected number of backgrounds is estimated to be 0.2 event for 1,367 live days. The detection efficiency of proton decays is 8.6%. No event is found in the signal region.

The upper limit of the number in the signal region \( x_{\text{limit}} \) is calculated by the following equation.

\[
\frac{\int_0^{x_{\text{limit}}} P(N_{\text{obs}}, N_{\text{exp}}(x)) dx}{\int_0^{\infty} P(N_{\text{obs}}, N_{\text{exp}}(x)) dx} = 0.90 \tag{53}
\]

\[
N_{\text{exp}}(x) = X_{BG} + x \tag{54}
\]

\( P(r, \mu) \) : Poisson distribution \( P(r, \mu) = \frac{\mu^r e^{-\mu}}{r!} \)

\( N_{\text{obs}} \) : the number of event in the signal region.

\( X_{BG} \) : the expected number of backgrounds.

The obtained \( x_{\text{limit}} \) is substituted in Equation (51). The lower limit of the partial lifetime for this decay mode is \( 1.0 \times 10^{33} \) years at 90% confidence level (C.L.). Figure 88 shows the \( N_{hit}\gamma \) distribution compared with M.C.. The characteristics of the remaining backgrounds are summarized in Table 15. The poor vertex reconstruction caused three background events.

![Figure 86: Real decay time \( t_{K^{+}\mu\nu} \) of \( K^+ \rightarrow \mu^+\nu \) versus \( t_{\mu}-t_{\gamma} \) distribution in \( p \rightarrow \pi K^+\) M.C.. Solid line shows \( t_{K^{+}\mu\nu} = t_{\mu}-t_{\gamma} \).](image)
Figure 87: $N_{\text{hit}}$ versus $t_{\gamma}-t_{\gamma}$ distributions for events that satisfy criteria (B1~B6). Each figure shows: (upper-left)atmospheric neutrino M.C., (upper-right)data and (lower-left)$p\rightarrow \nu K^+M.C$. The boxes show criterion (B7, B8).
Figure 88: $N_{\text{hit}\gamma}$ distributions for events that satisfy criteria (B1~B7). Each figure shows: (left) Full circles shows data and histogram shows atmospheric neutrino M.C. normalized by livetime and neutrino oscillation. (right) $p \rightarrow \nu K^+\text{M.C.}$. The region pointed by arrow shows criterion (B8).

Table 15: The characteristics of background events to $^{16}\text{O} \rightarrow \pi K^+\gamma N\gamma$, $K^+ \rightarrow \mu^+\nu$ search, based on 70 years M.C.

<table>
<thead>
<tr>
<th>neutrino energy</th>
<th>interaction mode</th>
<th>vertex error</th>
<th>reason for the remaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.43 GeV</td>
<td>$\nu p \rightarrow \nu p$</td>
<td>255 cm</td>
<td>proton-recoil event. 1.4 GeV/c proton makes $\pi^+$. Michel electron is in-gate type.</td>
</tr>
<tr>
<td>0.33 GeV</td>
<td>$\overline{\nu}_\mu p \rightarrow \mu^+ n$</td>
<td>195 cm</td>
<td>$\mu^+$ momentum is 224 MeV/c. $\mu^+$ has in-gate type Michel electron.</td>
</tr>
<tr>
<td>1.35 GeV</td>
<td>$\nu_{\mu} n \rightarrow \mu^- p \pi^0$</td>
<td>615 cm</td>
<td>proton-recoil event. $\mu^-$ is invisible. $\pi^0$ is absorbed. Michel electron is in-gate type.</td>
</tr>
<tr>
<td>0.40 GeV</td>
<td>$\nu_{\mu} n \rightarrow \mu^- p$</td>
<td>81 cm</td>
<td>$\mu^-$ momentum is 205 MeV/c.</td>
</tr>
<tr>
<td>0.26 GeV</td>
<td>$\overline{\nu}_\mu p \rightarrow \mu^+ n$</td>
<td>40 cm</td>
<td>$\mu^+$ momentum is 184 MeV/c. Michel electron is in-gate type. decay time is 110 ns.</td>
</tr>
</tbody>
</table>
Figure 89 shows the $N_{\text{hit}}$ distribution for p3/2 and s1/2 state of the $^{15}\text{N}^*$ proton hole. p3/2 and s1/2 contribution is 80% and 19%, respectively.

Figure 89: $N_{\text{hit}}$ distributions for $p \rightarrow \bar{p}K^+\text{M.C.}$ events that satisfy criteria (B1~B7). Solid line shows proton decay from p3/2 state. Dashed line shows proton decay from s1/2. The region pointed by arrow shows criterion (B8).
7.4 $p \rightarrow \bar{\nu}K^+, K^+ \rightarrow \pi^+\pi^0$ search

Figure 90 shows a schematic view of $p \rightarrow \bar{\nu}K^+, K^+ \rightarrow \pi^+\pi^0$. Most of the $K^+$ decays at rest. The $\pi^+$ decays to $\mu^+\nu$ and the $\mu^+$ decays to $e^+\nu_e$. The $\mu^+$ is invisible. Figure 91 shows an event display of typical $p \rightarrow \bar{\nu}K^+, K^+ \rightarrow \pi^+\pi^0$. M.C. The selection criteria (C1~C6) are required in this analysis.

- (C1) 2 ring
- (C2) e-like, e-like
- (C3) 1 Michel electron
- (C4) $175 < p_{\pi^0} < 250$ MeV/c
- (C5) $85 < m_{\pi^0} < 185$ MeV/c²
- (C6) $Q_{in} < 100$ P.E., $Q_{out} < 60$ P.E., $Q_{out} < 1.7 \times Q_{in} - 25$.

![Diagram of particle interactions](image)

Figure 90: A schematic view of $p \rightarrow \bar{\nu}K^+, K^+ \rightarrow \pi^+\pi^0$. Particles with a ring emit Cherenkov light and can be detected Cherenkov ring. Particle with a dotted ring ($\pi^+$) emits Cherenkov light, however a Cherenkov ring is not reconstructed due to its low momentum. Particles with dashed arrow are invisible.

![Event display](image)

Figure 91: An event display of typical $p \rightarrow \bar{\nu}K^+, K^+ \rightarrow \pi^+\pi^0$. M.C.

Expected to be mono-energetic at 205 MeV/c. To detect this $\pi^0$, criteria (C1,C2,C4,C5) are applied. Figure 92 shows the distribution of the particle identification likelihood. 94% of $p \rightarrow \bar{\nu}K^+, K^+ \rightarrow \pi^+\pi^0$ M.C. are determined as e-like, e-like events. $m_{\pi^0}$ and $p_{\pi^0}$ are the reconstructed $\pi^0$ mass and momentum, respectively. Figure 93 shows the number of Michel electrons. The distributions of the reconstructed $\pi^0$ momentum and mass are compared with M.C. in Figure 94, 95, respectively. Data agree with atmospheric neutrino M.C. within statistical errors. Moreover Figure 96 shows the distribution of the reconstructed $\pi^0$ mass versus the reconstructed $\pi^0$ momentum.

Though the detection efficiency of Michel electrons from the $\pi^+$ is low, Criterion (C3) is required for the backgrounds rejection. Because the only two rings of $\pi^+$ are reconstructed, most of the remaining backgrounds do not have Michel electrons. Figure 93 shows the multiplicity distribution of the Michel electrons. 80% of remaining backgrounds after applying criteria (C1,C2) are rejected.

Since the $K^+$ decays to a $\pi^+$ and a $\pi^0$ at rest, these two particles go back-to-back. The $\pi^0$ momentum is
Figure 92: The distributions of the particle identification likelihood of both rings for events that satisfy criterion (C1). Each figure shows: (left) Full circles shows data and histogram shows atmospheric neutrino M.C. normalized by livetime and neutrino oscillation. (right) p→πK+M.C. The regions pointed by arrow show e-like.

Figure 93: The distributions of the number of Michel electron for events that satisfy criteria (C1, C2). Each figure shows: (left) Full circles shows data and histogram shows atmospheric neutrino M.C. normalized by livetime and neutrino oscillation. (right) p→πK+M.C.. The regions pointed by arrow show criterion (C3).
Figure 94: The distributions of reconstructed $\pi^0$ momentum for events that satisfy criterion (C1~C3). Each figure shows: (left) Full circles show data and histogram shows atmospheric neutrino M.C. normalized by livetime and neutrino oscillation. (right) $p\to K^+\text{M.C.}$. The region pointed by arrow shows criterion (C4).

Figure 95: The distributions of reconstructed $\pi^0$ mass for events that satisfy criterion (C1~C3). Each figure shows: (left) Full circles show data and histogram shows atmospheric neutrino M.C. normalized by livetime and neutrino oscillation. (right) $p\to K^+\text{M.C.}$. The region pointed by arrow shows criterion (C5).
Figure 90: The distributions of the reconstructed $\pi^0$ mass versus the reconstructed $\pi^0$ momentum for events that satisfy (C1~C3). Each figure shows: (upper-left) atmospheric neutrino M.C., (upper-right) data, (lower-left) $p \rightarrow \pi K^+ M.C.$ The boxes show criteria (C4, C5).
The $\pi^+$ momentum is so close to the Cherenkov threshold that the Cherenkov ring is not detected in most cases. However, since many backgrounds survive criteria (C1\textasciitilde C5), we use P.E.s information from the $\pi^+$. Figure 97 shows the scheme of this search. The hit PMTs in the ID are separated into three regions, (a), (b) and (c), as shown in Figure 97. Region (a) is the area within the 90° half angle towards the two gamma-ray directions. Most of P.E.s from the two gamma-rays are detected in this region. Region (b) is the area within 40° half opening angle opposite to the $\pi^0$ direction. Since the $\pi^+$ and $\pi^0$ go back-to-back, the $\pi^+$ signal should lie here. In the region (b) we search for $Q_{in}$, which is the P.E.s corrected for light attenuation and PMT acceptance. Region (c) is the remaining area. The corrected P.E.s in the region (c) ($Q_{out}$) is used for the background rejection. If no particle emits except the backward direction, $Q_{out}$ is small. Using these $Q_{in}$ and $Q_{out}$, criterion (C6) is required. The solid angle for region (b) is $0.468\pi$ and for region (c) is $0.792\pi$. If the background event emits Cherenkov light uniformly, the ratio $Q_{in}/Q_{out}$ is determined to be 1.7 by the solid angle ratio. Therefore criterion $Q_{out} < 1.7 \times Q_{in} - 25$, is applied. $25$ is determined to obtain the best S/N ratio. Figure 98 shows the $1.7 \times Q_{in} - Q_{out}$ distribution. Moreover the distribution of $Q_{in}$ versus $Q_{out}$ are shown in Figure 99. 94% of remaining backgrounds are rejected by criterion (C6).

![Figure 98: $1.7Q_{in}-Q_{out}$ distribution for events that satisfy criteria (C1\textasciitilde C5, $Q_{in} < 100$ P.E., and $Q_{out} < 60$ P.E.). The region pointed by arrow shows $Q_{out} < 1.7 \times Q_{in} - 25$.](image)

No candidate event is found in the data sample as shown in Figure 99. This is consistent with the estimated background of 0.7 events. With the detection efficiency of 6.7% the corresponding lower limit is $8.1 \times 10^3$ years at 90% C.L. The characteristics of background contributions to $p \rightarrow \nu K^+$, $K^+ \rightarrow \pi^+\pi^0$ search are shown in Table 16. Half of backgrounds are due to $\pi^0$ and $\mu$ or $\pi^\pm$. In other backgrounds electrons and $\pi^\pm$ make the $\pi^0$-like event pattern.

<table>
<thead>
<tr>
<th>interaction mode</th>
<th># of event</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_\mu p \rightarrow \mu^- \pi^+$</td>
<td>3</td>
</tr>
<tr>
<td>$\nu_\mu p \rightarrow \mu^- \pi^0$</td>
<td>2</td>
</tr>
<tr>
<td>$\nu_\mu p \rightarrow e^- \pi^+$</td>
<td>2</td>
</tr>
<tr>
<td>$\nu_\mu n \rightarrow e^- \pi^+$</td>
<td>1</td>
</tr>
<tr>
<td>$\nu N \rightarrow \nu N m_\pi$</td>
<td>3</td>
</tr>
<tr>
<td>$\nu_\mu N \rightarrow \mu^- N m_\pi$</td>
<td>2</td>
</tr>
<tr>
<td>$\nu_\mu N \rightarrow e^- N m_\pi$</td>
<td>1</td>
</tr>
<tr>
<td>$\nu p \rightarrow \nu AK^+$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 16: The characteristics of background contributions to $p \rightarrow \nu K^+$, $K^+ \rightarrow \pi^+\pi^0$ search, based on 70 years M.C..
Figure 97: A scheme of the $Q_{in}$ and $Q_{out}$ search. Upper event display shows $p \rightarrow \bar{\nu}K^+, K \rightarrow \pi^+\pi^0$ M.C. The hit PMTs are divided into three regions using reconstructed information. Lower three figures show the divided regions. They are $\pi^0$ P.E.s region (lower-left), $\pi^+$ P.E.s region (lower-center) and other region (lower-right). $Q_{in}$ and $Q_{out}$ is searched for the region as shown in the lower-center and lower-right figures, respectively.
Figure 99: The distributions of the $Q_{in}$ versus $Q_{out}$ for events that satisfy criteria (C1~C5). Each figure shows (upper-left) atmospheric neutrino M.C., (upper-right) data and (lower-left) $p\rightarrow\pi K^+ M.C.$ The box shows criteria (C6).
7.5 **systematic uncertainty**

Systematic uncertainties of lifetime limit are discussed in this section. Uncertainty from the detector exposure is negligible. Therefore main uncertainty is caused by detection efficiency.

\[ p \rightarrow \pi K^+, K^+ \rightarrow \mu^+ \nu \text{ search} \]

Main uncertainty is caused by energy scale. A systematic uncertainty of energy scale is estimated to be ±2.5% by using various calibration sources (section 6.5).

\[ {}^{16}\text{O} \rightarrow \pi K^{+15}N\gamma, K^+ \rightarrow \mu^+ \nu \text{ search} \]

The detection efficiency of this search depends on the model of residual nucleus. However, the uncertainty of the nuclear gamma-ray emission probability isn’t considered in this estimation. Uncertainty from energy scale is ≤2%. Uncertainty from goodness defined by Equation (25) is estimated to 3% by stopping muon study. Moreover uncertainty from the vertex reconstruction is estimated to 5% by stopping muon study. Therefore uncertainty from fitting bias is estimated to 6%. The total systematic uncertainty for \[ {}^{16}\text{O} \rightarrow \pi K^{+15}N\gamma, K^+ \rightarrow \mu^+ \nu \text{ search} \] is 6%.

\[ p \rightarrow \pi K^+, K^+ \rightarrow \pi^+\pi^0 \text{ search} \]

The contribution from energy scale is estimated to be ≤2% by changing \( \pi^0 \) momentum and mass, \( Q_{in} \) and \( Q_{out} \) by ±2.5%. Uncertainty of \( \pi^{+16}\text{O} \) cross section is estimated to 20% by a beam test at KEK[52]. The contribution from uncertainty of \( \pi^{+16}\text{O} \) cross section is estimated to 5% by using this value. The systematic uncertainty related to the ring fit is estimated to be 7% by comparison with the manual ring fit result. The total systematic uncertainty for \[ p \rightarrow \pi K^+, K^+ \rightarrow \pi^+\pi^0 \text{ search} \] is 9%.

The systematic uncertainties in each search are summarized in Table 17. The lifetime limit is changed by only ≤3% even if systematic uncertainties are taken into account.

<table>
<thead>
<tr>
<th></th>
<th>( p \rightarrow \pi K^+, K^+ \rightarrow \mu^+ \nu \text{ search} )</th>
<th>( {}^{16}\text{O} \rightarrow \pi K^{+15}N\gamma, K^+ \rightarrow \mu^+ \nu \text{ search} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>energy scale</strong></td>
<td>≤2%</td>
<td></td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>6%</td>
<td>6%</td>
</tr>
<tr>
<td><strong>energy scale</strong></td>
<td>≤2%</td>
<td></td>
</tr>
<tr>
<td><strong>fitting bias</strong></td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>9%</td>
<td>5%</td>
</tr>
<tr>
<td><strong>ring fit</strong></td>
<td>7%</td>
<td></td>
</tr>
<tr>
<td><strong>( \pi^+ ) interaction</strong></td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td><strong>energy scale</strong></td>
<td>≤2%</td>
<td></td>
</tr>
</tbody>
</table>
7.6 combined analysis

The proton decay search via \( p \rightarrow \bar{\nu}K^+ \) was performed using three different event topologies. No evidence for proton decay was found. The results are summarized in Table 18.

The combined upper limit of the number of signal \( (x_{\text{limit}}) \) for \( p \rightarrow \bar{\nu}K^+ \) is calculated using the following equation.

\[
\int_0^{x_{\text{limit}}} \frac{\sum_{i=1}^n P(N_{\text{obs}}(i), N_{\text{exp}}(i, x))}{\sum_{i=1}^n P(N_{\text{obs}}(i), N_{\text{exp}}(i, x))} dx = 0.90 \quad (55)
\]

\[
N_{\text{exp}}(i, x) = X_{BG}(i) + \frac{\varepsilon B_m(i) \cdot T}{\sum_{j=1}^n \varepsilon B_m(i) \cdot T} x \quad (56)
\]

\( n \): the number of proton decay search \( (n=3 \text{ in our analysis}) \)

\( i \): In our analysis, \( i=1,2,3 \) correspond to \( \bar{\nu}K^+K^+ \rightarrow \mu^+\mu \) search, \( \frac{1}{2}O \rightarrow \bar{\nu}K^+K^+ \rightarrow \mu^+\mu \) search, \( p \rightarrow \bar{\nu}K^+K^+ \rightarrow \pi^+\pi^0 \) search, respectively.

\( P(r, \mu) \): Poisson distribution \( P(r, \mu) = (\mu e^{-\mu})/(r!) \)

\( N_{\text{obs}}(i) \): the number of observed events in \( i \)-th search

\( X_{BG}(i) \): the number of expected backgrounds in \( i \)-th search

\( \varepsilon B_m(i) \): the detection efficiency in \( i \)-th search

\( T \): detector exposure

The obtained \( x_{\text{limit}} = 6.4 \) is substituted in Equation (51). The combined lower limit for \( p \rightarrow \bar{\nu}K^+ \) is \( \tau/B(p \rightarrow \bar{\nu}K^+) > 2.2 \times 10^{31} \text{ years} (90\% \text{ C.L.}) \).
<table>
<thead>
<tr>
<th>method</th>
<th>efficiency</th>
<th>background</th>
<th>candidate</th>
<th>limit (10^{12} year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow \pi K^+, K^+ \rightarrow \mu^+ \nu$ search</td>
<td>35%</td>
<td>—</td>
<td>—</td>
<td>6.1</td>
</tr>
<tr>
<td>$^{16}O \rightarrow \pi K^+, K^+ \rightarrow \mu^+ \nu$ search</td>
<td>8.6%</td>
<td>0.2</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>$p \rightarrow \pi K^+, K^+ \rightarrow \pi^+ \pi^0$ search</td>
<td>6.7%</td>
<td>0.7</td>
<td>0</td>
<td>8.1</td>
</tr>
<tr>
<td>combined</td>
<td>50%</td>
<td>—</td>
<td>—</td>
<td>22</td>
</tr>
</tbody>
</table>
8 conclusion

8.1 summary

Proton decay search via \( p \to \pi K^+ \) was carried out using 84.1 kton·year exposure (1367 days) in Super-Kamiokande. No evidence was found in three different decay modes. The lower limit of the partial proton lifetime for \( p \to \pi K^+ \) mode was obtained. Combining three independent searches, we obtained the lifetime limit of:

\[
\tau/B(p \to \pi K^+) > 2.2 \times 10^{33} \text{ years (90\% C.L.)}
\]

The result of \( ^{16}\text{O} \to \pi K^{+15}N\gamma \), \( K^+ \to \mu^+\nu \) search depends on the model of residual nucleus, because the nuclear gamma-ray tags proton decays in this search. We use the gamma-ray emission probability cited by [25]. The most significant emission probability, whose branch is 6.3 MeV gamma-ray from \( p\chi/2 \) state, is 41% in \( ^{16}\text{O} \). The previous limit obtained from Super-Kamiokande is \( 6.7 \times 10^{32} \) years[54]. The minimal SUSY SU(5) models[3] is fully excluded by this work. Moreover this result gives the strong constraints on the SUSY SO(10) models, which predicts the lifetime in \( 10^{29-34} \) years [4][5].

8.2 future prospect

The proton lifetime predicted by the SUSY SO(10) models[4][5] is comparable with the limit obtained by the present work. Figure 100 shows the sensitivity as a function of exposure.

In Super-Kamiokande or a next-generation water Cherenkov detectors, such as Hyper-Kamiokande, the method using the prompt gamma-ray, \( ^{16}\text{O} \to \pi K^{+15}N\gamma \), \( K^+ \to \mu^+\nu \) is the most viable. The most significant background is caused by the poor vertex reconstruction. If the vertex is reconstructed properly, most background will be rejected. However, the ultimate background is coming from real \( K^+ \) productions from atmospheric neutrino(Section 3.2.3). The interaction sequence is

\[
\nu^{16}\text{O} \to \nu K^{+15}N\gamma \rightarrow K^+ \rightarrow \mu^+\nu \rightarrow p\pi^- (64\%) \rightarrow n\pi^0 (36\%)
\]

Most of these interactions can be distinguished from \( p \to \pi K^+ \), because a pion from \( \Lambda \) decay emits

![Figure 100: Sensitivity of the Super-Kamiokande detector for \( p \to \pi K^+ \) search. Each line shows sensitivities for partial lifetime at 90% C.L. using the analysis of this thesis. Line (a), (b) and (c) shows the sensitivity of \( ^{16}\text{O} \to \pi K^{+15}N\gamma \), \( K^+ \to \mu^+\nu \) search, \( p \to \pi K^+, K^+ \to \pi^+\pi^0 \) search and \( p \to \pi K^+, \pi^+ \to \mu^+\nu \) search, respectively. Thick line shows the sensitivity of combined analysis.](image-url)
Cherenkov light. However, since a significant fraction of pions are absorbed in oxygen nuclei, they pose a serious background. This type of background can be estimated using our simulation. 1,000 year M.C. of $K^+$ production is generated. In total 3.682 $K^+$ are found in the fiducial volume. The same analysis (section 7) is applied. The result is shown in Figure 101. 1 event/Mt-year still remain after the selection of $^{16}O \rightarrow \pi K^{+}N\gamma$, $K^+ \rightarrow \mu^+\nu_\mu$ while 0.7 events/Mt-year remain for $p \rightarrow \pi K^+$, $K^+ \rightarrow \pi^+\pi^0$. Hence a next generation detector, such as Hyper-Kamiokande, could reach the sensitivity $\sim 10^{34}$ years with little background.
Figure 101: Left figure shows $N_{d}$ versus $t_{\mu}-t_{\gamma}$ distributions for events that satisfy criteria (B1~B6). Right figure shows distribution of the $Q_{dm}$ versus $Q_{out}$ for events that satisfy criteria (C1~C5). Both figures show 1,000 year statistics of only K+ production mode of atmospheric M.C.. Boxed regions are current criteria.
References


