博士論文

Search for Muon Neutrino Oscillations in Kamiokande and Super-Kamiokande

(カミオカンデとスーパーカミオカンデにおけるミューオン・ニュートリノ振動の探索)

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平成 9 年
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Chapter 1

Introduction

1.1 Theoretical Background of Neutrino Oscillations

Elementary particle physics has been developed by interactive activities between theories and experiments. At present, the "Standard Model" formulated by Weinberg-Salam is accepted by many physicists not only because it has a mathematically beautiful formulation but also has been tested by many experiments to incredible accuracy. Is our goal then to complete the Standard Model? Let us look at neutrinos, here is essentially unsolved problems. Neutrino masses, if any, are abnormally small in comparison with other fermions. In case of the photon, its zero mass is assured by charge conservation law, but it cannot be applied to the neutrino.

On the other hand, many models which are trying to unify the weak, electromagnetic and strong interactions predict finite neutrino masses. In this viewpoint, neutrinos are the members of the same multiplet including quarks and charged leptons. So it may be natural that neutrinos have finite masses. Table 1.1 shows the list of fermion masses (or the upper limits) from the recent experimental results.

<table>
<thead>
<tr>
<th>$m_{\nu_e} \leq 15\text{eV}$</th>
<th>$m_{\nu_\mu} \leq 170\text{keV}$</th>
<th>$m_{\nu_\tau} \leq 24\text{MeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_e = 0.5 \text{ MeV}$</td>
<td>$m_\mu = 106 \text{ MeV}$</td>
<td>$m_\tau = 1.777 \text{ GeV}$</td>
</tr>
<tr>
<td>$m_\nu = 2\sim 8 \text{ MeV}$</td>
<td>$m_\mu = 1.0 \sim 6 \text{ GeV}$</td>
<td>$m_\tau = 180\pm 12 \text{ GeV}$</td>
</tr>
<tr>
<td>$m_{\nu_\mu} = 5\sim 15 \text{ MeV}$</td>
<td>$m_\mu = 100\sim 300 \text{ MeV}$</td>
<td>$m_\tau = 4.1\sim 4.5 \text{ GeV}$</td>
</tr>
</tbody>
</table>

Table 1.1: List of fermion masses [1].

Many experiments have been carrying out to measure neutrino masses. In a direct method, the upper limit of the electron neutrino mass was derived from precise measurement of the end-point energy of the tritium $\beta$ decay spectrum. The upper limit of the muon neutrino mass comes from measuring the momentum of muons produced in the decay of pions at rest. The upper limit of the tau neutrino mass was obtained by the end-point shape of the invariant mass spectrum of 5 pions decay made of $\tau^- \rightarrow 3\pi^- + 2\pi^+ + \nu_\tau$.

On the other hand, another method to test the finite neutrino mass was considered in relatively early time by assuming the mixing of neutrino flavors in the weak interaction. If neutrinos have the finite mass, it is possible the mass eigenstates are different from the weak interaction flavor eigenstates. In such a situation, a neutrino changes its flavor to another one while it is traveling. This phenomenon is called "neutrino oscillations".

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CHAPTER 1. INTRODUCTION

Intuitively, the neutrino oscillations can be explained as follows: each neutrino mass eigenstate propagates by a different phase velocity, so the phase of each flavor eigenstate is gradually shifting. Therefore a neutrino appears with different flavor eigenstates from its initial one along traveling.

The probability that one neutrino changes to another flavor neutrino depends on the difference of masses of two neutrinos and the mixing angle parameter between them. So if the neutrino oscillations is measured, one can get the evidence of the finite neutrino masses.

1.1.1 Neutrino Oscillations in Vacuum

In general, the flavor eigenstates, $\nu_e, \nu_\mu$ and $\nu_\tau$ are not the same as the mass eigenstates, $\nu_1, \nu_2, \nu_3$ for the massive neutrinos. The transformation between those eigenstates is written by,

$$|\nu_\alpha\rangle = U_{\alpha j}|\nu_j\rangle,$$  \hspace{1cm} (1.1)

where $U$ is the transformation matrix between mass eigenstates and flavor eigenstates. $\alpha$ is the index for flavor eigenstates, $\alpha = e, \mu, \tau$ and $j$ is the index for mass eigenstates, $j = 1, 2, 3$.

Neutrino oscillations in vacuum is one of remarkable features of Quantum Mechanics. Time evolution of states obey the Schrödinger equation:

$$i \frac{d}{dt}|\nu_j\rangle = E_j|\nu_j\rangle,$$ \hspace{1cm} (1.2)

where $E_j$ is the energy of $\nu_j$. The solution for the neutrino wave function can be written as

$$|\nu_j(t)\rangle = e^{-iE_jt}|\nu_j(0)\rangle.$$ \hspace{1cm} (1.3)

Neutrinos are generated as a flavor eigenstate at the weak decay of particle and are observed as a flavor eigenstate via weak interactions, so both of initial and final states are flavor eigenstates. Hence the Schrödinger equation, Eq.(1.2), is rewritten using Eq.(1.1) as

$$i \frac{d}{dt}|\nu_\alpha\rangle = U_{\alpha j} \ E_j \ U_{j\alpha}^\dagger \ |\nu_\alpha\rangle,$$ \hspace{1cm} (1.4)

and its solution, Eq.(1.3), is presented by

$$|\nu_\alpha(t)\rangle = U_{\alpha j} \ e^{-iE_jt} \ U_{j\alpha}^\dagger \ |\nu_\alpha(0)\rangle.$$ \hspace{1cm} (1.5)

If an $\nu_\alpha$ is produced at $t = 0$, the probability of detecting this neutrino at $t = t$ can be written as

$$P(\nu_\alpha \rightarrow \nu_\alpha) \ = \ |\langle \nu_\alpha(t)|\nu_\alpha(0)\rangle|^2$$

$$\ = \ |\langle \nu_\alpha(0)|U_{\alpha j} \ e^{-iE_jt} \ U_{j\alpha}^\dagger \ |\nu_\alpha(0)\rangle|^2.$$ \hspace{1cm} (1.6)

To simplify the problem, we consider only two flavors. For example, if one assumes the neutrino mixing between $\nu_\mu$ and $\nu_\tau$, a transformation matrix is expressed as

$$U = \begin{bmatrix} \cos \theta^\circ \nu & \sin \theta^\circ \nu \\ -\sin \theta^\circ \nu & \cos \theta^\circ \nu \end{bmatrix},$$ \hspace{1cm} (1.7)
where $\theta'_v$ is the mixing angle in vacuum between $\nu_\mu$ and $\nu_\tau$ like the Cabibbo angle in the case of the quark flavor mixing. Then Eq.(1.4) is expressed as

$$i \frac{d}{dt} \left[ \begin{array}{c} |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{array} \right] = U \left[ \begin{array}{cc} E_2 & 0 \\ 0 & E_3 \end{array} \right] U^\dagger \left[ \begin{array}{c} |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{array} \right]$$

$$= \left( \frac{E_2 + E_3}{2} \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) + \frac{E_3 - E_2}{2} \left[ \begin{array}{cc} -\cos 2\theta'_v & \sin 2\theta'_v \\ \sin 2\theta'_v & \cos 2\theta'_v \end{array} \right] \left[ \begin{array}{c} |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{array} \right].$$

(1.8)

The term $(E_2 + E_3)/2$ is a common phase, so it can be omitted. Hence Eq.(1.8) can be written by

$$i \frac{d}{dt} \left[ \begin{array}{c} |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{array} \right] = \frac{E_3 - E_2}{2} \left[ \begin{array}{cc} -\cos 2\theta'_v & \sin 2\theta'_v \\ \sin 2\theta'_v & \cos 2\theta'_v \end{array} \right] \left[ \begin{array}{c} |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{array} \right].$$

(1.9)

The probability of Eq.(1.6), is expressed as

$$P(\nu_\mu \rightarrow \nu_\mu) = \left| \left[ 1 \begin{array}{cc} \cos \theta'_v & \sin \theta'_v \\ -\sin \theta'_v & \cos \theta'_v \end{array} \right] e^{-iE_2t} \left[ \begin{array}{cc} 0 & 0 \\ 0 & e^{-iE_3t} \end{array} \right] \left[ \begin{array}{cc} \cos \theta'_v & -\sin \theta'_v \\ \sin \theta'_v & \cos \theta'_v \end{array} \right] \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \right|^2$$

$$= \left| e^{-iE_2t} \cos^2 \theta'_v + e^{-iE_3t} \sin^2 \theta'_v \right|^2.$$  

(1.10)

If neutrino masses are sufficiently smaller than their momenta, the following approximation can be applied,

$$E_j = (p^2 + m_j^2)^{1/2} \approx p + \frac{m_j^2}{2p},$$

(1.11)

where $m_j$ is the mass of $\nu_j$ and $p$ is the momentum of $\nu_\mu$.

Applying the above approximation Eq.(1.9) is reduced to

$$i \frac{d}{dt} \left[ \begin{array}{c} |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{array} \right] = \frac{\pi}{L_v} \left[ \begin{array}{cc} -\cos 2\theta'_v & \sin 2\theta'_v \\ \sin 2\theta'_v & \cos 2\theta'_v \end{array} \right] \left[ \begin{array}{c} |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{array} \right].$$

(1.12)

Here, Eq.(1.10) is also reduced to

$$P_v(\nu_\mu \rightarrow \nu_\mu) \approx 1 - \sin^2 2\theta'_v \sin^2 \frac{\Delta m^2 t}{4p}$$

$$= 1 - \sin^2 2\theta'_v \sin^2 \frac{\pi L}{L_v},$$

(1.13)

where $L$ is the propagation length in the time interval of $t$ ($L = t$ for neutrinos in natural units), $L_v$ is the oscillation length in vacuum defined as

$$L_v \equiv \frac{4\pi E}{\Delta m^2} = 2.48 \frac{E \text{(GeV)}}{\Delta m^2 \text{(eV}^2)} \text{(km)},$$

$$\Delta m^2 \equiv m_3^2 - m_2^2.$$

(1.14)

(1.15)

Eq.(1.13) indicates that if $\nu_\mu$ and $\nu_\tau$ have finite different masses and there is a non-zero mixing angle between them, the number of observed $\nu_\mu$ events are less than that of generated $\nu_\mu$ events.
1.1.2 Neutrino Oscillations in Matter

Neutrino oscillations in matter was first proposed by S.P. Mikheyev and A.Yu. Smirnov based on the theory advocated by L. Wolfenstein, hence, that is often called the MSW effect \( [2], [3] \).

Electron neutrinos propagating through matter receive an additional potential energy of \( V_e \) by the charged current interaction with electrons in matter (Fig. 1.1):

\[
V_e = \sqrt{2} G_F N_e ,
\]

where \( G_F \) is the Fermi coupling constant, \( N_e \) is the electron density in matter.

![Feynman Diagrams](image)

Figure 1.1: Feynman diagrams of the neutrino scattering with electron, (a) a charged current interaction of \( \nu_e \), and (b) a neutral current interaction of \( \nu_e, \nu_\mu, \nu_\tau \).

Adding \( V_e \) to Eq. (1.4), the neutrino propagation in matter is given as

\[
i \frac{d}{dt} |\nu_\alpha\rangle = \left( U_{\alpha j} E_j U_{j\alpha}^\dagger + V_e \right) |\nu_\alpha\rangle .
\]

When a mixing angle for \( \nu_e \) and \( \nu_\mu \) is expressed by \( \theta_e \), a transformation matrix is defined by

\[
U = \begin{pmatrix}
\cos \theta_e & \sin \theta_e \\
-\sin \theta_e & \cos \theta_e
\end{pmatrix},
\]

and then Eq. (1.17) is represented as

\[
i \frac{d}{dt} \begin{bmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{bmatrix} = \left( U \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} U^\dagger + \begin{bmatrix} V_e & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{bmatrix} .
\]

Omitting common phase, Eq. (1.19) reduces to

\[
i \frac{d}{dt} \begin{bmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{bmatrix} = \frac{\pi}{L_e} \begin{bmatrix} \frac{L_e}{E_1} - \cos 2\theta_e & \sin 2\theta_e \\ \sin 2\theta_e & \cos 2\theta_e \end{bmatrix} \begin{bmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{bmatrix} ,
\]

where \( L_e \) is the oscillation length in matter defined as

\[
L_e \equiv \frac{\sqrt{2} \pi}{G_F N_e} = 1.624 \times 10^4 \text{ (km)} ,
\]
where $N_e$ is the electron density in matter in units of the Avogadro’s number per cm$^3$. Fig.1.2 shows the electron density distribution in the earth as a function of the earth radius.

The mixing matrix in matter, $U_m$, can be parameterized analogous to the vacuum mixing parameterization as

$$U_m = \begin{bmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{bmatrix}, \quad (1.22)$$

where $\theta_m$ is the mixing angle in matter.

Similarly to Eq.(1.12), the Schrödinger equation of neutrino in matter is expressed as

$$i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = \frac{\pi}{L_m} \begin{bmatrix} -\cos 2\theta_m & \sin 2\theta_m \\ \sin 2\theta_m & \cos 2\theta_m \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}, \quad (1.23)$$

Comparing Eq.(1.20) and Eq.(1.23), the relation between the oscillation lengths in vacuum and in matter is given by

$$L_m = \frac{L_v}{\sqrt{\sin^2 2\theta_v + \left(\frac{L_v}{L_m} - \cos 2\theta_v\right)^2}}, \quad (1.24)$$

and the mixing angles in vacuum and in matter have the following relation,

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_v}{\sin^2 2\theta_v + \left(\frac{L_v}{L_m} - \cos 2\theta_v\right)^2}, \quad (1.25)$$
when $m_2 > m_1$.

Assuming $\Delta m^2$ is positive, Eq.(1.25) shows that the neutrino mixing is resonantly enhanced for neutrinos at $L_e/L_\nu = \cos 2\theta_\nu$, even if the intrinsic mixing, $\sin^2 2\theta_\nu$, is small. This is the key point of the MSW effect.

In terms of the electron density, the resonance condition is represented as

$$ N_e = N_{e,\text{crit}} = \frac{\Delta m^2}{2\sqrt{2} G_F E} \cos 2\theta_\nu, \quad (1.26) $$

where $N_{e,\text{crit}}$ is the critical electron density of the resonance condition, $\Delta m^2 = m_2 - m_1$. Numerically the resonance condition is represented as

$$ 1.53 \times 10^{-4} \times \frac{E \text{ (GeV)} \cdot N_e \text{ (cm$^3$)}}{\Delta m^2 \text{ (eV$^2$)}} = \cos 2\theta_\nu. \quad (1.27) $$

Fig.1.3 shows how the effective neutrino mass squared and the effective mixing angle change according to the electron density in matter. As can be seen in Fig.1.3, if $N_e \ll N_{e,\text{crit}}$, $\theta_m \simeq \theta_\nu$, neutrinos oscillate with a oscillation length of $L_\nu$, as in vacuum. For $N_e \gg N_{e,\text{crit}}, \theta_m \simeq \pi/2$, the oscillation length in matter $L_m$, which is defined in Eq.(1.24) is much shorter than $L_\nu$ for a large $N_e$.

![Figure 1.3](image)

Figure 1.3: (a) Effective neutrino mass squared in the medium with electron density $N_e$. $N_{e,\text{crit}}$ is the crossing point defined by Eq.(1.26). (b) Mixing angle $\theta_m$ in the medium as a function of $N_e$, corresponding to the case shown in (a).

If $\nu_e$ is produced in the region $N_e > N_{e,\text{crit}}$ and propagates into the region $N_e < N_{e,\text{crit}}$, the state follows the upper branch given in Fig.1.3-(a), then $\nu_e$ fully converts into $\nu_\mu$, provided that the density gradient, $d(\ln N_e)/dr$, is sufficiently small that neutrino conversion occurs adiabatically. This adiabatic condition may be derived from

$$(\text{energy gap}) \times (\text{transition time}) \gg \hbar, \quad (1.28)$$

where the energy gap $\delta E$ is

$$ \delta E = \frac{1}{2E} (\Delta m^2) \sin 2\theta, \quad (1.29) $$
and the transition time in the level crossing region is

\[
\delta t = \frac{\delta r}{c} = \left( \frac{dN_e}{N_e \, dr} \right)^{-1} \frac{\delta N_e}{N_e} = \left( \frac{1}{N_e} \frac{dN_e}{dr} \right)^{-1} \frac{\delta A}{A},
\]

(1.30)

where \( A \equiv 2\sqrt{2} E G_F N_e \). Since “resonance” occurs at \( A = \Delta m^2 \cos 2\theta \) and its width is \( \delta A \sim \Delta m^2 \sin 2\theta \), we find that Eq.(1.28) is written

\[
\frac{1}{N_e} \frac{dN_e}{dr} \ll \frac{\Delta m^2 \sin^2 2\theta_e}{2E \cos 2\theta_v}.
\]

(1.31)

The probability for \( \nu_e \rightarrow \nu_e \) is easily calculated by averaging out the time-varying part,

\[
P(\nu_e \rightarrow \nu_e) = \sin^2 \theta_e \sin^2 \tilde{\theta}_v + \cos^2 \theta_e \cos^2 \tilde{\theta}_v = \frac{1}{2}(1 + \cos 2\theta_v \cos 2\tilde{\theta}_v),
\]

(1.32)

where \( \tilde{\theta}_v \) is the mixing angle at the initial point.

If the adiabatic condition Eq.(1.31) is not satisfied, the state of the upper branch undergoes a transition to the lower branch while passing through the crossing point with the probability given by

\[
P_f = \exp\left(-\frac{\pi}{2} \gamma\right),
\]

(1.33)

where

\[
\gamma = \frac{\Delta m^2 \sin^2 2\theta_e}{2E \cos 2\theta(1/N_e)(dN_e/dr)}
\]

(1.34)

is the ratio of the right-hand side to the left-hand side of inequality Eq.(1.31). This is a straightforward application of the well-known Landou-Zener formula for level crossings [4].

With this \( P_f \), we write the probability for \( \nu_e \rightarrow \nu_e \) for general case

\[
P(\nu_e \rightarrow \nu_e) = \sin^2 \theta_e \sin^2 \tilde{\theta}_v + \cos^2 \theta_e \cos^2 \tilde{\theta}_v (1 - P_f) + (\sin^2 \theta_e \cos^2 \tilde{\theta}_v + \cos^2 \theta_e \sin^2 \tilde{\theta}_v)P_f
\]

\[= \frac{1}{2} + (\frac{1}{2} - P_f) \cos 2\theta_v \cos 2\tilde{\theta}_v.
\]

(1.35)

For \( N_e \rightarrow \infty, \cos 2\tilde{\theta}_v \rightarrow -1 \), and this reduces to

\[
P(\nu_e \rightarrow \nu_e) = \sin^2 \theta_e + \cos 2\theta_v P_f.
\]

(1.36)

In summary, a virtually complete conversion of \( \nu_e \) to \( \nu_\mu \) takes place when the three conditions are satisfied:

(i) \( N_e > N_{e,exit} \) : see Eq.(1.26), which gives \( \Delta m^2 \lesssim \text{const} \) (resonance condition)

(ii) \( \Delta m^2 \sin^2 2\theta_v \gtrsim \text{const} \) : see Eq.(1.31), (for a small \( \theta_v \); adiabatic condition)
(iii) $\sin^2 \theta_v < 1$: see Eq. (1.35).

These three conditions complete a rectangular triangle (which we refer to as the MSW triangle) in the $(\Delta m^2, \sin^2 \theta_v)$ plane (see Fig. 1.4).

For antineutrinos, the potential energy by the charged current interaction with positrons in matter has an opposite sign,

$$V_e = -\sqrt{2} G_F N_e .$$

(1.37)

So $\sin^2 \theta_m$ and $L_m$ became as follows,

$$L_m = \frac{L_v}{\sqrt{\sin^2 2\theta_v + (\frac{L_v}{L_e} + \cos 2\theta_v)^2}} ,$$

(1.38)

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_v}{\sin^2 2\theta_v + (\frac{L_v}{L_e} + \cos 2\theta_v)^2} .$$

(1.39)
1.2 Searches for Neutrino Oscillations

Experiments for neutrino oscillations can be done with various neutrino sources:

- Artificial sources:
  - Neutrinos from nuclear reactors ($\bar{\nu}_e$)
  - Low energy neutrinos from meson factories ($\nu_e, \nu_\mu, \bar{\nu}_\mu$)
  - High energy neutrinos from accelerators ($\nu_\mu, \bar{\nu}_\mu$)

- Natural sources:
  - Neutrinos generated inside the sun by nuclear fusions – Solar neutrinos ($\nu_e$)
  - Neutrinos generated from interactions between cosmic-rays and the atmosphere – Atmospheric neutrinos ($\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$)

These neutrino sources are summarized in Table 1.2, together with their energy regions, and the possible typical observation distances $L$, and the typical mass squared difference squared $\Delta m^2$ to which they are sensitive (Assuming the best possible experimental conditions with small statistical errors or large mixing angles).

<table>
<thead>
<tr>
<th>$\nu$ source</th>
<th>$E_\nu$ (MeV)</th>
<th>$L$ (m)</th>
<th>$\Delta m^2$ (eV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor</td>
<td>1</td>
<td>$10^2$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Meson Factory</td>
<td>40</td>
<td>$10^2$</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>Accelerator</td>
<td>$10^3$</td>
<td>$10^4$</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>Solar</td>
<td>$0.1 \sim 10$</td>
<td>$10^{11}$</td>
<td>$10^{-10} \sim 10^{-4}$</td>
</tr>
<tr>
<td>Atmospheric</td>
<td>$10^3 \sim 10^6$</td>
<td>$10^4$</td>
<td>$10^{-4} \sim 1$</td>
</tr>
</tbody>
</table>

Table 1.2: Sensitivity of various neutrino sources to neutrino oscillations.

1.2.1 Reactor and Accelerator Neutrino Experiments

In this subsection, we review the neutrino oscillations experiments which use artificial sources.

1. Reactor Experiments: Reactor neutrino experiments use electron antineutrinos produced in the core of nuclear reactors. The signal of $\bar{\nu}_e$ is detected by the reaction of $\bar{\nu}_e + p \rightarrow e^+ + n$. To eliminate accidental $\bar{\nu}_e$ events, the reactor-off data are subtracted from the reactor-on data. The existing experiments are Gösgen[24], Bugey [25] and Chooz[26] which already presented their results. Palo Verde will take data soon, Kam-LAND $^1$ [27] will start in the year 2001. The positron spectrum from the reaction $\bar{\nu}_e + p \rightarrow e^+ + n$ has been measured at $L = 37.9m$ from the core of the Gösgen power reactor to search for neutrino oscillations of the type $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ in the low $\Delta m^2$ parameter range. Fig.1.5 is energy spectra of detected positrons which are generated by electron antineutrinos produced by Gösgen reactor. The results up to now are consistent with

$^1$Kamioka Liquid scintillator AntiNeutrino Detector. The experiment is projected by Tohoku University.
the no oscillation hypothesis. Upper limits of $\Delta m^2 \leq 0.016$ eV$^2$ (90% CL) for full mixing, and $\sin^2 2\theta \leq 0.17$ (90% CL) in the limit of large $\Delta m^2$ were obtained. The ratio of the integrated experimental yield to that predicted for the case of no oscillations was $1.05 \pm 0.02$ (stat) $\pm 0.05$ (syst; 68% CL) in the Gösgen experiment.

Bugey, using detection modules filled with $^6$Li-loaded liquid scintillator reported on high statistics measurements of neutrino energy spectra carried out at 15, 40 and 95 meters away from a 2800 Megawatt reactor. No oscillations have been observed. Exclusion zones for oscillation parameters are deduced from the observed consistency of the spectra at the three distances. The minimum excluded values of $\Delta m^2$ and $\sin^2 2\theta$ parameters are $1 \times 10^{-2}$ eV$^2$ and $2 \times 10^{-2}$ (at 90% CL), respectively.

Initial results are presented from Chooz, a long-baseline reactor-neutrino vacuum-oscillations experiment. Electron antineutrinos from the reactors were detected by a liquid scintillation calorimeter located at a distance of about 1 km. The detector was constructed in a tunnel protected from cosmic-rays by a 300 MWE rock overburden. From the statistical agreement between detected and expected neutrino event rates, the Chooz experiment finds (at 90% CL) no evidence for neutrino oscillations in the $\bar{\nu}_e$ disappearance mode for the parameter region given approximately by $\Delta m^2 > 0.9 \times 10^{-3}$ eV$^2$ for maximum mixing and $\sin^2 2\theta > 0.18$ for large $\Delta m^2$.

\begin{figure}[h]
\centering
\includegraphics{figure1.5.png}
\caption{Results of the Gösgen reactor experiment. (a) Positron energy spectra for reactor-on and reactor-off. Bin width = 0.305 MeV. The errors shown are statistical. The contribution of the accidentals is indicated by the dashed curve. (b) Experimental positron spectrum obtained by subtracting reactor-off from reactor-on spectra. The solid curve represents the predicted positron spectrum assuming no neutrino oscillations. (figure is taken from [24].)}
\end{figure}

2. Meson Factory Experiments: The meson factory experiments use low energy neutrinos (several tens of MeV) which are generated by the decay of stopping $\pi^+$ mesons. The on-going
experiments are KARMEN² [19], LSND³[20].

KARMEN is situated at the beam stop neutrino source ISIS. It provides $\nu_\mu$, $\nu_e$, and $\bar{\nu}_\mu$ in equal intensities from the $\pi^+ \to \mu^+$-decay at rest. The oscillations $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ are investigated with a 56 t liquid scintillation calorimeter at a mean distance of 17.6 m from the $\nu$-source. No evidence for oscillations was found with KARMEN, resulting in 90% CL exclusion limits of $\sin^2 2\theta < 8.5 \times 10^{-3}$ ($\bar{\nu}_\mu \to \bar{\nu}_e$) and $\sin^2 2\theta < 4.0 \times 10^{-2}$ ($\nu_\mu \to \nu_e$) for $\Delta m^2 > 1 \text{ eV}^2$.

LSND carried out at the Los Alamos Meson Factory searches for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations by using $\nu_\mu$ from $\mu$-decay at rest. The $\nu_e$ are detected via the reaction $\bar{\nu}_e p \to e^+ n$, correlated with a $\gamma$ from $np \to d\gamma$ ($E_\gamma = 2.2 \text{ MeV}$). The use of tight cuts to identify $e^+$ events with correlated $\gamma$ rays yields 22 events with $e^+$ energy between 36 and 60 MeV and only $4.6 \pm 0.6$ background events. If attributed to $\nu_\mu \to \nu_e$ oscillations, this corresponds to an oscillation probability of $(0.3 \pm 0.12 \pm 0.05)\%$.

3. High Energy Accelerator Experiments: High energy accelerator experiments use high energy neutrinos (several GeV) which are generated by high energy $\pi^\pm$, $K^\pm$ mesons in flight decays. These high energy mesons are produced by high energy proton beams focused on a fixed target. The produced mesons are momentum selected to make a narrow neutrino energy spectrum – called “narrow-band neutrino beam”, or are not selected by momentum to get a high intensity of neutrino flux – called “wide-band neutrino beam”. The typical neutrino beam lines are listed in Table 1.3.

<table>
<thead>
<tr>
<th>accelerator name</th>
<th>proton energy</th>
<th>neutrino mean energy</th>
<th>decay pipe length</th>
<th>neutrino flight length</th>
<th>experiment/detector name</th>
<th>comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN-SPS</td>
<td>450(GeV)</td>
<td>27(GeV)</td>
<td>414(m)</td>
<td>822(m)</td>
<td>CHORUS NOMAD</td>
<td>wide-band (running)</td>
</tr>
<tr>
<td>FNAL-Tevatron</td>
<td>800(GeV)</td>
<td>140(GeV)</td>
<td>542(m)</td>
<td>1480(m)</td>
<td>CCFR</td>
<td>narrow-band (running)</td>
</tr>
<tr>
<td>FNAL-Main-Injector</td>
<td>120(GeV)</td>
<td>12(GeV)</td>
<td>800(m)</td>
<td>950(m) 700(km)</td>
<td>COSMOS MINOS</td>
<td>wide-band (will run 2000)</td>
</tr>
<tr>
<td>KEK-PS</td>
<td>12(GeV)</td>
<td>1(GeV)</td>
<td>200(m)</td>
<td>200(km)</td>
<td>Super-Kamiokande</td>
<td>wide-band (will run 1998)</td>
</tr>
</tbody>
</table>

Table 1.3: On-going/under-construction neutrino beam lines. FNAL-MI to MINOS and KEK-PS to Super-Kamiokande will have such a long neutrino flight path, they are called “Long-Base-Line” experiments.

The existing experiments are CHORUS-NOMAD, CDHSW⁴[23], BNL-E776[21], CCFR ⁵[22].

CDHSW has searched for $\nu_\mu$ oscillations by comparing the rates of $\nu_\mu$, charged-current interactions in two detectors located 130 and 885 m from the target, which was struck by a 19.2 GeV/c proton beam from the CERN Proton Synchrotron. No evidence for $\nu_\mu$ oscillations was found. At the 90% CL, $\Delta m^2$ values between 0.26 and 90 eV² are excluded for maximal mixing. The most restrictive limit on the neutrino mixing-angle parameter $\sin^2 2\theta$ is 0.053 at $\Delta m^2 = 2.5 \text{ eV}^2$.

²Karlsruhe Rutherford Medium Energy Neutrino experiment
³Liquid Scintillator Neutrino Detector
⁴CERN, Dortmund, Heidelberg, Saclay, Warsaw
⁵Cincinnati, Columbia, Fermi, Rochester
BNL-E776 at the Brookhaven National Laboratory has searched for the appearance of $\nu_e(\bar{\nu}_e)$, 1 km from the source of a wide-band $\nu_\mu(\bar{\nu}_\mu)$ beam. The experiment used a total of $3 \times 10^{19}$ protons on target from the Alternating Gradient Synchrotron. No excess of $\nu_e$ or $\bar{\nu}_e$ over the expected background was detected. The 90% CL limits obtained are $\Delta m^2 \leq 7.5 \times 10^{-2}$ eV$^2$ for maximal mixing, and $\sin^2 2\theta \leq 0.003$ for large $\Delta m^2$. CCFR at Fermilab presented limits on $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$ oscillations. Neutrino energies range from 30 to 600 GeV with a mean of 140 GeV, and $\nu_\mu$ flight lengths vary from 0.9 to 1.4 km. The result excludes oscillations in the region with $\sin^2 2\theta > 1.8 \times 10^{-3}$ for large $\Delta m^2 (> 1000$ eV$^2$) and $\Delta m^2 > 1.6$ eV$^2$ for $\sin^2 2\theta = 1$. This result is the most stringent limit to date for $\Delta m^2 > 25$ eV$^2$ and it excludes the high $\Delta m^2$ oscillation region favored by the LSND experiment. The $\nu_\mu$ to $\nu_e$ cross section ratio was measured as a test of $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$ universality to be $1.026 \pm 0.025$ (stat) $\pm 0.049$ (syst).

![Diagram](image)

**Figure 1.6**: Left figure is calculated spectra for $\nu_\mu, \nu_e, \bar{\nu}_\mu, \bar{\nu}_e$ per proton on target (POT) per m$^2$ at the BNL-E776 experiment, (a) the horn at positive polarity and (b) the horn at negative polarity. Right figure is the calculated neutrino energy spectra for the CCFR experiment. (figures are taken from [21], [22].)

90% CL exclusion curves for $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations from various accelerator and reactor experiments are presented in Fig.1.7, Fig.1.8, respectively.
Figure 1.7: 90% CL exclusion curves and limits of $\nu_e \leftrightarrow \nu_\mu$ oscillations from various neutrino experiments. Here, Palo Verde and Kam-LAND contour plots show the expected ones. (from G.Gratta, stanford-HEP-97-03.)

Figure 1.8: 90% CL exclusion curves and limits of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations from CHORUS, NOMAD, CDHSW experiments.
1.2.2 Solar Neutrino Experiments

The sun is an intense source of low energy ($E_{\nu} \lesssim 1.5\text{MeV}$) electron neutrinos ($\nu_e$) that are products of the nuclear fusion processes in its higher temperature central region. The underlying nuclear fusion processes are in the following.

- **$p$-$p$ Chain**

  \[ p + p \rightarrow ^2\text{H} + e^+ + \nu_e \ (E_{\nu} < 0.42\text{MeV}) \]  
  \[ \text{or} \]
  \[ p + e^- + p \rightarrow ^2\text{H} + \nu_e \ (E_{\nu} = 1.44\text{MeV}) \]
  \[ ^2\text{H} + p \rightarrow ^3\text{He} + \gamma \]  
  \[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p \]

- **CNO Cycle**

  \[ ^{12}\text{C} + p \rightarrow ^{13}\text{N} + \gamma \]  
  \[ ^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e \ (E_{\nu} < 1.2\text{MeV}) \]  
  \[ ^{13}\text{C} + p \rightarrow ^{14}\text{N} + \gamma \]  
  \[ ^{14}\text{N} + p \rightarrow ^{15}\text{O} + \gamma \]  
  \[ ^{15}\text{O} \rightarrow ^{15}\text{C} + e^+ + \nu_e \ (E_{\nu} < 1.7\text{MeV}) \]  
  \[ ^{15}\text{N} + p \rightarrow ^{12}\text{C} + ^4\text{He} \].

Processes (1.40) to (1.56) produce electron neutrinos which are called, respectively, $pp$, $pcp$ and $^7\text{Be}, ^8\text{B}, ^{13}\text{N}$ and $^{15}\text{O}$ neutrinos. $pcp$ and $^7\text{Be}$ neutrinos are monoenergetic. Other neutrinos have continuous energy spectra. Fig.1.9 shows the calculated fluxes of solar neutrinos on the earth.

In the late 1960s, a unique experiment which aimed to detect solar neutrinos were started, and it first succeeded in the detection of the astronomical neutrinos. The experiment was started by R.Davis et al. in the Homestake Gold Mine at Lead, South Dakota, and its goal was to observe the flux of the electron neutrinos. In order to detect the solar neutrinos, they used a radiochemical method based upon the inverse beta process of $\nu_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar}$, of which the energy threshold was 0.814 MeV. The total solar neutrino flux for the chlorine experiment was predicted to be 7.5
SNU (Solar Neutrino Unit: 1 SNU $\equiv 10^{-36}$ captures/target atom/sec). The solar neutrinos were detected at first in 1968, and the observed rate was below 3 SNU. The Homestake experiment proved that the origin of solar energy was nuclear fusion. However, it also reported the unexpected result in the framework of the solar model. They observed less than half of the solar neutrino flux predicted from the solar model. This experiment continued to observe solar neutrinos and reported the deficit for more than twenty years. The deficit of the solar neutrino flux has been called the "solar neutrino problem", and well known in the field of both particle physics and astrophysics.

The second experiment to observe the solar neutrino flux was Kamiokande-II, which started its observation in January 1987. Kamiokande-II was an imaging water Cherenkov detector with 4500 tons of water, and detected solar neutrinos above 7.5 MeV via the reaction $\nu_e + e^- \rightarrow e^- + \nu_e$. Kamiokande-II was the first experiment which had the ability to measure the direction and the energy of solar neutrinos in real time. Thus, it was first revealed from the data of Kamiokande-II that the neutrinos undoubtedly come from the direction of the sun. Fig. 1.10 shows the directional distribution of solar neutrino events observed by Kamiokande-II and the next period of experiment Kamiokande-III. The $^8$B solar neutrino flux observed by Kamiokande-II,III in 1966 live days was $0.48 \pm 0.03$(stat.) $\pm 0.07$(syst.) times the prediction by Bahcall and Ulrich in 1988. Thus, the deficit of the solar neutrinos was confirmed by Kamiokande-II,III.

Thereafter, two radiochemical experiments to measure the $pp$ neutrinos, namely SAGE and GALLEX were started and their first results were reported 1991 and 1992, respectively. An inverse $\beta$-decay reaction of $\nu_e + ^{71}$Ga $\rightarrow e^- + ^{71}$Ge is used in both experiments, and its energy threshold is 0.2332 MeV which makes those experiments sensitive to the $pp$ neutrinos. The observed $pp$ neutrino fluxes were $20_{-13}^{+15}$(stat.) $\pm 9$(syst.) (SNU) by SAGE(1991) and $83 \pm 19$(stat.) $\pm 8$(syst.) (SNU) by GALLEX(1992), though the total rate for the $^{71}$Ga experiment is predicted as 132 SNU. The $pp$
and pcp neutrino fluxes are thought to be theoretically robust and all the predicted fluxes of pp and pcp neutrinos are essentially the same among recent Standard Solar models (SSMs), and the sum of those neutrino fluxes is estimated to be 74 ± 1 SNU. If the standard electroweak theory and the prediction of pp and pcp neutrino fluxes are correct, the results from 71Ga experiments imply the deficit of other solar neutrinos such as the 7Be or 8B neutrinos.

Table 1.4 and Table 1.5 show the summary of experimental results and the two different SSM predictions on the solar neutrino flux, where the unit is SNU for 37Cl and 71Ga experiments and $\nu/cm^2/sec$ for $e^-(H_2O)$ target experiment, respectively.

Fig.1.11 shows the allowed $\nu_e \leftrightarrow \nu_\mu$ oscillation parameter regions obtained by the Homestake, Kamiokande, and Ga experiments, and the combined data, where SSM are (a) by Bahcall & Pinsonneault and (b) by Turck-Chieze & Lopes.

<table>
<thead>
<tr>
<th>Target(Exp.)</th>
<th>Data</th>
<th>SSM prediction</th>
<th>Data/SSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{37}$Cl(Homestake)</td>
<td>2.28 ± 0.23</td>
<td>8.0 ± 3.0</td>
<td>0.29 ± 0.03</td>
</tr>
<tr>
<td>$^{71}$Ga(SAGE)</td>
<td>$73^{+18}_{-16}$</td>
<td>131.5$^{+124}_{-17}$</td>
<td>0.56$^{+0.14}_{-0.12}$-0.06</td>
</tr>
<tr>
<td>$^{71}$Ga(Gallex)</td>
<td>79 ± 10 ± 6</td>
<td>131.5$^{+124}_{-17}$</td>
<td>0.60 ± 0.08 ± 0.05</td>
</tr>
<tr>
<td>e$^-(H_2O, \text{Kamiokande})$</td>
<td>$(2.7 \pm 0.2 \pm 0.3) \times 10^6$</td>
<td>$(5.7 \pm 2.5) \times 10^6$</td>
<td>0.48 ± 0.03 ± 0.06</td>
</tr>
</tbody>
</table>

Table 1.4: SSM = Bahcall and Pinsonneault Model
<table>
<thead>
<tr>
<th>Target (Exp.)</th>
<th>Data</th>
<th>SSM prediction</th>
<th>Data/SSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{37}$Cl (Homestake)</td>
<td>$2.28 \pm 0.23$</td>
<td>$6.4 \pm 1.4$</td>
<td>$0.36 \pm 0.04$</td>
</tr>
<tr>
<td>$^{71}$Ga (SAGE)</td>
<td>$73^{+18}_{-10}$</td>
<td>$123 \pm 7$</td>
<td>$0.50^{+0.12}_{-0.13} \times 0.08$</td>
</tr>
<tr>
<td>$^{71}$Ga (Gallex)</td>
<td>$79 \pm 10 \pm 6$</td>
<td>$123 \pm 7$</td>
<td>$0.64 \pm 0.08 \pm 0.05$</td>
</tr>
<tr>
<td>$e^-(H_2O, \text{Kamiokande})$</td>
<td>$(2.7 \pm 0.2 \pm 0.3) \times 10^9$</td>
<td>$(4.4 \pm 1.1) \times 10^9$</td>
<td>$0.62 \pm 0.04 \pm 0.08$</td>
</tr>
</tbody>
</table>

Table 1.5: SSM = Turck-Chieze and Lopes Model

Figure 1.11: The allowed parameter regions of the Homestake, Kamiokande, and Ga experiments, and the combined data. Here SSM’s are (a) by Bahcall & Pinsonneault and (b) by Turck-Chieze & Lopes. The SSM uncertainties are included in the analysis. For the Bahcall & Pinsonneault SSM, the $1\sigma$ value of the SSM uncertainties is assumed to be $1/3$ of their ‘effective 3$\sigma$’ error. The earth effect on the regeneration of $\nu_e$ is included. The excluded region by the Kamiokande day/night data are also shown. (figures are taken from Ref.[7].)
1.2.3 Atmospheric Neutrino Experiments

Atmospheric neutrinos are produced through decays of charged pions and kaons which are secondary particles in the primary cosmic-ray interactions in the atmosphere. Pions, which are the dominant sources of atmospheric neutrinos, decay before interacting with air nuclei due to the small density at high altitude. Consequently the following decay chain is the main process of atmospheric neutrino production:

\[
p(\text{He}) + \text{air nucleus} \rightarrow \pi^\pm + X, \quad (1.57)
\]

\[
\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu), \quad (1.58)
\]

\[
\mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu). \quad (1.59)
\]

Thus, it is expected that the flux ratio of \((\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)\) is \(\sim 2\). This flux ratio’s calculations from different groups agree to within 5%, although the absolute flux values are with a 20% uncertainty.

There are two experimental methods of studying atmospheric neutrinos in massive underground and underwater detectors:

1. Detecting neutrino interactions occurring in the detector, which is called fully or partially contained events, depending on whether all produced particles stop inside the detector or not. See Fig. 1.12.

2. Detecting upward-going muons produced by neutrino interactions in nearby rock surrounding the detector.

![Schematic figure of fully and partially contained events. The vertex points of neutrino interactions are within the detector. “Fully” and “partially” are categorized by their stopping points.](image)

Figure 1.12: Schematic figure of fully and partially contained events. The vertex points of neutrino interactions are within the detector. “Fully” and “partially” are categorized by their stopping points.
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In Kamiokande event types are grouped by the mean energy of neutrinos: “sub-GeV” ($< E > \sim 0.7\text{ GeV}$), where fully-contained events are the main component; “multi-GeV” ($< E > \sim 6\text{ GeV}$), where fully-and partially-contained events are main components; and $< E > \sim 100\text{ GeV}$ upward-going muons. In this subsection, “sub-GeV” and “multi-GeV” data are shown. Upward-going muons which are taken in this thesis for the neutrino oscillation analysis are described in the Section 1.2.4.

Up to now there are five results of atmospheric neutrinos taken in different underground detectors. Two are the water Cerenkov type of experiments, IMB and Kamiokande. The other three are tracking detectors, NUSEX, Fréjus and Soudan-II. All experiments furnish the ability to identify the particle type of electron-like or muon-like. Thus, it is possible to compare the $R(\mu/e)$ ratio observed in all these experiments, where $R(\mu/e)$ is defined as follows,

$$R(\mu/e) = \frac{(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)_{\text{Data}}}{(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)_{\text{MC}}}.$$  \hspace{1cm} (1.60)

Table 1.6 lists the observed and expected number of events, and $R(\mu/e)$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Exposure (kton-year)</th>
<th>Data (events)</th>
<th>MC (events)</th>
<th>ratio of Data and MC $R(\mu/e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kamiokande</td>
<td>7.7</td>
<td>248</td>
<td>234</td>
<td>227.6 : 356.8 : 0.60$^{+0.08}_{-0.06} \pm 0.05^a$</td>
</tr>
<tr>
<td></td>
<td>8.2(full), 6.0(part)</td>
<td>98</td>
<td>135</td>
<td>66.5 : 162.2 : 0.57$^{+0.07}_{-0.05}$</td>
</tr>
<tr>
<td>IMB-3</td>
<td>7.7</td>
<td>325</td>
<td>182</td>
<td>257.3 : 268.0 : 0.54$^{+0.12}_{-0.05}$</td>
</tr>
<tr>
<td>Nusex</td>
<td>0.74</td>
<td>18</td>
<td>32</td>
<td>20.5 : 36.8 : 0.99$^{+0.08}_{-0.05}$ small</td>
</tr>
<tr>
<td>Fréjus</td>
<td>2.0</td>
<td>75</td>
<td>125</td>
<td>81.4 : 136.2 : 1.00$^{+0.08}_{-0.05}$</td>
</tr>
<tr>
<td>Soudan-II</td>
<td>1.52</td>
<td>49.4</td>
<td>37.0</td>
<td>45.3 : 47.1 : 0.72$^{+0.10}_{-0.07}$</td>
</tr>
</tbody>
</table>

Table 1.6: Results of the contained event analysis of each experiment. $a$: sub-GeV, $b$: multi-GeV.

Fig.1.13 shows the zenith angle ($\cos \theta$) distribution of e-like and $\mu$-like events from the Kamiokande experiment. Data points look somewhat deviated from the MC predictions. The dependence of the ratio $R(\mu/e)$ on $\cos \theta$ illustrates this more clearly as shown in Fig.1.13(c). One sees a gradual decrease with respect to $\cos \theta$ from 1 to -1. This dependence is well-fitted by the expectation from neutrino oscillations as shown by histograms in Fig.1.13(c), where the dashed histogram corresponds to the best-fit value for $\nu_\mu \leftrightarrow \nu_e$ oscillations, and the dotted one for $\nu_\mu \leftrightarrow \nu_\mu$ oscillations.

In terms of IMB-3 [11], [12] experiments, the result supports a small $R(\mu/e)$ ratio. On the other hand, two tracking experiments of NUSEX [15] and Fréjus [14] show $R(\mu/e) \sim 1$, which means that there is no atmospheric neutrino anomaly. Since the statistical significance is not high in NUSEX and Fréjus, these results are still consistent with those of Kamiokande and IMB-3 at the $3 \sigma$ level. The Soudan-II result [13] is also smaller than 1, although its statistical significance is not enough. It is noted that the detection threshold momentum of Soudan-II is $\sim 100\text{ MeV/c}$ and dE/dx information is available, unlike NUSEX and Fréjus.
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Figure 1.13: Zenith angle distribution of $e$-like (a) and $\mu$-like (b) multi-GeV events from the Kamiokande experiment. The histograms are MC prediction. The zenith angle distribution of $R(\mu/e)$ is shown in (c), where the histograms are the expectations from the neutrino oscillations with the best fit values of $(\Delta m^2, \sin^2 2\theta)$. (figure is taken from [16].)

Figure 1.14: 90% CL allowed parameter regions from the Kamiokande experiment for the cases of $\nu_\mu \leftrightarrow \nu_e$ (left) and $\nu_\mu \leftrightarrow \nu_\tau$ (right). The best-fit values are indicated by dash-crosses (sub-GeV data), full-crosses (multi-GeV data) and stars (sub- and multi-GeV data combined). The other curves are the 90% CL excluded regions from the previous experiments. (figure is taken from [16].)
1.2.4 Upward Through Going Muons

In Section 1.2.3, we mentioned upward-going muons originate from muon neutrinos which are generated predominantly in the atmosphere. In this section, we describe the features of upward-going muon events. The previous and on-going experiments of upward-going muons are described in Chapter 6.

Upward-going muons are grouped by two types in terms of their detection. One is “through-going” muons which are punching through the detector and the other is “stopping” muons which are stopped in the detector. Fig.1.15 shows schematic diagram of an upward-through-going muon event and an upward-stopping muon event.

![Schematic diagram of upward through-going and stopping muon events](image)

Figure 1.15: Schematic figure of upward stopping and through-going muon events. The vertex points of neutrino interactions are in the rock outside the detector. “Stopping” and “through-going” are categorized by their stopped points.

The reason why we select only “upward” muons is that almost all “downward” muons are cosmic-ray muons which are passing through the rock covering the detector and we cannot distinguish the neutrino induced muons from them. Moreover why we use muons is that electrons generated from electron neutrinos can not reach to the detector due to their short radiation length.

The generation of an atmospheric neutrino and their detection as an upward-through-going muon is represented in the schematic diagram in Fig.1.16, where a muon neutrino (or muon antineutrino) interacts with a nucleon in the rock at nearby the detector through the charged current weak interaction,

\[ \nu_\mu (\bar{\nu}_\mu) + N \rightarrow \mu^- (\mu^+) + N' + X. \quad (1.61) \]

The detailed description of the generation of upward-through-going muons is in Chapter 5, where the expected flux of upward-through-going muons detected in the underground detector is calculated.

---

6 The dominant component of cosmic-ray muons are atmospheric muons which are generated directly in the atmosphere by the reaction of Eq. (1.58).
Figure 1.16: Schematic figure for generation and detection of an upward through-going muon. In this example, a prompt cosmic-ray – H, He, etc. – interacts with a nucleus in the atmosphere – N, O – and generates a muon neutrino by the following reactions, \( A_{CR} + A_{air} \rightarrow \pi^+ + \) anything, \( \pi^+ \rightarrow \mu^+ + \nu_\mu \). A produced muon neutrino goes through the earth and interacts with a nucleus in the rock near the detector by CC weak interaction, \( \nu_\mu + N \rightarrow \mu^- + N' + \) anything. A produced muon travels in the rock and penetrates through the detector.

The mean energy of detected upward-through-going muons is relatively higher than that of contained event’s muons because they travel \( 0 \sim \) several 100 m in the rock. The energy spectra of parent muon neutrinos which generate upward-through-going/stopping muons are shown in Fig.1.17 together with those of contained event’s muons.

Here we summarize the features of upward-through-going muon events detected in deep underground water Cherenkov detector in contrast to contained events.

1. The only particle we must identify is single muon which is generated by CC interaction, Eq.(1.61) because neutrino-induced electron and pion and leptons decayed from it are stopped in the rock before they reach the detector.

2. There are no background events except for cosmic-ray muons contaminating from an almost horizontal downward direction.

3. The neutrino direction is determined accurately.
4. The information of muon energy deposited is obtained only within the detector, so the energy of the parent neutrino for each event is unknown.

From the feature 1 and 2, event identification becomes very simple and reliable, so the selection efficiency of upward through going muons is very good \(^7\). Although feature 1 is a weak point of upward through going muon events because when we perform oscillation analysis, we can’t use the \(\bar{\nu}_e/\nu_\mu\) flux ratio to compare to the expected one, but the gradient of zenith angle distributions has a small uncertainty so the oscillation analysis, we normalize the absolute value of the expected flux when comparing with the observed one.(see Chapter 6.)

Feature 3 is caused by two reasons. One is that the scattering angles between parent neutrinos and induced muons are very small because we use high energy\((\sim 100\text{GeV})\) neutrinos. Another is that the angular resolution is very accurate \(^8\) because the determination of muon direction is reliable especially for upward-through-going muons. (see Chapter 3.)

Feature 4 is because we don’t know the total energy deposited from neutrino induced muons because we can’t know the points where they are created and stop. Therefore, in the oscillation analysis, we have not compared the energy spectrum of fluxes but the angular distribution of them between the observed and expected flux.

---

\(^7\)more than 99 % for Super-Kamiokande

\(^8\) \(\sim 1.7^\circ\) for Super-Kamiokande
1.3 Oscillation Search by Upward-Through-Going Muons

We reviewed various experiments of neutrino oscillations in the previous sections. In the case of \( \nu_e \leftrightarrow \nu_\mu \) oscillations, \( \Delta m^2 \gtrsim 10^{-3} \text{ eV}^2 \) regions are searched for by reactor, meson factory and high energy accelerator neutrino experiments. \( \sin^2 2\theta \sim 10^{-3} \) parameter regions remain. On the other hand, \( 10^{-10} \leq \Delta m^2 \leq 10^{-4} \text{ eV}^2 \) regions are searched for by the solar neutrinos. \( \Delta m^2 \sim 10^{-5} \text{ eV}^2 \) at \( \sin^2 2\theta \sim 0.5 \) or \( \sin^2 2\theta \sim 10^{-2} \) parameter regions are remained.

In the case of \( \nu_\mu \leftrightarrow \nu_\tau \) oscillations, it can not be searched for by either reactor experiments nor solar neutrino experiments because the source neutrinos consist of only \( \bar{\nu}_e \) or \( \nu_e \), respectively. \( \Delta m^2 \gtrsim 10^{-1} \text{ eV}^2 \) is searched for by meson factory and high energy accelerator neutrino experiments. \( \sin^2 2\theta' \sim 10^{-3} \) parameter regions are remained.

Both for \( \nu_e \leftrightarrow \nu_\mu \) and \( \nu_\mu \leftrightarrow \nu_\tau \) oscillations, \( 10^{-4} \leq \Delta m^2(\Delta m'^2) \leq 1 \text{ eV}^2 \) parameter regions are searched for by atmospheric neutrino experiments. \( \sin^2 2\theta(\sin^2 2\theta') \gtrsim 0.4 \), \( \Delta m^2(\Delta m'^2) \sim 10^{-2} \) parameter regions remain.

In this thesis, upward-through-going muons originating from atmospheric muon neutrinos are taken for analysis of muon neutrino oscillations, and the neutrino oscillation hypothesis in the \( 10^{-3} \leq \Delta m^2(\Delta m'^2) \leq 1 \text{ eV}^2 \) parameter regions is tested. Fig.1.18 shows oscillation parameter regions searched by various neutrino experiments. The hatched regions can be searched for using upward-through-going muons in this thesis.

![Figure 1.18: The oscillation parameter regions searched for various neutrino experiments. The hatched regions are searched by using upward-through-going muons in this thesis.](image-url)
Chapter 2

Kamiokande and Super-Kamiokande Detectors

2.1 Kamioka Underground Laboratory

The Kamioka Underground Laboratory is located in the Mozumi Mine in Kamioka Town, which is operated by Kamioka Mining and Smelting Company. Kamioka Town, about 250 km west of Tokyo, lies in the northern region of Gifu Prefecture, bordering on Toyama Prefecture. The exact location of the laboratory is latitude 36.42°N and longitude 137.31°E; the geomagnetic latitude is 25.8°N. The laboratory has an elevation of 350 m, but it lies nearly under the top of Mt. Ikenoyama, 1,350 m high, as shown in Fig. 2.1. Therefore, the laboratory has a rock overburden of about 1,000 m in the vertical direction.

2.2 Brief History

The original motivation which stimulated the construction of the Kamiokande detector was the search for nucleon decay. As yet, it has not been discovered, with only the lower limits of the nucleon partial lifetimes having been determined. However, it was realized at the very beginning of the detector operation that the energy spectrum of decay electrons from the stopping muons could be measured down to a kinematic energy of ~ 12 MeV. The trigger rate below this energy rapidly increased because of radioactive impurities dissolved in the water. Therefore, if they could be reduced, the Kamiokande detector could be converted to a real-time solar neutrino detector.

The Kamiokande detector has been continually improved. The first phase of the experiment (July 1983 - November 1985) is called Kamiokande-I. It was initially operated with no anticounter layer, which was constructed later in October - December 1984 (top and bottom sections, both inside the main detector tank) and in October 1985 (barrel section, between the rock cavity and the main detector tank).

The second phase, Kamiokande-II, was aimed at observing solar neutrinos, and has been operated since November 1985 with a new electronics system capable of recording the arrival time of each photomultiplier tube (PMT) signal in addition to its charge. The continued effort to purify water

1The mine is closed now.
2Kamioka Nucleon Decay Experiment
3Kamioka Neutrino Detection Experiment
and, in particular, that of removing radioactive contamination from water should be specially noted, since it has been essential for the successful observation of low energy neutrinos.

In June 1988 the detector performance of Kamiokande-II was improved by doubling the PMT gain. The effects of the PMT gain increase are: (i) improved energy resolution for low energy electrons and (ii) improvement in the event vertex reconstruction. These improvements were achieved owing to the increase (~ 20%) of hit PMTs corresponding to the unit deposited energy by electron. This led to a significant reduction of background events.

The second phase was completed in April 1990. Up to then, since more than 10% of the PMTs had been out of order due to various reasons, the detector was emptied and dead PMTs were replaced. At this occasion, the entire electronics were replaced with a more compact, low-power system. A reflector was also introduced to each PMT in the inner detector to collect more light. Observation was resumed in December 1990; this phase is called Kamiokande-III.

In December 1991, construction of Super-Kamiokande started and official runs of Super-Kamiokande started in April 1996. Kamiokande-III was running until July 1996, but actual analysis was terminated, in terms of upward through going muons, in May 1995.

In this thesis, data of Kamiokande-III and Super-Kamiokande are analysed. Data of Kamiokande-II was already analysed [50], but it was used to combine with the Kamiokande-III result in this thesis.
2.3 The Detector

2.3.1 Ring-Imaging Water Cherenkov Detector

The Kamiokande and Super-Kamiokande detectors are cylindrical large Ring-imaging Water Cherenkov Detector. In this section, the principle of Ring-imaging Water Cherenkov Detector will described.

A charged particle propagating in water emits cherenkov photons when its velocity \( v \) exceeds the velocity of light in water \( c/n \):

\[
n\beta > 1, \beta \equiv \frac{v}{c}\]

(2.1)

where \( n(=1.344) \) is the refraction index of water. The light emission process is analogous to shock wave production. Cherenkov photons are emitted in a circular cone with a half opening angle of \( \theta \) to the particle trajectory,

\[
\cos \theta = \frac{1}{\beta^n}.
\]

(2.2)

In the case of the relativistic limit \( (\beta=1) \), \( \theta \) has an asymptotic value of 41.9°. The spectrum of cherenkov light is quantum-mechanically calculated and the number of photons emitted per unit path length per unit wave length is,

\[
\frac{dN}{dx \lambda} = \frac{2\pi \alpha}{\lambda^2} \left( 1 - \frac{1}{n^2 \beta^2} \right)
\]

(2.3)

where \( \alpha \) is the fine structure constant, \( x \) the path length of the charged particle and \( \lambda \) the wave length of cherenkov light. The threshold momentum of cherenkov light emission depends on its mass due to the condition of Eq. (2.1).

A ring-imaging water cherenkov detector detects cherenkov lights emitted in a massive detector volume of water by charged particles, with PMTs arrayed two-dimensionally on the detector walls surrounding the water. A cherenkov ring image is projected onto the detector walls. The particle trajectory and energy are determined, using the charge and timing information of each hit PMT.

In the case of atmospheric neutrinos, a typical reaction ( quasi-elastic scattering) is as follows,

\[
\nu + N \rightarrow \ell + N'
\]

where \( \nu \) is incident neutrino, \( N \) and \( N' \) are nucleons, \( \ell \) is the accompanied lepton( \( e \) or \( \mu \) ).
2.3.2 Detector Overview

The components of detector are almost the same between Kamiokande and Super-Kamiokande. The detectors consist of a cylindrical steel tank filled with pure water, water-air-purification system, 20-inch/8-inch diameter PMTs attached on wall of detector, electronics system, and online data acquisition (DAQ) system.

Detector itself is divided in two parts. One is the inner detector (main detector) in which we observe the desired events such as solar/atmospheric neutrinos or proton decay, and another is the outer detector (anti detector) to veto incoming events from outside of the detector for contained events analysis. For the detector through going muon events analysis, signal of outer detector is used as trigger instead of veto.

The specification of Kamiokande-(II,III) and Super-Kamiokande detectors are summarized in Table 2.1. The schematic views of detector are shown in Fig 2.2 for Kamiokande-(II,III), Fig 2.3 for Super-Kamiokande. The inside photograph of Super-Kamiokande is shown in Fig. 2.4.

In the following text, the detail of detector specification is described about Super-Kamiokande. That of Kamiokande already has many descriptions, for example Ref.[31],[5],[32], so it is not referred to in this thesis.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Super-Kamiokande</th>
<th>Kamiokande-(II,III)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total size</td>
<td>42m x 39mφ</td>
<td>16.1m x 15.6mφ</td>
<td></td>
</tr>
<tr>
<td>Total mass</td>
<td>50,000 t</td>
<td>4,500 t</td>
<td></td>
</tr>
<tr>
<td>Number of PMTs</td>
<td>11,146 (20 inchφ)</td>
<td>948 (20 inchφ)</td>
<td>inner detector</td>
</tr>
<tr>
<td></td>
<td>1,885 (8 inchφ)</td>
<td>123 (20 inchφ)</td>
<td>anti detector</td>
</tr>
<tr>
<td>Photosensitive coverage</td>
<td>40%</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>PMT timing resolution</td>
<td>3 nsec</td>
<td>4 nsec</td>
<td></td>
</tr>
<tr>
<td></td>
<td>at 1 photoelectron</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy resolution</td>
<td>2.6%/√E</td>
<td>3.6%/√E</td>
<td>e of E (GeV)</td>
</tr>
<tr>
<td></td>
<td>2.5%</td>
<td>4%</td>
<td>μ(&lt;1 GeV)</td>
</tr>
<tr>
<td></td>
<td>16%/√E</td>
<td>20%/√E</td>
<td>e (&lt;20 MeV)</td>
</tr>
<tr>
<td>Position resolution</td>
<td>30 cm</td>
<td>110 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>~ 10 cm</td>
<td>15 cm</td>
<td>10 MeV</td>
</tr>
<tr>
<td>Angular resolution</td>
<td>28°</td>
<td>28°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>~ 1°</td>
<td>~ 2°</td>
<td>10 MeV</td>
</tr>
<tr>
<td></td>
<td>Through going μ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E_trig(termination)</td>
<td>4~5 MeV</td>
<td>5.2 MeV</td>
<td></td>
</tr>
<tr>
<td>E_{th}(analysis)</td>
<td>5 MeV</td>
<td>7.5 MeV</td>
<td>solar μ</td>
</tr>
<tr>
<td>e/μ separation</td>
<td>99%</td>
<td>98±1 %</td>
<td>0.03 &lt; p_e &lt; 1.33 GeV/c</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.2 &lt; p_μ &lt; 1.5 GeV/c</td>
</tr>
</tbody>
</table>

Table 2.1: Performance of Super-Kamiokande detector and Kamiokande-(II,III) detector.
Figure 2.2: Schematic view of Kamiokande-(II,III) Detector

Figure 2.3: Schematic view of Super-Kamiokande Detector
Figure 2.4: The inside view of Super-Kamiokande. This photograph was taken when the detector was under construction (Feb. 1996). Pure water was filled to the 3/5 level of the tank at this time. The man on the boat was wiping off dust on the surface of PMTs.
2.3.3 20 Inch-Diameter PMT

20 inch-diameter PMTs attached on the surface of inner detector were developed for Kamiokande and improved for Super-Kamiokande by HAMAMATSU Photonics Co. Ltd. in cooperation with members of Kamiokande[33]. Fig.2.5 presents a cross-section picture of 20 inch-diameter PMT for Super-Kamiokande. The gain of this PMT is about $10^7$ with the applied voltage of about 2000V. The strong feature of this PMT is it has acceptance of large area and wide angle due to it's big diameter and sphere shaped photosensitive area. The quantum efficiency of this PMT is good enough in the range of 300 nm ~ 600 nm which corresopnding to wave length of cherenkov light in the water (Fig.2.6). To get good energy resolution for low energy events (~ 5 MeV), a 1 p.e. level sensitivity to the output signal is required. The 1 p.e. peak in the pulse height distribution can be distinguished from dark current(Fig.2.7). Since photocasode and first dinode has 30 cm distance, trajectory of photoelectrons are affected by external magnetic field such as geomagnetic field, and it may cause bad time resolution. In order to compensate the geomagnetic field, electric wires are laid on the surface of the cavity around detector to compensate for the magnetic field.

![Figure 2.5: Cross section view of 20 inch-in-diameter PMT](image)
Figure 2.6: Quantum Efficiencies of several 20 inch PMTs

Figure 2.7: Pulse height distribution corresponding to 1 p.e. of 20 inch PMT.
2.3.4 Water Purification System and Radon Free Air System

In order to collect Cherenkov photons efficiently, the water, as a mediator of Cherenkov photons, must be kept as transparent as possible. Furthermore, radioactive elements dissolved in the water, which is serious background for solar neutrino events, should be removed. For such purposes, a water purification system and radon free air system is necessary.

Fig. 2.8 shows the water purification and radon free air system for Super-Kamiokande.

The 50,000 tons of water used in Super-Kamiokande is made from the water in the mine and it goes through the water purification system to get rid of impurities. When pure water was filled in the tank, it is circulated among the detector and water purification system to get higher purity of water.

![Block diagram for water purification system and radon free air system](image)

Figure 2.8: Block diagram for water purification system and radon free air system
2.3.5 Electronics and DAQ

Super-Kamiokande has many channels of PMT signals, therefore the electronics and DAQ system have a complicated hierarchy. The block diagram of Electronics and DAQ (Data AcQuisition system) are shown in Fig.2.9. It is explained how PMT's signal go to data taking workstation as an event information in the following text.

Getting PMT's signal and converting them to charge and timing information are performed by ATM (Analog Timing Module). ATM has functions of ADC (Analog to Digital Converter), TDC (Timing to Digital Converter), Amplifier and Discriminator. It is based on TKO (TRISTAN/KEK Online) specification and designed for Super-Kamiokande.

Each ATM has 12 inputs from PMTs, and signal of each input is amplified and divided into several signals. One is discriminated and the TTL logic output of all channels are summed to make a HITSUM signal. The other two signals are converted by TAC/QAC (timing-to-analog converter / charge-to-analog converter) to a signal with a pulse height corresponding to it's timing/charge quantity. The HITSUM signal for each ATM goes through a trigger making module and given to a workstation to make a global trigger. The signal of the global trigger is divided and goes through GONG (Go or Not Go) module and given to all ATMs. When an ATM gets a global trigger signal, it converts the timing/charge signal to a digital value by ADC and write them to FIFO (First In First Out) memories. The timing/charge information from the FIFO are gotten by the VME interface board called SCH (super control header) and saved in the memory buffers called SMP (super memory partner) and given to data taking workstations.

There are 8 data taking workstations and data of each workstation is transported through FDDI and collected to one workstation and reconstructed as an event information.

2.3.6 Real Time Data Reductions

In Super-Kamiokande, the data transfer size is huge (6 Gbyte/hour) because of the large number of PMT channels (11146), and high trigger counting ratio (~ 10Hz). Therefore, data of Super-Kamiokande is reduced by offline computers in real time. Fig.2.10 is a block diagram of the real time data reductions. In the following text, each process of the real time reduction is explained.

The reconstructed data are transferred from online computer in the mine to the offline computer in the office out of the mine by FDDI. Then data are reformatted to ZEBRA \(^4\) format and saved on a ram disk temporarily.

The reformated files are loaded on the memory of a super-computer and converts the timing/charge information from ADC/TDC counts to real physical unit of nano sec/photonlectron. We called this procedure TQREAL process.

After TQREAL process, the data are distributed to several workstations and subjected to the various reductions for each analysis, USA, LOW, MU, MID, SN, etc.

USA is the copy process to tape which belongs to the offsite analysis group \(^5\), LOW is the reduction process for the low energy analysis – i.e. solar neutrino analysis, MU is the reduction process of muon events for upward through going muons and the radioactive spallation cut of low energy analysis, MID is the reduction process for the middle energy analysis – i.e. atmospheric neutrino analysis as contained events, and the nucleon decay analysis. SN is a watch process for super nova.

After those reduction processes, the data are saved in the MTL (Magnetic Tape Library).

\(^4\) Fortran based data bank system software belong to the CERN program library.

\(^5\) This analysis group is composed of members from universities and laboratories in United States.
Figure 2.9: DAQ Block Diagram of Super Kamiokande
Online Format Data

Reformat

Ram Disk

Super Computer VPX 210

Super Computer Super SUPARK 20

TQREAL

Flow

MTL

Disk

US storage

Work Stations Super SPARK 20

USA LOW MU MID SN etc

USA LOW MU MID SN etc

Figure 2.10: Realtime Offline Reduction Process
Chapter 3

Event Selection

In this thesis, upward through going muon events which originate from atmospheric neutrinos are used to search for muon neutrino oscillations. The method to select upward through going muon events is described in this chapter.

3.1 Event Selection Overview

Kamiokande and Super-Kamiokande detectors consist of pure water for a target material and a lot of PMTs for detecting Cherenkov light. The information used for event reconstruction is as follows:

1. Charge of PMT signal
2. Timing of PMT signal
3. Geometrical pattern of hit PMTs

Generally speaking, charge information is used to classify the event energy. On the other hand, timing information and geometrical pattern are used to reconstruct the event vertex point and its direction.

The primitive quantities for event selection are:

- **Total Q**
  - Sum of charge of all PMTs in the inner detector. The unit of charge is photoelectrons. It is also called “Total PE”.

- **Max Q**
  - The charge of the PMT which has the maximum charge among all the PMTs.

- **Nhit**
  - Number of hit PMTs in one event, where “hit PMTs” means PMTs whose charges are greater than the threshold charge. A threshold charge is defined for each analysis.

- **GDNS**
  - Standard quantity to obtain goodness in the directional fitting in which the relative timing information of the PMTs is used. The definition of GDNS is in section 3.6.2.

In the case of selection of the upward through going muon events, other quantities(functions) for muon directional fitting are defined. They are defined in section 3.3.1.
CHAPTER 3. EVENT SELECTION

The event selection process is roughly divided into three steps:

1. Total Q cut.

2. Automatic directional fitting and then zenith angle selection, $\cos \theta < 0.1$, where $\theta$ is defined in Fig. 3.1.

3. Eye scan cut to get rid of junk events and then manual directional fitting to obtain the correct muon direction.

![Diagram of zenith and azimuth angles](image)

Figure 3.1: Definition of zenith and azimuth angle.

The flow chart to select the upward through going muon events is shown in Fig. 3.2 for Kamiokande-III and Fig. 3.3 for Super-Kamiokande. Each step is explained in the latter sections.
Figure 3.2: The flow chart of event selection for the upward through going muon events in Kamiokande-III.
Figure 3.3: The flow chart of event selection for the upward through going muon events in Superk-Kamiokande.
3.2 Typical Muon Events

In order to see the geometrical pattern of Cherenkov light, we use the detector-spread-shaped event display. For example, a detector-punch-through muon event like Fig.3.4 looks like Fig.3.5 by the Kamiokande-III event display, and like Fig.3.6 by the Super-Kamiokande event display.

The spread-shape of the inner detector is represented by two circles and one rectangle which correspond to the top, bottom and barrel surface of the cylinder-shaped detector. The spread-shape of the anti detector is also shown at the upper left position of the event display which has a similar shape to the inner one. "O" marks on the detector surface represent PMTs attached on the detector surface. The size of the "O" is proportional to the pulse height of corresponding PMT. "×" marks on the detector surface represent dead PMT positions.

In Fig.3.5, we can see a ring-shaped Cherenkov light pattern from a cosmic-ray muon. In this event, the entrance and exit positions of the muon are fitted and marked by "×". The edge line of Cherenkov ring is also drawn by assuming 42° opening angle from the fitted muon track. In the spread-shape of the anti detector, we can see the hit PMTs corresponding to the entrance and exit positions at the inner detector.

Fig.3.6 shows the event display of Super-Kamiokande. This event is a typical single-track cosmic-ray muon. The display form is similar to Kamiokande-III except that dead PMT's are marked by a small "×".

Muon events are categorized by their geometrical pattern of Cherenkov light as follows:

1. single-track muon event with no shower interactions – so called clean event.
   (Fig.3.5 for Kam-3, Fig.3.6 for S-Kam)

2. multiple-bundle muon event – so called multi event.
   (Fig.3.7, Fig.3.8 for S-Kam)

3. muon which stopped in the inner detector – so called stopping event.
   (Fig.3.9 for S-Kam)

4. muon event with hard electromagnetic and/or hadronic showers – so called hard event.
   (Fig.3.10 for S-Kam)

5. muon event crossing the detector edge – so called edge clipping event.
   (Fig.3.11, Fig.3.12 for S-Kam)
Figure 3.4: Schematic view of detector-punch-through muon event. Cherenkov light of a muon makes a ring pattern in the detector.
Figure 3.5: The detector-spread event display of Kamiokande-III. This is a typical single track cosmic-ray muon. In this event, the entrance and exit position of the muon are fitted and marked by “*”. The edge line of Cherenkov ring is also drawn by assuming 42° opening angle from fitted muon track. “O” marks on detector’s surface represent PMTs attached on detector wall. The size of “O” is proportional to pulse height of corresponding PMT. “x” marks on detector’s surface represent dead PMTs.
Figure 3.6: The detector-spread event display of Super-Kamiokande. This is a typical event of a single track cosmic-ray muon. The display form is similar to Kamiokande-III’s one except dead PMT’s are marked by small “*”.
Figure 3.7: Two track cosmic-ray muon event (Super-Kamiokande).
Figure 3.8: Four track cosmic-ray muon event. Edge lines of Cherenkov rings are not drawn. (Super-Kamiokande)
Figure 3.9: Typical event of a stopping cosmic-ray muon (Super-Kamiokande).
Figure 3.10: Typical event of a hard showered cosmic-ray muon (Super-Kamiokande).
Figure 3.11: Typical event of an edge-clipping cosmic-ray muon (Super-Kamiokande).
Figure 3.12: Another event of an edge-clipping cosmic-ray muon (Super-Kamiokande).
In order to categorize the event types, relative timing information T and charge information Q of PMTs are useful. Fig. 3.13 shows T and Q distributions of a clean cosmic-ray muon event. Relative timing distribution has two components. One is a peak distribution in the 950 ~ 1150 nsec which corresponds to the direct Cherenkov light. The other is a broad tail shown in T ≥ 1150 nsec which corresponds to the light reflected on the PMTs or the detector surface.

In terms of the charge distribution, a peak near Q ∼ 0 p.e. is from hit PMTs from reflection light or dark pulse. For detector punch-through-muon events, the charge of the PMTs which is close to the muon exit position saturates at ~ 250 p.e.

Fig. 3.14 shows the T and Q distributions of a stopping event. Since there is no exit point in the stopping muon event, we can see no charges saturate.

Fig. 3.15 shows the T and Q distributions of a hard event. The relative timing distribution doesn’t have a broad tail of reflected light because almost all the PMT’s hits from direct light are due to the cascade shower. Also the charge distribution has no reflection component. A lot of Q’s are saturated.

Figure 3.13: Upper histogram is relative timing distribution of inner PMTs for a typical single track cosmic-ray muon event which corresponds to the event of Fig. 3.6 (Super-Kamiokande). Unit of T is nano seconds. Lower histogram is the charge distribution of inner PMTs for the same event. Unit of Q is photoelectron.
CHAPTER 3. EVENT SELECTION

Figure 3.14: $T$ and $Q$ distribution of typical stopping cosmic-ray muon event. This event corresponds to the event of Fig. 3.9 (Super-Kamiokande).

Figure 3.15: $T$ and $Q$ distribution of typical hard showered cosmic-ray muon event. This event corresponds to the event of Fig. 3.10 (Super-Kamiokande).
3.3 Automatic Muon Directional Fitting

In order to select upward through going muon events it is necessary to fit the muon track. The muon track is determined by the muon entrance and exit positions on the inner detector wall. There are two ways to fit the entrance and exit positions. One is by human's eyes where two positions are determined manually by viewing the event with the detector-spread event display. The other is automatic track finding using $T, Q$ information. Since almost all of the muon events are cosmic-ray muons coming from above the horizontal direction ($\cos \theta > 0$), we have to eliminate these events as efficiently as possible. In the pre-reduction process for selecting upward through going muon events, automatic muon directional fitting is applied. The automatical method to determine the muon direction has basically 3 steps for both Kamiokande-III and Super-Kamiokande.

1. muon entrance finding
2. muon exit finding
3. muon track determination

For Super-Kamiokande, the muon event rate becomes much higher than that of Kamiokande, because of large detector acceptance \(^1\). So two muon fitting processes are applied (see Fig.3.3). The former is called the 1st muon fitting which runs fast (1 ~ 2 sec/event) so that it is used in realtime along with the online data taking. The latter is called the 2nd muon fitting which finds the muon track more precisely in the offline analysis but more slowly (1 ~ 10 sec/event).

In the following sections, automatic muon fitting for Kamiokande-III and Super-Kamiokande are explained.

3.3.1 Muon Fitting for Kamiokande-III

Processes of the muon fitting for Kamiokande-III are in the following.

1. muon entrance finding:
   Candidates of muon entrance the position are determined by positions of PMTs which are hit relatively early.

2. muon exit finding:
   Candidates of muon exit position are determined by the positions of PMTs which have maximum charges.

3. muon track determination:
   Apply fitting evaluation functions to each muon track which is reconstructed by entrance-exit candidates, then a track which reduces the best evaluation value is selected.

There are two fitting evaluation functions performed at the muon track determination. One is based on charge information and is called $R_{CONE}$ which means Ratio of hit PMT numbers inside and outside of Cherenkov cone. $R_{CONE}$ is defined by the following formula,

$$R_{CONE} = 1 - \frac{N_{\text{hit}42}}{N_{\text{PMT}42}} + \frac{N_{\text{hit}70}}{N_{\text{PMT}70}}.$$  \hspace{1cm} (3.1)

\(^1\)In Kamiokande-III counting rate of cosmic-ray muon event is about 0.37 Hz, while in Super-Kamiokande it is about 2.0 Hz.
where $N_{\text{hit}42}$ is the number of hit PMTs in 42° Cherenkov cone, $N_{\text{PMT}42}$ is the number of PMTs in 42° Cherenkov cone, $N_{\text{hit}70}$ is the number of hit PMTs between 70° cone and 42° cone, $N_{\text{PMT}70}$ is number of PMTs between 70° cone and 42° cone. The threshold charge of $N_{\text{hit}}$ is set to an empirical formula, $\sqrt{\text{total } Q/100}$. The term $N_{\text{hit}42}/N_{\text{PMT}42}$ represents the “filledness” of the Cherenkov ring and the term of $N_{\text{hit}70}/N_{\text{PMT}70}$ represents the “sharpness” of the edge of the Cherenkov ring. A smaller $R_{\text{CONE}}$ means a better muon fit.

Another fitting evaluation function is based on the relative hit timing information and called TOF-$\sigma$ which means deviation ($\sigma$) of the timing distribution subtracted Time of Flight of the muon and Cherenkov light.

TOF-$\sigma$ is defined by the following formula,

$$\text{TOF-}\sigma = \text{RMS}(T_i - \text{TOF}_i)$$

$$\text{TOF}_i = \frac{l_{\mu(i)}}{c} + \frac{l_{\rho(i)}}{cn}$$

where index $i$ is the PMT number, $T_i$ is the timing information of the $i$-th PMT, $l_{\mu(i)}$ is the distance from the entrance point of the muon to the emission point of the Cherenkov photon which will reach the $i$-th PMT, $l_{\rho(i)}$ is the distance from the emission point of the Cherenkov photon to the $i$-th PMT, $c$ is the light velocity in vacuum, $n$ is the refractive index of water. Fig.3.16 shows how to define $l_{\mu(i)}$ and $l_{\rho(i)}$. RMS is the root mean square function defined by

$$\text{RMS}(x_i) = \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}}$$

A smaller TOF-$\sigma$ means a better muon fit.

Figure 3.16: Positional correlation between muon and emitted Cherenkov photon.

These functions have good convergence to the true position of the muon entrance/exit point. Fig.3.17 and Fig.3.18 shows convergence of $R_{\text{CONE}}$, TOF-$\sigma$ for Kamiokande-III, respectively. The position which makes $R_{\text{CONE}}$ and TOF-$\sigma$ a minimum value is taken as best fit position for the muon entrance/exit.
Figure 3.17: The upper figure shows the convergence of $R_{\text{CONE}}$ to the muon entrance. The lower figure shows the convergence of $R_{\text{CONE}}$ to the muon exit. (Kamiokande-III)

Figure 3.18: The upper figure shows convergence of TOF-\(\sigma\) to the muon entrance. The lower figure shows convergence of TOF-\(\sigma\) to the muon exit. (Kamiokande-III)
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When one selects upward through going muon events, cosmic-ray muons are eliminated by their zenith angles, $\cos \theta \leq 0$, so good angular resolution for zenith angle directions is necessary. The zenith angle deviation is defined as

$$\Delta \theta = |\theta_{MF} - \theta_{MC}| \quad (3.5)$$

where $\theta_{MF}$ is the zenith angle of the muon reconstructed by automatic muon fitting, $\theta_{MC}$ is the zenith angle of the muon for the MC initial parameter.

$\Delta \theta$ distribution is fitted by a gaussian function,

$$G_{\text{zenith}}(\Delta \theta) = c(\Delta \theta) \cdot \exp \left( -\frac{1}{2} \left( \frac{\Delta \theta}{\sigma(\Delta \theta)} \right)^2 \right) \quad (3.6)$$

$$c(\Delta \theta) = a_0 \cdot e^{-a_1 \Delta \theta^2} \quad (3.7)$$

$$\sigma(\Delta \theta) = a_0 + a_2 \cdot \Delta \theta^2 \quad (3.8)$$

where scale factor $c(\Delta \theta)$ and deviation sigma $\sigma(\Delta \theta)$ are dependent on $\Delta \theta$, therefore, there are 4 free parameters, $a_0, a_0, a_1, a_2$.

Fig. 3.19 shows the zenith angle deviation of muon fitting. From this distribution we have,

$$a_0 = 473 \pm 12, \quad a_0 = 2.66 \pm 0.06^\circ$$

$$a_1 = 0.0072 \pm 0.0009, \quad a_2 = 0.0165 \pm 0.0015$$

The zenith angle resolution of muon fitting is now reduced by the following formula,

$$\sigma_\theta = \frac{\int c(x) \cdot \sigma(x) \, dx}{\int c(x) \, dx} \quad (3.9)$$

We got

$$\sigma_\theta = 3.68 \pm 0.07^\circ \quad (3.10)$$

Because the Kamiokande-III/Super Kamiokande detectors have cylindrical shape, the geometrical acceptance is not symmetric for zenith angle direction. We have to check zenith angle dependence of angular resolution. Fig. 3.20 shows that dependence. We can see no significant zenith angle dependence.

Moreover muon track reconstruction is suspected to depend on the muon track length for the same reason. We also check the track length dependence of the angular deviation. Fig. 3.21 shows that dependence.

We can see the inverse proportional correlation between the track length and the zenith angle deviation. The origin of this dependence is that if the muon track length is short, the number of PMTs used in the muon track reconstruction become relatively few (see Fig. 3.12), and that makes the large deviation to fitting evaluation functions defined by Eq.(3.1), Eq.(3.2).

The correlation becomes constant when the track length is more than 7 m. Therefore we adopted a 7 m track length cut for the final upmu samples.
Figure 3.20: Zenith angle dependence of zenith angle resolution $\theta$ (Luminosity=III).

Figure 3.19: Zenith angle dependence of zenith angle resolution $\theta$ (Luminosity=III).
Track length dependence of zenith angle deviation

Figure 3.2: Track length dependence of the zenith angle deviation. We applied a thin track length cut to get good zenith angle resolution. (Kamokande-III)

The angular deviation of muon hitting is estimated by using muon events which are created by Monte Carlo detector simulation.

The angular deviation of muon hitting is shown in Fig. 3.2. We can see angular deviation becomes halved in comparison with Kamokande-II, since because PMT coverage density of Super-Kamiokande is two times denser than that of Kamokande-II.

The algorithm of muon track determination is as follows:

1.Muon entrance hitting:
   - The candidates of muon entrance position are determined by the positions of PMT's which have relatively early arrival times.

2.Muon exit hitting:
   - The candidates of muon exit position are determined by the positions of PMT's which have matched candidates with early entrance hitting. The algorithm is iterated until all candidates exit to one PMT.

3.Muon track determination:
   - Linear change of center of mass position to the true muon entrance/exit points are very good (see Fig. 3.2.3, Fig. 3.2.4)
   - The algorithm of muon track determination for 1st muon hitting is simple and fast. It finds the track length dependence of zenith angle deviation

muon track length (m)

zenith angle deviation (deg.)
The zenith angle deviation of 1st muon fitting is shown in Fig. 3.25. The zenith angle deviation is fitted by function $G_{\text{zenith}}(\Delta \theta)$ which is defined by Eq. (3.6).

From this distribution we have,

\begin{align*}
\alpha_0 &= 255 \pm 12, & \sigma_0 &= 1.24 \pm 0.04^\circ \\
\alpha_1 &= 0.0131 \pm 0.0023, & \alpha_2 &= 0.0264 \pm 0.0022
\end{align*}

We have the zenith angle resolution for 1st muon fitting by Eq. (3.9)

\[ \sigma_\theta = 2.23 \pm 0.05^\circ. \] (3.11)

### 3.3.3 2nd Muon Fitting for Super-Kamiokande

The process of 2nd muon fitting for Super-Kamiokande is as follows,

1. **Muon entrance finding:**
   The candidates of muon entrance position are determined by the PMT positions which are hit relatively early.

2. **Muon exit finding:**
   The candidates of muon exit position are determined by the positions of PMTs which have maximum charges.

3. **Muon track determination:**
   Muon entrance/exit candidates positions are clustered and for each cluster, fitting evaluation functions defined by Eq. (3.1), (3.2) are applied, then the track which reduces the best evaluation value is selected.

The fitting evaluation functions, $R_{\text{CON}}, R_{\text{TOF-}}$, are same as those of Kamiokande-III. The convergence of the fitting evaluation function for 2nd muon fitting of Super-Kamiokande is shown in Fig. 3.26 and Fig. 3.27. We can see good convergence both of $R_{\text{CON}}$ and $R_{\text{TOF}}$.

The zenith angle deviation for 2nd muon fitting of Super-Kamiokande is shown in Fig. 3.28. From this distribution we got,

\begin{align*}
\alpha_0 &= 273 \pm 13, & \sigma_0 &= 1.14 \pm 0.04^\circ \\
\alpha_1 &= 0.0189 \pm 0.0039, & \alpha_2 &= 0.0313 \pm 0.0032
\end{align*}

We got zenith angle resolution,

\[ \sigma_\theta = 1.97 \pm 0.06^\circ. \] (3.12)

Zenith angle dependence of angular resolution for 2nd muon fitting is checked. Fig. 3.29 show that dependence. We can see no significant zenith angle dependence on it.

Muon track length dependence of the zenith angle deviation is shown in Fig. 3.30. We can see an inverse proportional correlation between track length and zenith angle deviation. Although the correlation becomes constant when the track length is more than 7 m. Therefore we adopted a 7 m track length cut to 2nd muon fitting.
Figure 3.22: Single muon event generated by detector MC simulation. Muon entrance and exit positions marked by “☆” are extracted from initial position and momentum of the muon in this MC event. The edge line of the Cherenkov ring is drawn in this event display.
Figure 3.23: Only PMTs which are hit relatively early are drawn in this event display. The data are generated by MC (same to Fig.3.22). We can see clearly that the position of fast hit PMTs well agrees with the position of the muon entrance.
Figure 3.24: Only PMTs which are charge saturated (Q > 235) are drawn in this event display. The data are generated by MC (same to Fig. 3.22). We can see clearly that the position of the saturated Q PMTs well agrees with the position of muon exit.
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Figure 3.25: Zenith angle deviation of muon direction reconstructed by automatic 1st muon fitting compared with MC muon direction as reference. (Super-Kamiokande)

Figure 3.26: The upper figure shows convergence of $R_{\text{CONE}}$ to the muon entrance. The lower figure shows convergence of $R_{\text{CONE}}$ to the muon exit. (Super-Kamiokande)
Figure 3.27: The upper figure shows the convergence of TOF-\(\sigma\) to the muon entrance. The lower figure shows the convergence of TOF-\(\sigma\) to the muon exit. (Super-Kamiokande)

Figure 3.28: The zenith angle deviation of muon direction reconstructed by the automatic 2nd muon fitting compared with MC muon direction as reference. (Super-Kamiokande)
We applied a $\tau$ in track length cut to get good zenith angle resolution. Figure 3.30: The track length dependence of the zenith angle deviation of 2nd muon fitting.
3.4 Total Q and Event Classification

As has been discussed previously, events taken by Kamiokande-III and Super-Kamiokande are classified by Total Q. Fig. 3.31 and Fig. 3.32 shows Total Q distribution of Kamiokande-III and Super-Kamiokande respectively.

There are two peaks in the total Q distribution. The lower one dominantly consists of $\beta$, $\gamma$-rays from radioactive sources in the water or from excited nuclei which are spalled by cosmic-ray muons. The Cherenkov light intensity of this peak corresponds to energy deposits of electrons which have $1 \sim 50$ MeV. Solar neutrino events are hidden in this energy region.

The valley region between two peaks dominantly consists of Cherenkov light from cosmic-ray muons with track lengths which are relatively short such as detector edge clipping muons and stopping muons. Cherenkov light from electrons, muons and pions which are generated in the detector by atmospheric neutrinos are also in this region. The energy deposit of electrons and muons in this region is $100$ MeV $\sim 10$ GeV. The fully contained events of atmospheric neutrinos and, if they exist, proton decay events are hidden in this energy region.

The higher peak consists of the Cherenkov light from cosmic-ray muons which punch through detector. The energy deposit of muons in this region is $1$ GeV $\sim 100$ GeV. The upward through going muon events are hidden in this energy region.

Total Q distribution is proportional to the muon track length because the muon energy deposit is proportional to it’s track length. Fig. 3.33 and Fig. 3.34 shows the correlation between total Q and muon track length in Kamiokande-III and Super-Kamiokande. Distributions of total Q divided by the track length are also shown in the figures.

The peak value is fitted by a gaussian function, and it reduces to this formula between total Q and muon track length,

Kamiokande-III:

$$\text{total Q(p.e.)} = (13.7 \pm 1.4) \times \text{track length(cm)} ,$$  \hspace{1cm} (3.13)

Super-Kamiokande:

$$\text{total Q(p.e.)} = (22.5 \pm 2.2) \times \text{track length(cm)} .$$ \hspace{1cm} (3.14)

The reason for the difference in the coefficient between Kam-3 and S-Kam is that PMT gain and PMT coverage density is increased from Kam-3 to S-Kam.
Figure 3.31: Total Q distribution of Kamiokande-III.

Figure 3.32: Total Q distribution of Super-Kamiokande.
Figure 3.33: The upper figure shows the ratio of Total Q and muon track length, and the lower figure shows the correlation plot between Total Q and the muon track length in Kamiokande-III.

Figure 3.34: Same as Fig.3.33 in Super-Kamiokande.
Actually, a muon deposits its energy not only by ionization but also by electromagnetic cascade shower – bremsstrahlung, direct pair production and photo-nuclear interactions. This additional energy deposition appears as a Landau fluctuation tail in the Total Q/muon track length distribution.

Selecting upward through going muon events, we perform a total Q cut to select detector through going muon events.

For Kamiokande-III, we set total Q cut value as

$$6000\text{(p.e.)} < \text{total Q} < 30000\text{(p.e.)}. \quad (3.15)$$

The lower limit, 6000 p.e., corresponds to $4 \sim 5$ m muon track length is deduced from Eq.(3.13). From study of muon directional fitting we found the angular resolution of muon directions is bad when the muon track length is short, so we adapted 7m track length cut (see Fig.3.21). At the pre-reduction phase, we took $4 \sim 5$ m cut for safety.

The upper limit, 30000 p.e., corresponds to $20 \sim 25$ m muon track length which is as long as maximum passing length for Kamiokande-III detector. Although actually, the total Q of hard EM showered muon events is larger than 30000 p.e., so hard events may be lost in this cut. We estimate how many hard events are lost by MC events. Fig.3.35 shows the efficiency of total Q cut for various muon energies.

![Energy dependence of total Q cut efficiency](image)

Figure 3.35: Efficiency of total Q cut of Kamiokande-III for various muon energies.

For Super-Kamiokande, we set total Q cut value for the 1st reduction as

$$5000\text{(p.e.)} < \text{total Q} . \quad (3.16)$$
and for the 2nd reduction as
\[
12000(\text{p.e.}) < \text{total } Q .
\] (3.17)

The 5000 p.e. cut for the 1st reduction corresponds to \(2 \sim 2.5\) m muon track length is deduced from Eq. (3.14). The saved data passing this reduction is used not only for upward thorough going muon analysis but also other muon analyses.

The 12000 p.e. cut for the 2nd reduction corresponds to \(4.9 \sim 5.9\) m muon track length. The angular resolution of muon direction is bad when muon track length is short. The 7m track length cut is adopted (see Fig. 3.30). At the pre-reduction phase, we took \(4.9 \sim 5.9\) cut for safety. In Super-Kamiokande, the upper limit of total Q cut is not set in order to remain hard showered events. So the efficiency of the total Q cut in Super-Kamiokande is 100%.

### 3.5 Other Q Cuts

For upward going muons coming from out side the detector, PMTs in anti detector must hit. On the other hand, muons from fully contained events are generated in the inner detector, so the PMTs in the anti detector must not hit.

In Kamiokande-III, to eliminate contained events, we set following condition after the total Q cut,
\[
0(\text{p.e.}) < \text{anti total } Q .
\] (3.18)

In Super-Kamiokande, anti total Q cut is not applied because in our analysis, especially in the early time of the experiment, the DAQ system of the anti detector was not stable. Nevertheless, a maximum Q cut,
\[
235(\text{p.e.}) < \text{Max } Q .
\] (3.19)

is applied in the 1st reduction of Super-Kamiokande. This condition is set by assuming the charge of PMTs at the muon exit must saturate if it is a penetrating muon event. For this condition, almost all single-ring stopping muon events are rejected.

### 3.6 Muon Zenith Angle Selection

#### 3.6.1 Zenith Angle Selection for Kamiokande-III

After total Q cut and anti total Q cut, automatic muon fitting described in section 3.3.1 is applied. Goodness of muon fitting is estimated by the following conditions,
\[
R_{\text{Cone}} < \frac{2000}{\text{total } Q} ,
\] (3.20)
\[
\text{TOF} - \sigma < 7 \text{ nsec} ,
\] (3.21)

these conditions are deduced empirically.

After estimating goodness of muon fitting, zenith angle selection of the remaining events is performed,
\[
\theta > 80^\circ .
\] (3.22)

All remained bad condition events are saved. Finally, good condition events are selected as follows,
\[
\theta > 90^\circ .
\] (3.23)
3.6.2 Zenith Angle Selection for Super-Kamiokande

After total Q cut and max Q cut, 1st muon fitting described in section 3.3.2 is performed. Goodness of muon fitting is estimated by the following conditions,

\[ L_{\text{enter}} > 3(m) \quad \& \quad L_{\text{exit}} < 3(m) \quad (3.24) \]
\[ \& \quad \text{GDNS} > 0.88 \quad (3.25) \]

where \( L_{\text{enter}} \) is the distance from the max Q PMT position to the muon entrance position, \( L_{\text{exit}} \) is the distance from the max Q PMT position to the muon exit position. These conditions are applied because the max Q PMT position is identical to muon exit position. GDNS is defined in the following formula,

\[ \text{GDNS} = \sum_i \frac{1}{\sigma_i^2} \exp \left( -\frac{(t_i - T)^2}{2 \cdot (\sigma \cdot a)} \right) / \sum_i \frac{1}{\sigma_i^2} \quad (3.26) \]

where \( t_i = T_i - \text{TOF}_i \) is defined in Eq.(3.2), (3.3), \( T \) is time when muon enter into inner detector, \( \sigma_i \) is timing resolution of \( i \)-th PMT, \( \sigma \) is mean of \( \sigma_i \), \( a \) is scaling factor. GDNS represents \( \chi^2 \) estimation quantity which are normalized by exponential function, so it has range of 0 ~ 1. GDNS \( \rightarrow 1 \) means better fitting.

After estimating goodness of muon fitting, remained good condition events are performed zenith angle selection as follows,

\[ \cos \theta < 0.1 \quad . \quad (3.27) \]

All remained bad condition events are saved.

After 1st muon fitting, total Q cut of 2nd reduction is applied, then 2nd muon fitting is applied. All fitted events are selected by zenith angle same as 1st reduction,

\[ \cos \theta < 0.1 \quad . \quad (3.28) \]

3.7 Eye Scan & Manual Muon Fitting

Even though zenith angle selection is performed, some events which are not upward through going muons remain.

Almost all such events are cosmic-ray muons with bad fits. Others are pedestal events \(^2\), PMT flashing events \(^3\) and contained events.

Table 3.1, 3.2 shows the fraction of event type remained before the eye scan phase for Kamiokande-III and Super-Kamiokande.

Upward through going muon events selected by eye scan are finally fitted manually. The manual muon directional fitting is performed by selecting muon entrance/exit points by looking at the detector-spread event display. We estimate angular deviation of manual directional fitting by comparing with MC muon events.

Zenith angle deviation of manual fitting is defined as follows

\[ \Delta \theta_{\text{MA}} = |\theta_{\text{MA}} - \theta_{\text{MC}}| \quad , \quad (3.29) \]

\(^2\)Event that a part of channels do not take real signal but null signal to take the pedestal of their channels.

\(^3\)Event that one PMT is flashing because it’s dinode is damaged. If it is often flashing, that channel will be killed.


<table>
<thead>
<tr>
<th>event type</th>
<th>ratio(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cosmic-ray muon</td>
<td></td>
</tr>
<tr>
<td>clean event</td>
<td>28</td>
</tr>
<tr>
<td>multi event</td>
<td>27</td>
</tr>
<tr>
<td>edge clipping event</td>
<td>25</td>
</tr>
<tr>
<td>hard event</td>
<td>14</td>
</tr>
<tr>
<td>stopping event</td>
<td>3</td>
</tr>
<tr>
<td>others</td>
<td>2</td>
</tr>
<tr>
<td>upward through going muon</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.1: Left event type after automatic zenith angle selection in Kamiokande-III.

<table>
<thead>
<tr>
<th>event type</th>
<th>ratio(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cosmic-ray muon</td>
<td></td>
</tr>
<tr>
<td>multi event</td>
<td>55</td>
</tr>
<tr>
<td>edge clipping event</td>
<td>12</td>
</tr>
<tr>
<td>hard event</td>
<td>13</td>
</tr>
<tr>
<td>stopping event</td>
<td>7</td>
</tr>
<tr>
<td>clean event</td>
<td>1</td>
</tr>
<tr>
<td>others</td>
<td>7</td>
</tr>
<tr>
<td>upward through going muon</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.2: Left event type after automatic zenith angle selection in Super-Kamiokande.

where $\theta_{\text{MA}}$ is zenith angle of muon reconstructed by manual muon fitting, $\theta_{\text{MC}}$ is zenith angle of muon for MC initial parameter.

Fig. 3.36 and Fig. 3.37 are zenith angle deviations of Kamiokande-III and Super-kamiokande, respectively. We fitted these deviation distribution by function $G_{\text{zenith}}(\Delta \theta)$ defined by Eq.(3.6) with the condition $a_1 = a_2 = 0$ because they have no broad tail components so that man doesn’t act so terrible mistake in the muon directional fitting.

For Kamiokande-III, the fitting results are $c_0 = 30.5 \pm 3.9$, $\sigma_0 = 1.89 \pm 0.08$°.

Zenith angle resolution, $\sigma_{\theta}^{\text{MA}}$, is

$$\sigma_{\theta}^{\text{MA}} = 1.89 \pm 0.08° . \quad (3.30)$$

For Super-Kamiokande, the fitting results are $c_0 = 67.1 \pm 6.5$, $\sigma_0 = 0.90 \pm 0.06$°.

Zenith angle resolution, $\sigma_{\theta}^{\text{MA}}$, is

$$\sigma_{\theta}^{\text{MA}} = 0.90 \pm 0.06° . \quad (3.31)$$
Figure 3.36: Zenith angle deviation of manual directional reconstruction for Kamiokande-III.

Figure 3.37: Zenith angle deviation of manual directional reconstruction for Super-Kamiokande.
3.8 Summary of the Event Selection

In Kamiokande-III, total 188 events of upward through going muon events are found in live time 1332.5 days. Table 3.3 shows the summary of event selection for upward through going muon in Kamiokande-III.

<table>
<thead>
<tr>
<th></th>
<th>number of event</th>
</tr>
</thead>
<tbody>
<tr>
<td>total events</td>
<td>$1.4 \times 10^8$</td>
</tr>
<tr>
<td>after total Q cut</td>
<td>$2.9 \times 10^4$</td>
</tr>
<tr>
<td>after zenith angle cut $90^\circ &lt; \theta$</td>
<td>23248</td>
</tr>
<tr>
<td></td>
<td>$80^\circ &lt; \theta \leq 90^\circ$</td>
</tr>
<tr>
<td>after eye scan $(\cos \theta &lt; -0.04)$</td>
<td>188</td>
</tr>
</tbody>
</table>

Table 3.3: Summary of event selection of upward through going muon events in Kamiokande-III

In Super-Kamiokande, total 413 events of upward through going muon events are found in live time 363 days. Table 3.4 shows the summary of event selection for upward through going muon in Super-Kamiokande.

<table>
<thead>
<tr>
<th></th>
<th>number of event</th>
</tr>
</thead>
<tbody>
<tr>
<td>total events</td>
<td>$3.5 \times 10^8$</td>
</tr>
<tr>
<td>after 1st reduction</td>
<td>$2.1 \times 10^5$</td>
</tr>
<tr>
<td>after 2nd reduction $\cos \theta \leq 0$</td>
<td>6629</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \cos \theta &lt; 0.1$</td>
</tr>
<tr>
<td>after eye scan</td>
<td>413</td>
</tr>
</tbody>
</table>

Table 3.4: Summary of event selection of upward through going muon events in Super-Kamiokande

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4 In case of Kamiokande-III, the selection efficiency of horizontal direction is not so good, we adapted $\cos \theta < -0.04$ (see Section 4.3).

5 Actually, we have two independent analysis in Kamiokande-III (Ref.[32]), the final data are combined both of them.
Chapter 4

Observed Flux of Upward Through Going Muons

The calculation of observed upward through going muon flux requires the following terms,

- **Detector Live Time**: The observation time which is used for analysis of upward through going muons.

- **Effective Area**: Projected area of the detector on a plane normal to the direction of the muon track, where the muon track length is longer than 7 m.

- **Selection Efficiency**: Efficiency of selecting upward through going muons for one direction.

The differential flux of upward through going muons is defined as follows,

\[
\left( \frac{d\Phi}{d\cos \theta} \right)_i = \frac{N_i}{T \cdot \Omega_i \cdot S_i \cdot \varepsilon_i}, \quad (4.1)
\]

\[
\Omega_i = \frac{2\pi}{N_{\text{bin}}} = 2\pi \times \begin{cases} 0.1, & \text{for } i=10 \text{ in Kam-II,III} \\ 0.06, & \text{otherwise} \end{cases}, \quad (4.2)
\]

\[
i = \text{Int} \left( (\cos \theta_j + 1) \times 10 \right) + 1, \quad (4.3)
\]

where the index \( i \) represents the \( i \)-th bin of zenith angle, \( \text{Int}(x) \) is the conversion function from real numbers to integers, \( \cos \theta_j \) is the cosine of the zenith angle of the \( j \)-th upward through going muon, \( T \) is the detector live time, \( \Omega_i \) is the solid angle of the \( i \)-th bin, \( N_{\text{bin}}(= 10) \) is the number of bins, \( S_i \) is the effective area of the \( i \)-th bin, \( \varepsilon_i \) is the selection efficiency of the \( i \)-th bin, \( N_i \) is the number of upward through going muons in the \( i \)-th bin.

The total flux of upward through going muons is defined by averaging differential fluxes,

\[
\Phi_{\text{tot}} = \sum_{i=1}^{N_{\text{bin}}} \frac{1}{N_{\text{bin}}} \left( \frac{d\Phi}{d\cos \theta} \right)_i. \quad (4.4)
\]

\(^1\)This function round off under floating point part of \( x \).


4.1 Live Time Calculation

The total live time for upward through going muon analysis is calculated by

\[ T = \sum_i (T_{\text{run}}^i - T_{\text{dead}}^i - T_{\text{ped}}^i), \]  

(4.5)

where the index \( i \) represents the \( i \)-th observation run, \( T_{\text{run}}^i \) is the run time, \( T_{\text{dead}}^i \) is the electronics dead time, and \( T_{\text{ped}}^i \) is the pedestal event time.

In Kamiokande-III, the total live time is

\[ T = 1332.5 \text{ days}. \]

The efficiency of live time \( T/T_{\text{real}} \) is 86.8%, where \( T_{\text{real}} \) is the elapsed time from the beginning analysis run. Fig. 4.1 shows the long term variation of the live time efficiency of Kamiokande-III.

![Live Time Efficiency of Kamiokande-III](image)

Figure 4.1: Long term variation of live time efficiency of Kamiokande-III.
In Super-Kamiokande, the total live time \(^2\) is

\[ T = 363.3 \text{ days}. \]

The efficiency of the live time – \( T/T_{\text{real}} \) is 81.3%. Fig.4.2 shows the long term variation of the live time efficiency of Super-Kamiokande.

Figure 4.2: Long term variation of live time efficiency of Super-Kamiokande.

\(^2\)This is current status. Observation in Super-Kamiokande is still continuing.
4.2 Effective Area

The effective area for an upward through-going muon from a given direction is defined to be the projected area of the detector on a plane normal to the direction of the muon track. In this analysis, upward through-going muons with a track length longer than 7m are selected, so the projected area with the track length longer than 7m is defined as the effective area. Fig. 4.3 shows a schematic illustration of the effective area.

The calculation method of the effective area is as follows. A large enough plane is taken near the detector. Two-dimensional grid points are defined on this plane every 10cm step, and a line is drawn from each point vertically to the detector. If the length of the line cutting the two intersecting points with the inner detector walls is longer than 7m, the corresponding grid point is counted as part of the detection area with the track length longer than 7m. Therefore, the effective area becomes (number of grid points) \times (100cm^2).

The zenith angle in the direction of the area is changed degree by degree and this calculation procedure is repeated. The calculation result of the effective area as a function of cosine zenith angle is shown in Fig. 4.4.

Figure 4.3: Schematics of the effective area for upward through-going muons which track length is longer than 7m.
Figure 4.4: Zenith angle dependence of effective area of 7m track length cut for upward through going muon.

### 4.3 Selection Efficiency

The selection efficiency of the zenith angle cut by muon fitting of the $i$-th cosine bin is defined as follows,

$$
\varepsilon_i = \frac{N_i^{\text{left}}}{N_i},
$$

where the index $i$ means the $i$-th bin of the cosine zenith angle, $N_i$ is the number of events in the $i$-th bin, $N_i^{\text{left}}$ is the number of remaining events. $N_i^{\text{left}}$ is estimated by using the MC data.

<table>
<thead>
<tr>
<th>bin</th>
<th>$i=1$</th>
<th>$i=2$</th>
<th>$i=3$</th>
<th>$i=4$</th>
<th>$i=5$</th>
<th>$i=6$</th>
<th>$i=7$</th>
<th>$i=8$</th>
<th>$i=9$</th>
<th>$i=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos\theta$</td>
<td>-0.95</td>
<td>-0.85</td>
<td>-0.75</td>
<td>-0.65</td>
<td>-0.55</td>
<td>-0.45</td>
<td>-0.35</td>
<td>-0.25</td>
<td>-0.15</td>
<td>-0.05</td>
</tr>
<tr>
<td>$N_i$</td>
<td>471</td>
<td>507</td>
<td>529</td>
<td>522</td>
<td>598</td>
<td>676</td>
<td>746</td>
<td>870</td>
<td>972</td>
<td>1244</td>
</tr>
<tr>
<td>$N_i^{\text{left}}$</td>
<td>469</td>
<td>507</td>
<td>525</td>
<td>517</td>
<td>591</td>
<td>675</td>
<td>744</td>
<td>839</td>
<td>966</td>
<td>1135</td>
</tr>
<tr>
<td>$\varepsilon_i$</td>
<td>0.996</td>
<td>1.000</td>
<td>0.992</td>
<td>0.990</td>
<td>0.988</td>
<td>0.999</td>
<td>0.997</td>
<td>0.987</td>
<td>0.994</td>
<td>0.912</td>
</tr>
</tbody>
</table>

Table 4.1: Selection efficiency of zenith angle cut by muon fitting of Kamiokande-III
Table 4.1 shows the selection efficiency for each cosine bin in Kamiokande-III. The selection efficiency at the most horizontal cosine bin ($i = 10$) is not so good compared to other bins, so we change the binning from $-0.1 < \cos \theta < 0$ to $-0.1 < \cos \theta < -0.04$. Therefore, in the case of Kamiokande-III, the solid angle of the most horizontal bin is changed to $\Omega_{i=10} = 2\pi \times 0.06$ (see Eq.(4.2)). The selection efficiency of $i = 10$ bin becomes the following:

<table>
<thead>
<tr>
<th>$\cos \theta$</th>
<th>$\varepsilon_{i=10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td></td>
</tr>
</tbody>
</table>

In the case of Super-Kamiokande, the event selection process has two parts and we estimate each selection efficiency independently and multiply them, $\varepsilon_i = \varepsilon_{i,1} \times \varepsilon_{i,2}$, where $\varepsilon_{i,1}, \varepsilon_{i,2}$ are selection efficiencies of 1st and 2nd zenith angle cut respectively. Table 4.2 show the selection efficiency of 1st and 2nd zenith angle cut, where $N_{i,1}, N_{i,2}$ and $N_{i,1}^{\text{left}}, N_{i,2}^{\text{left}}$ are the total number of events and number of remaining events of 1st, 2nd zenith angle cut, respectively. The selection efficiency of the most horizontal bin is not so bad even if we take $-0.1 < \cos \theta < 0$ for the most horizontal bin ($i=10$).

<table>
<thead>
<tr>
<th>bin</th>
<th>$i=1$</th>
<th>$i=2$</th>
<th>$i=3$</th>
<th>$i=4$</th>
<th>$i=5$</th>
<th>$i=6$</th>
<th>$i=7$</th>
<th>$i=8$</th>
<th>$i=9$</th>
<th>$i=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos \theta$</td>
<td>-0.95</td>
<td>-0.85</td>
<td>-0.75</td>
<td>-0.65</td>
<td>-0.55</td>
<td>-0.45</td>
<td>-0.35</td>
<td>-0.25</td>
<td>-0.15</td>
<td>-0.05</td>
</tr>
<tr>
<td>$N_{i,1}$</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td>$N_{i,2}$</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td>$N_{i,1}^{\text{left}}$</td>
<td>9986</td>
<td>9979</td>
<td>9980</td>
<td>9975</td>
<td>9971</td>
<td>9974</td>
<td>9964</td>
<td>9944</td>
<td>9892</td>
<td>9767</td>
</tr>
<tr>
<td>$N_{i,2}^{\text{left}}$</td>
<td>9984</td>
<td>9981</td>
<td>9984</td>
<td>9980</td>
<td>9972</td>
<td>9969</td>
<td>9967</td>
<td>9947</td>
<td>9916</td>
<td>9788</td>
</tr>
<tr>
<td>$\varepsilon_{i,1}$</td>
<td>0.999</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.997</td>
<td>0.997</td>
<td>0.996</td>
<td>0.994</td>
<td>0.989</td>
<td>0.977</td>
</tr>
<tr>
<td>$\varepsilon_{i,2}$</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.997</td>
<td>0.997</td>
<td>0.997</td>
<td>0.995</td>
<td>0.992</td>
<td>0.979</td>
</tr>
<tr>
<td>$\varepsilon_i$</td>
<td>0.997</td>
<td>0.996</td>
<td>0.996</td>
<td>0.996</td>
<td>0.994</td>
<td>0.994</td>
<td>0.993</td>
<td>0.989</td>
<td>0.981</td>
<td>0.956</td>
</tr>
</tbody>
</table>

Table 4.2: Selection efficiency of 1st, 2nd and combined zenith angle cut of Super-Kamiokande

### 4.4 Contamination by Cosmic-Ray Muons

One of the features which is worthy of special mention for upward through going muon events is that there are no background events except cosmic-ray muons which come from an almost horizontal downward direction contaminating the neutrino induced upward going muon events. So it is important to estimate the contamination of cosmic-ray muon events.

The method of how to estimate the contamination of cosmic-ray muons is explained for the Super-Kamiokande detector in the following text.

---

[3] Actually, we have two independent analysis in Kamiokande-III (Ref.[32]), the selection efficiency are combined both of them.
At first we checked the angular distribution of cosmic-ray muon by both of real observed data and MC simulated data. Fig.4.5 shows azimuth ($\phi_{\text{ter}}$)\(^4\) and zenith ($\cos \theta$) angle distribution of cosmic-ray muon observed in the period of $\sim 35$ minutes.

We can see that the distribution of the observed cosmic-ray muons agrees well with MC simulated one which take into account the slant depth of rock around the detector. Especially, in terms of azimuth angle direction, the geometrical thickness of the mountain is reflected in the rate of cosmic-ray muons arriving at the detector. On the other hand, the rate of neutrino induced muon events is not affected by the mountain shape. Fig.4.6 is $\phi$-$\cos \theta$ distribution of upward through going muon and horizontal cosmic-ray muon events observed in this analysis. The two thick clusters in the region of $\cos \theta > 0$ and $60^\circ < \phi < 240^\circ$ are cosmic-ray muons from relatively thin rock depth directions.

To distinguish neutrino induced events and cosmic-ray muon events, we divide them into two regions as follows,

\[
\text{region (1)} : \quad \phi \leq 60^\circ, \phi > 240^\circ , \\
\text{region (2)} : \quad 60^\circ < \phi \leq 240^\circ .
\]

\(^4\phi_{\text{ter}}\) is terrestrial coordinate azimuth angle, on the other hand $\phi$ is detector coordinate azimuth angle defined in Fig.3.1. There is correlation between $\phi$ and $\phi_{\text{ter}}$, $\phi_{\text{ter}} = 221^\circ - \phi$. 

---

Figure 4.5: Azimuth angle distribution of cosmic-ray muon direction integrated over all zenith angle (upper figure) and zenith angle distribution of cosmic-ray muon direction integrated over all azimuth angle (lower figure) observed in Super-Kamiokande. Cross point represents real data and histogram represents MC simulated data which are taken account of slant depth of rock around detector.
Fig. 4.7 shows $\phi$ projection of Fig. 4.6. Vertical axis is logarithm scale. The border line of the divided regions is drawn in it.

For each $\phi$ region, the $\cos\theta$ distribution around the horizontal direction is plotted in Fig. 4.8. It is suspected that almost all of events in region (1) are neutrino induced muons because the rate is the same order both of $\cos\theta \leq 0$ and $\cos\theta > 0$ muons. On the other hand, $\cos\theta > 0$ muons in region (2) show an exponentially increasing rate proportional to $\cos\theta$, so they are suspected to be cosmic-ray muons. The slope of the zenith angle distribution of cosmic-ray muons in region (2) is fitted by an exponential function, $e^{a+b(\cos\theta)}$, and we get $a = 1.06 \pm 0.12$, $b = 81.7 \pm 2.0$ with fitting condition $\chi^2/n = 0.55$. The contaminating cosmic-ray muon events are estimated by integrating the exponential slope in the upward going range,

$$
N_{\text{cont}} = \int_{-1}^{0} e^{a+b(\cos\theta)} d(\cos\theta) .
$$

We got the number of contaminating cosmic-ray muon events,

$$
N_{\text{cont}} \simeq 3.5 \pm 0.3 .
$$

Figure 4.6: $\phi$-$\cos\theta$ distribution of upward through going muon events and cosmic-ray muon events ($\cos\theta < 0.08$),
Figure 4.7: $\phi$ projection of Fig. 4.6. It is divided into two regions, region (1): $\phi \leq 60^\circ$, $\phi > 240^\circ$, region (2): $60^\circ < \phi \leq 240^\circ$.

Figure 4.8: $\cos \theta$ distribution around horizontal direction in (1) – plotted by filled triangle, and region (2) – plotted by circle.
4.5 Systematic Errors

One of sources of the experimental systematic error originates from the uncertainty of the parent neutrino direction and deviation of the muon track direction. Fig. 4.9 shows a schematic view of the source of angular uncertainties. $\Delta \theta_{\nu\mu}$ stands for the scattering angle between muon neutrino and muon produced by charged current neutrino-nucleon interaction. $\Delta \theta_{\text{mul}}$ corresponds to the deflected angle caused by coulomb multiple scattering of a muon traveling through the rock.

![Diagram of angular uncertainties](image)

Figure 4.9: Definition of angular uncertainties, which affect the observed upward through-going muon flux as systematic errors.

$\Delta \theta_{\nu\mu}$ depends on the parent neutrino energy and $\Delta \theta_{\text{mul}}$ depends on the induced muon energy. Unfortunately, the muon energy at the production point and the parent neutrino energy are unknown because we can only measure the muon energy deposited in the inner detector. Therefore, the MC technique was introduced in order to estimate these uncertainties.

$\Delta \theta_{\nu\mu}$ is approximately given by Berezenskii et al. [35]:

\[
\Delta \theta_{\nu\mu} \sim 2.6^\circ \sqrt{\frac{100\text{GeV}}{E_\nu}},
\]

(4.9)

where $E_\nu$ is the parent neutrino energy.

$\Delta \theta_{\nu\mu}$ is also estimated by PYTHIA 5.71 [34] MC event generator. Fig. 4.10 shows $\Delta \theta_{\nu\mu}$ as a function of neutrino energy. Eq.(4.9) agrees well with PYTHIA’s estimation.

Then, $\Delta \theta_{\text{mul}}$ in the matter for muons is estimated approximately as follows:

\[
\Delta \theta_{\text{mul}} = \frac{13.6\text{MeV}}{\beta c p_\mu(\text{MeV/c})} \frac{x(\text{cm})}{X_0(\text{cm})} \left[1 + 0.038 \ln \frac{x(\text{cm})}{X_0(\text{cm})}\right],
\]

(4.10)
Figure 4.10: Deviation of scattering angle between muon neutrinos and generated muons, $\Delta \theta_{\mu}$, as a function of neutrino energy.

where $p_{\mu}$ and $X_0$ correspond to muon momentum and radiation length in the standard rock ($X_0 = 10.5$ cm), respectively. Fig.4.11 shows the angular deviation of the generated muon direction due to Coulomb multiple scattering as Eq.(4.10).

Assuming these approximation, $\Delta \theta_{\nu}$ and $\Delta \theta_{\mu}$ are simultaneously estimated by comparing the direction of muons entering the detector with the input neutrino direction.

The deviation, $\sqrt{\Delta \theta_{\nu}^2 + \Delta \theta_{\mu}^2}$, is roughly estimated to be,

$$\sqrt{\Delta \theta_{\nu}^2 + \Delta \theta_{\mu}^2} \sim 4.1^\circ.$$  (4.11)

These uncertainties smeared the angular distribution of the observed upward through going muons. These angular uncertainties affect the total upward through going muon flux by 1.6%.

The cosmic-ray muons which are contaminating from almost horizontal downward direction are also included as systematic errors. In this analysis of Kamiokande-III the zenith angle cut is performed as $\cos \theta < 0$, but another analysis([32]) used $\cos \theta < 0.05$. From that analysis the systematic error from the cosmic-ray background is obtained approximately $-10\%$ at most horizontal bin and 0% at the other bins. In the case of Super-Kamiokande, a zenith angle cut is performed as $\cos \theta < 0.1$, then we estimated 3.54 events as cosmic-ray background (see Section 4.4). The systematic error is $-4.2\%$ at the most horizontal bin and 0% at the other bins.

There are two more systematic errors which are concerned with the most horizontal bin especially in Kamiokande-III. One is the uncertainty of selection efficiency which is caused by the difference of the estimation method between MC data and real data (non-biased single track cosmic-ray muon events). At the most horizontal bin the uncertainty of the selection efficiency is larger than the other bins because in the real data the number of events of the most horizontal bin is much less than the other bins. The systematic errors from the uncertainty of the selection efficiency are $^{+2.8}_{-0.3}\%$ in the most horizontal bin and $^{+0.6}_{-1.2}\%$ in the other bins. Another systematic error which is concerned with
Figure 4.11: Angular deviation by Coulomb multiple scattering as a function of muon energy.

The most horizontal bin is the uncertainty of the number of events by the $\cos \theta = -0.04$ cut. This is estimated by varying the cut value $\cos \theta = -0.02, -0.03, -0.04, -0.05, -0.06$. The systematic error of the uncertainty of $\cos \theta = -0.04$ cut is $\pm 18.2\%$.

Conceivable systematic sources in the determination of the upward through going muon flux is summarized in Table 4.3.

<table>
<thead>
<tr>
<th>Error source</th>
<th>Error Kam-3 (%)</th>
<th>Error S-Kam (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>live time $^{a}$</td>
<td>±0.4</td>
<td>±0.4</td>
</tr>
<tr>
<td>effective area $^{a}$</td>
<td>±0.3</td>
<td>±0.1</td>
</tr>
<tr>
<td>7 m track length cut $^{a}$</td>
<td>±0.5</td>
<td>±0.1</td>
</tr>
<tr>
<td>$\sqrt{\Delta \theta^2_{\nu} + \Delta \theta^2_{\text{mul}}}^{a}$</td>
<td>±1.6</td>
<td>±1.6</td>
</tr>
<tr>
<td>cosmic-ray $\mu$ contamination $^{c}$</td>
<td>-10</td>
<td>-4.2</td>
</tr>
<tr>
<td>selection efficiency $^{b}$</td>
<td>+0.6, -1.2</td>
<td>$\ll 1$</td>
</tr>
<tr>
<td>selection efficiency $^{c}$</td>
<td>+2.8, -8.3</td>
<td>$\ll 1$</td>
</tr>
<tr>
<td>$\cos \theta = -0.04$ cut dependence $^{c}$</td>
<td>+18.2, -4.4</td>
<td>none</td>
</tr>
<tr>
<td>total (i=1~9 bin)</td>
<td>+1.8, -2.1</td>
<td>+1.7, -1.7</td>
</tr>
<tr>
<td>total (i=10 bin)</td>
<td>+18.5, -13.8</td>
<td>+1.7, -4.5</td>
</tr>
</tbody>
</table>

Table 4.3: Conceivable systematic errors. $^{a}$ i=1~10 bin, $^{b}$ i=1~9 bin, $^{c}$ i=10 bin.
4.6 Observed Flux

4.6.1 Upward Through Going Muon Flux of Kamiokande-II+III

In the case of Kamiokande-II,III, an additional condition of zenith angle cut, $\cos \theta < -0.04$, is adapted because of two reasons. One is that the selection efficiency of upward through going muons in the almost horizontal direction is not so good (see Section 4.3). Another is the estimation of contamination of cosmic-ray muons is not so clear. After adding the previous conditions, we get 185 events of upward through going muons in Kamiokande-II in 1124 live days, and 188 events in Kamiokande-III in 1332 live days.

The compilation of Kamiokande-II and Kamiokande-III results in 373 events in 2456 live days. The total flux of upward through going muons is calculated to be,

$$\Phi_{total} = 1.99 \pm 0.10 (\text{stat})^{+0.07}_{-0.06} (\text{syst}) \times 10^{-13} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}.$$ 

The angular distribution of flux is shown in Fig.4.12.

![Zenith Angle Dist. of Upward-Through-Going Muon Flux](image)

Figure 4.12: Zenith angle distribution of compilation of observed upward through going muon flux in Kamiokande-II and Kamiokande-III.
4.6.2 Upward Through Going Muon Flux of Super-Kamiokande

In total 363 live days in Super-Kamiokande, 413 events of upward through going muon events are observed.

Observed total flux is calculated to be,

$$\Phi_{\text{total}} = 1.72 \pm 0.08(\text{stat}) \pm 0.01(\text{syst}) \times 10^{-13} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}.$$

Angular distribution of flux is shown in Fig.4.13.

![Zenith Angle Dist. of Upward-Through-Going Muon Flux](image)

Figure 4.13: Zenith angle distribution of observed upward through going muon flux in Super-Kamiokande.
Chapter 5

Expected Flux of Upward Through Going Muons

In order to search for the neutrino oscillations using the upward through going muon, it is necessary to compare observed upward through going muon flux with expected one which are calculated by numerically. So it is important to estimate expected upward through going muon flux as carefully as possible. The method of numerical calculation has been established in the various experiments which treated neutrino-induced muon. In this chapter, the method to calculate expected upward through going muon fluxes and to apply it for the Kamiokande and Super-Kamiokande detector are described.

5.1 Calculation of Upward Through Going Muon Flux

Neutrino-induced upward through going muons are produced in the following way. Atmospheric neutrinos are produced by interacting of the primary components of cosmic-ray flux with the earth’s atmosphere, giving rise to pions and kaons which subsequently decay to neutrinos. Produced neutrinos go through the earth and make reaction with nuclei in the rocks near the Kamiokande/Super-Kamiokande detector by charged current weak interaction $\nu_\mu(\bar{\nu}_\mu) + N \rightarrow \mu^{-}(\mu^{+}) + \text{anything}$. Produced muons travel in the rocks losing their energy from as far away as several kilometers and punch through the detector. Therefore, an analytical approach to calculate upward through going muon flux originating from atmospheric muon neutrinos requires basic knowledge as follows,

1. Atmospheric muon neutrino flux

2. Charged current cross section between neutrino and nucleon

3. Muon range in the rock

The calculation procedure was initially developed by M.Honda and M.Mori when they calculated the upward going muon flux from super nova SN1987A[36]. Some parts were modified in order to adapt it to the atmospheric neutrino case.

The flux of induced muons which are generated by parent neutrinos of energy $E_\nu$ is given by

$$\frac{d^2 \sigma}{d\omega d\Omega} \cdot \frac{d^2 \phi_\nu(E_\nu, \cos \theta)}{dE_\nu d\Omega}$$

(5.1)
where \( d^3\sigma/dxdy \) is the differential charged current cross section as a function of the Bjorken scaling parameters, \( x \) and \( y \). \( d^2\phi_\nu(E_\nu, \cos \theta)/dE_\nu d\Omega \) is the differential spectrum of the parent neutrinos as a function of the neutrino energy, \( E_\nu \), and the zenith angle, \( \theta \).

To be detected as through going muons, the produced muon must survive after traveling a distance \( X \). \( g(X, E_\mu, E_{th}) \) is defined as the probability that a muon generated with an energy of \( E_\mu \) survives with energy larger than \( E_{th} \) after traversing the distance of \( X \). The upward going muon flux at the detector with energy larger than \( E_{th} \) is given by

\[
\frac{d\phi_\mu(E_{th}, \cos \theta)}{d\Omega} = \int_0^\infty dX \int_{E_{th}}^\infty dE_\mu \int_0^1 dy \int_0^1 dx \cdot \frac{d^2\sigma}{dx dy} \cdot \frac{d^2\phi_\nu(E_\nu, \cos \theta)}{dE_\nu d\Omega} \cdot g(X, E_\mu, E_{th})
\] (5.2)

The function \( g(X, E_\mu, E_{th}) \) can be written as

\[
g(X, E_\mu, E_{th}) = \psi(R(E_\mu, E_{th}) - X)
\] (5.3)

where \( R(E_\mu, E_{th}) \) is the range of the muons in the rock, in other words, a distance that the muon travels while its energy decreases from \( E_\mu \) to \( E_{th} \), and \( \psi(x) \) is the Heavside step function:

\[
\psi(x) = \begin{cases} 
1 & \text{if } x \geq 0 \\
0 & \text{if } x < 0
\end{cases}
\] (5.4)

The integral over \( X \) can be simply replaced by \( R(E_\mu, E_{th}) \), and \( d\phi_\mu(E_{th}, \cos \theta)/d\Omega \) is reduced to be

\[
\frac{d\phi_\mu(E_{th}, \cos \theta)}{d\Omega} = \int_{E_{th}}^\infty dE_\mu \int_0^1 dy \int_0^1 dx \cdot \frac{d^2\sigma}{dx dy} \cdot \frac{d^2\phi_\nu(E_\nu, \cos \theta)}{dE_\nu d\Omega} \cdot R(E_\mu, E_{th})
\] (5.5)

Since the neutrino flux and the muon range do not depend on the scaling parameter \( x \), and the integral over \( y \) is independent of the neutrino energy spectrum. Equation (5.5) can be transformed as follows,

\[
\frac{d\phi_\mu(E_{th}, \cos \theta)}{d\Omega} = \int_{E_{th}}^\infty \left[ \int_0^1 \left[ \int_0^1 \frac{d^2\sigma}{dx dy} \right] dx \right] \cdot R(E_\mu, E_{th}) dy \cdot \frac{d^2\phi_\nu(E_\nu, \cos \theta)}{dE_\nu d\Omega} dE_\nu
\] (5.6)

If a function \( P(E_\nu, E_{th}) \) is defined as,

\[
P(E_\nu, E_{th}) = \int_0^1 \left[ \int_0^1 \frac{d^2\sigma}{dx dy} \right] dx \cdot R(E_\mu, E_{th}) dy
\]

\( d\phi_\mu(E_{th}, \cos \theta)/d\Omega \) can be written as,

\[
\frac{d\phi_\mu(E_{th}, \cos \theta)}{d\Omega} = \int_{E_{th}}^\infty P(E_\nu, E_{th}) \cdot \frac{d^2\phi_\nu(E_\nu, \cos \theta)}{dE_\nu d\Omega} dE_\nu
\] (5.7)

The function \( P(E_\nu, E_{th}) \) is interpreted as the total cross section convoluting the daughter muon ranges for neutrinos of energy \( E_\nu \), and is also interpreted as the probability that the neutrinos of energy \( E_\nu \) are observed as muons at the detector with energy larger than \( E_{th} \). One name this function the “observation probability”.

It should be noted that this equation is very useful in the actual calculation of the neutrino-induced muon flux for the following reason. One wants to calculate the upward through going muon flux for many assumptions of the neutrino energy spectrum, especially, in the neutrino oscillation
analysis, the muon neutrino flux is affected by the neutrino oscillation parameters; the mass difference squared \( \Delta m^2 \) and mixing angle \( \theta _{\nu} \) between the two neutrinos. Therefore the calculation of the upward through going muon flux should be repeated for a given \( \Delta m^2 \) and \( \theta _{\nu} \). If we calculated all these processes by the full Monte Carlo simulation, the calculation would take a huge computer power beyond reach. The neutrino cross sections and the muon range are already included in the function \( P(E_{\nu}, E_{th}) \), and once this function is calculated, the muon flux with an energy threshold larger than \( E_{th} \) can be obtained by the single integral. So this analytic approach is very efficient from another viewpoint of avoiding meaningless repetition of the calculation.

In the discussion above, \( \nu _{\mu} \) and \( \bar{\nu} _{\mu} \) are not distinguished in Eq.(5.8). Indeed, we have to take this distinction into account, because \( \nu _{\mu} \) has different reaction cross section from \( \bar{\nu} _{\mu} \). Hence, Eq.(5.8)

\[
\frac{d\phi _{\mu}(E_{th}, \cos \theta)}{d\Omega} = \int_{E_{th}}^{\infty} \left[ P_{\nu _{\mu}}(E_{\nu}, E_{th}) \cdot \frac{d^2\phi _{\nu _{\mu}}(E_{\nu}, \cos \theta)}{dE_{\nu}d\Omega} + P_{\bar{\nu} _{\mu}}(E_{\nu}, E_{th}) \cdot \frac{d^2\phi _{\bar{\nu} _{\mu}}(E_{\nu}, \cos \theta)}{dE_{\nu}d\Omega} \right] dE_{\nu} \tag{5.9}
\]

The distinction between \( \mu^+ \) and \( \mu^- \) is not necessary, because water Cherenkov detector can not discriminate \( \mu^+ \) from \( \mu^- \).

### 5.2 Atmospheric Muon Neutrino Flux

Atmospheric neutrino fluxes have been calculated from the incident beam of primary cosmic-rays by Butkevich et al.[38], Bartol[1][41], and Honda et al.[42].

In this section, recent status of the atmospheric neutrino fluxes calculated by Butkevich, Bartol and Honda are presented. The comparison among their and the uncertainty of flux calculations are also discussed.

#### 5.2.1 Primary Cosmic-Ray Fluxes

Primary cosmic-ray fluxes are relatively well known in the energy region less than 100 GeV by which the low energy atmospheric \( \nu \)-fluxes ( \( < 3 \) GeV ) mainly produced. Webber and Lezniak have compiled the energy spectrum of the cosmic-rays for the hydrogen, helium, and CNO nuclei in the range 10 MeV ~ 1,000 GeV for three levels of solar activity. A similar compilation has been made by others for hydrogen and helium nuclei, which agrees well with what of Webber and Lezniak.

Fig.5.1 shows Observed fluxes of cosmic-ray protons, helium nuclei, and CNOs from the compilation of Webber and Lezniak.

The geomagnetic field determines the minimum energy with which a cosmic-ray can arrive at the earth. For the cosmic-ray nucleus, the minimum energy of cosmic-rays arriving at the earth is determined by the minimum rigidity (rigidity cutoff) rather than the minimum momentum. cosmic-rays with energy greater than 100 GeV, which are responsible for > 10 GeV atmospheric neutrino fluxes, are not affected by solar activity and by geomagnetic field. There are few measurements of the cosmic-ray chemical composition at these energies, especially above 1 TeV. Honda et al. compiled the available data and parametrized the observed fluxes for > 100 GeV with a single power function, and showed the result in Table 5.1.

---

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Figure 5.1: Observed fluxes of cosmic-ray protons, helium nuclei, and CNOs from the compilation of Webber and Leziak. Solid lines are parametrization of Honda, et al. for solar mid, dashed lines for solar min., and dotted lines for solar max. (figure is taken from [44])

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$A$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>$(6.65 \pm 0.13) \times 10^{-2}$</td>
<td>$-2.75 \pm 0.020$</td>
</tr>
<tr>
<td>He</td>
<td>$(3.28 \pm 0.05) \times 10^{-3}$</td>
<td>$-2.64 \pm 0.014$</td>
</tr>
<tr>
<td>CNO</td>
<td>$(1.40 \pm 0.07) \times 10^{-4}$</td>
<td>$-2.50 \pm 0.06$</td>
</tr>
<tr>
<td>Ne-S</td>
<td>$(3.91 \pm 0.03) \times 10^{-5}$</td>
<td>$-2.49 \pm 0.04$</td>
</tr>
<tr>
<td>Fe</td>
<td>$(1.27 \pm 0.11) \times 10^{-5}$</td>
<td>$-2.56 \pm 0.04$</td>
</tr>
</tbody>
</table>

Table 5.1: Compiled cosmic-ray spectrum in the form: $A(E/100 \text{GeV})^\gamma$. (table is taken from [44])
5.2.2 Production and Decay of Hadrons

As cosmic-rays propagate in the atmosphere, they produce $\pi$'s and K's in interactions with air nuclei. These mesons create atmospheric $\nu$'s when they decay as follows:

\[
A_{cr} + A_{ar} \rightarrow \pi^\pm, K^\pm, K^0, \ldots
\]

\[
\pi^+ \rightarrow \mu^+ + \nu_\mu,
\]

\[
\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu,
\]

\[
\pi^- \rightarrow \mu^- + \bar{\nu}_\mu,
\]

\[
\mu^- \rightarrow e^- + \nu_e + \nu_\mu.
\]

The calculations of the cosmic-ray protons with air nuclei consists of a number of Monte Carlo codes corresponding to different primary energies. Honda et al. employed the NUCRIN Monte Carlo code for the hadronic interaction of cosmic-rays for $E_{lab} < 5$ GeV, and LUND code - FRITIOF version 6.3 - for $5 \text{ GeV} < E_{lab} < 500$ GeV. Above 500 GeV, the original code developed by Kasahara et al. (COSMOS) was used. The K/$\pi$ ratio is taken 7% at 10 GeV, 11% at 100 GeV, and 14% at 1,000 GeV in laboratory energy.

Except for rare decays, all the decay modes of $\pi$ and K mesons are considered. Charmed meson production are ignored, since the contribution of charmed particle to atmospheric neutrinos becomes sizable only for $E_\mu > 100$ TeV.

In the two body decay of charged $\pi$'s and K's, the resulting $\mu^\pm$ is fully polarized against toward the direction of $\mu$ motion in the charged $\pi$ or K rest frame.

5.2.3 Comparison of Several Calculations

We use the atmospheric neutrino fluxes calculated by Butkevich [38], Bartol [41] and Honda [45] in this analysis.

Fig.5.2 and Fig.5.3 shows energy dependence of atmospheric $\nu_\mu + \bar{\nu}_\mu$ fluxes and $\nu_\mu / \bar{\nu}_\mu$ flux ratios respectively from horizontal direction (cos $\theta = 0.1$) and vertical direction (cos $\theta = 1.0$). Fig.5.4 shows zenith angle dependence of atmospheric $\nu_\mu + \bar{\nu}_\mu$ fluxes in neutrino energy is 1 GeV and 100 GeV.

5.2.4 Uncertainties of Atmospheric Neutrino Fluxes

The systematic error in the calculation of atmospheric $\nu$ fluxes comes mainly from the incompleteness of the knowledge of the primary cosmic-ray fluxes. Even at low energies, where the primary cosmic-ray fluxes are rather well studied, it is difficult to determine the absolute value due to the uncertainties in the instrumental efficiency ( ~ 12%) and exposure factor (2 - 3%). In compilation, the error in the fit is ~ 10% for the nucleon flux at 100 GeV and ~ 20% at 100 TeV. Assuming ~ 10% uncertainty below 100 GeV, the systematic error in the atmospheric $\nu$-fluxes is estimated to be ~ 10% at < 3 GeV, increasing to ~ 20% at 100 GeV, and remains almost constant up to 1,000 GeV.

The interaction model is another source of systematic errors. In comparison, the agreement of the LUND model and the COSMOS code with the experimental data is < 10%. The authors of the NUCRIN code claim that the agreement is within 10 - 20%. The hadronic interaction below 5 GeV contributes at most 5% to the production of atmospheric $\nu$-fluxes at 1 GeV. The systematic error caused by the hadronic interaction model is estimated to be ~ 10% above 1 GeV. One-dimensional
Figure 5.2: Energy dependence of atmospheric $\nu_\mu + \bar{\nu}_\mu$ fluxes. upper figure is from horizontal direction ($\cos \theta = 0.1$) and lower one is from vertical direction ($\cos \theta = 1.0$).

approximation is justified only at high energies. It is expected to be accurate above 3 GeV. Since the calculation of rigidity cutoff is very simplified in this scheme, this may result in a systematic error in the absolute value of the atmospheric neutrino fluxes of 10 - 20 % at 100 MeV and 5 % at 1 GeV. The statistics of the Monte Carlo calculation also causes an error in the atmospheric neutrino fluxes. The uncertainty due to the statistics is estimated to be < 5 % up to 100 - 300 GeV for $\nu_\mu$ and $\bar{\nu}_\mu$, and up to 30 - 100 GeV for $\nu_e$ and $\bar{\nu}_e$, depending on the zenith angle. The errors increase to ~ 10 % at the highest energy for each kind of $\nu$'s. Combining all the systematic and nonsystematic errors, the total error is estimated as 15 % from 1 GeV to 100 GeV, and 20 - 25 % at the highest energy.
Figure 5.3: Energy dependence of atmospheric $\nu_\mu/\bar{\nu}_\mu$ flux ratios. Upper figure is from horizontal direction ($\cos \theta = 0.1$) and lower one is from vertical direction ($\cos \theta = 1.0$).
Figure 5.4: Zenith angle dependence of atmospheric $\nu_\mu + \bar{\nu}_\mu$ fluxes. Upper figure shows neutrino energy is 1 GeV and lower one is 100 GeV.
5.3 Neutrino Nucleon Cross Section

5.3.1 Cross Section Formula for CC Interaction

The relevant physical process of charged current neutrino interaction with rock is as follows:

\[ \nu_\mu (\bar{\nu}_\mu) + N \rightarrow \mu^- (\mu^+) + \text{anything} \]  \hspace{1cm} (5.10)

where N represents an nucleon in rock. The differential charged current cross section for this process is expressed in terms of the Bjorken scaling parameter, \( x = Q^2/2M(E_\nu - E_\mu) \) and \( y = 1 - (E_\mu/E_\nu) \).

\[ \frac{d^2 \sigma}{dx dy} = \frac{G_F^2 M E_\nu}{\pi} \frac{M_W^4}{(M_W^2 + Q^2)^2} \left[ y^2 x F_1 + (1 - y - \frac{M x y}{2 E_\nu}) F_2 \pm (y - \frac{y^2}{2}) x F_3 \right] \]  \hspace{1cm} (5.11)

where the last term is positive for \( \nu \) and negative for \( \bar{\nu} \). \(-Q^2\) is the invariant momentum transfer between the incident neutrino and outgoing muon. \( M \) and \( M_W \) are the nucleon and intermediate \( W^\pm \) boson masses. \( G_F \) is the Fermi constant. And \( F_1(x, Q^2), F_2(x, Q^2) \) and \( F_3(x, Q^2) \) are the structure functions given by quark distribution functions.

For \( \nu_\mu + n(p) \rightarrow \mu^- + X \),

\[ F_2 = 2 x F_1 = 2 x \left[ f_u(f_u) + f_s + f_b + f_{\bar{u}}(f_{\bar{u}}) + f_{\bar{s}} + f_{\bar{b}} \right] \]  \hspace{1cm} (5.12)

\[ F_3 = 2 \left[ f_u(f_u) + f_s + f_b - f_{\bar{u}}(f_{\bar{u}}) - f_{\bar{s}} - f_{\bar{b}} \right] \]  \hspace{1cm} (5.13)

And for \( \bar{\nu}_\mu + p(n) \rightarrow \mu^+ + X \),

\[ F_2 = 2 x F_1 = 2 x \left[ f_u(f_u) + f_c + f_{\bar{t}} + f_{\bar{d}}(f_{\bar{d}}) + f_{\bar{s}} + f_{\bar{b}} \right] \]  \hspace{1cm} (5.14)

\[ F_3 = 2 \left[ f_u(f_u) + f_c - f_{\bar{t}} - f_{\bar{d}}(f_{\bar{d}}) - f_{\bar{s}} - f_{\bar{b}} \right] \]  \hspace{1cm} (5.15)

5.3.2 QCD Parton Distribution Functions

QCD parton distribution function models has been calculated by many physicists and groups. In previous analysis for upward through going muon, Y.Oyama\[50\] used quark distribution function given by Field and Feynman\[51\] and Eichten et al(EHLQ)[52] to integrate the differential charged current cross section. These are out of date and we did not use them. New parametrization of quark distribution ready to use for high momentum transfer. They are calculated by CTEQ \(^2\)[46], MRS \(^3\)[47], GRV \(^4\)[48].

Fig.5.5 shows recent experimental results of quark distribution function measured by \( e^+\mu \) collider HERA, comparing with MRS(A),MRS(A'),GRV94 prediction. Fig.5.6 shows data by fixed target muon beam experiments of CERN SPS comparing with CTEQ3 prediction.

\(^2\)Coordinated Theoretical/Experimental Project on QCD Phenomenology and Tests of the Standard Model
\(^3\)Martin, Roberts, Stirling
\(^4\)Glück, Reya, Vogt
Figure 5.5: ZEUS measurement of $F_2(x, Q^2)$ in the small $x$ regime compared with the predictions of the MRS(A) partons, the MRS(A') and MRS(G) partons and GRV(94) partons (figure is taken from [19]).
Figure 5.6: Comparison of the CTEQ3 fit with $F_2^{pN}$ data of BCDMS and NMC experiments. The absolute vertical scale is not labeled since an offset factor has been applied to the various $x$ bins to avoid overlap. (figure is taken from [46]).
5.3.3 Comparison with Experimental Data

Using formula (5.11) and quark distribution functions GRV94(DIS), CTEQ3(M), MRS(A'), we calculate $\nu_\mu N$ and $\bar{\nu}_\mu N$ total cross section for various neutrino energy. In this calculation, it should be taken care of the valid range of momentum transfer $Q^2$ for each quark distribution function. There is relation between $x, y$ and $Q^2$ as follows,

$$xy = \frac{Q^2}{2ME_\nu} \quad (5.16)$$

where $M$ is mass of relevant nucleon, and $E_\nu$ is energy of incident neutrino. Actually two of $x, y, Q^2$ are free parameter. Therefore, when integrating over $x$ and $y$, valid range of $Q^2$ make a effective contribution to this calculation. Especially for low energy neutrino ($E_\nu < \text{few GeV}$), minimum momentum transfer $Q^2_0$ is sensitive to integration. We treat $Q^2$ less than $Q^2_0$ as constant value $Q^2_0$.

CTEQ3(M) and MRS(A') sets $Q^2_0 = 4 \text{ GeV}^2$. On the other hand, GRV94(DIS) sets $Q^2_0 = 0.4 \text{ GeV}^2$. Each group which make quark distribution function determined the value of $Q^2_0$ from minimum $Q^2$ of experimental data they fitted.

Fig.5.7 shows calculated total cross section per neutrino energy of $\nu_\mu N$ and $\bar{\nu}_\mu N$ reactions together with experimental data as a function of neutrino energy.

5.4 Muon Energy Loss in Rock

High energy muons passing through matter lose energy due to electro-magnetic processes — mainly ionization (order $\alpha^2$), bremsstrahlung (order $\alpha^3$), and direct pair production (order $\alpha^4$) — and photonic-nuclear interactions. Fig.5.8 shows $dE/dx$ of muons in standard rock\(^5\). The dash line is $dE/dx$ contribution of ionization considering density effect calculated by Sternheimer et al. [53]. The solid line is $dE/dx$ including all contribution — ionization, bremsstrahlung, direct pair production, photo-nuclear interactions — calculated by Lohmann et al.[54].

Classically muon energy loss is written as $dE/dx = -\alpha - \beta E$, where $\alpha \approx 2 \text{ (MeV/g/cm}^2\text{)}$ and $\beta \approx 3.9 \times 10^{-6} \text{ (cm}^2\text{/g)}$. This formula is not accurate enough to obtain a reliable result in the energy range involved in this analysis. The detailed calculation on muon energy loss in rock is given by Bezrukov and Bugaev[55] in a polynomial-fit form. Also Lohmann et al. give numerical results. The range of a muon is calculated as

$$R(E_\mu, E_{th}) = \int_{E_\mu}^{E_{th}} \frac{-dE}{dE/dx} \quad (5.17)$$

and results using above energy loss formulas are compared in Fig.5.9. Bezrukov and Lohmann formulas are almost indistinguishable in the energy range of interest.

In this analysis, Lohmann’s $dE/dx$ is taken to calculate muon range.

\(^5\) $Z=11, A=22$. Ionization potential and density corrections as for calcium carbonate.
Figure 5.7: $\nu_\mu N$ and $\bar{\nu}_\mu N$ total cross section per neutrino energy using GRV94(DIS), CTEQ3(M), MRS(A') quark distribution functions. Minimum momentum transfer is set to $Q_0^2 = 4$ GeV$^2$ for CTEQ3M and MRSA', $Q_0^2 = 0.4$ GeV$^2$ for GRV94DIS. Various experiment data is also plotted on it. Range of neutrino energy is $0 < E_\nu < 40$ GeV in left figure, and $40 < E_\nu < 250$ GeV in right one.
Figure 5.8: $dE/dx$ in standard rock

Figure 5.9: Range of muon in the standard rock.
5.5 Application to Kamiokande/Super-Kamiokande Detector

In Equation (5.9), the analytical method to calculate the expected upward thorough going muon flux is expressed as a function of threshold muon energy deposited $E_{\text{th}}$ in the detector and zenith angle.

Prior to the calculation, the threshold muon energy $E_{\text{th}}$ corresponding to the threshold muon track length $7 \text{m}$ in this analysis should be estimated. The muon threshold energy in the Kamiokande-(II,III) detector was expressed as a function of muon track length $x$ by Oyama [50] as follows expression,

$$E_{\text{th}}(x) = 0.0025x^2 + 0.2x + 0.1975.$$  \hspace{1cm} (5.18)

From this formula, the energy threshold corresponding to $7 \text{m}$ track length is reduced to $\sim 1.7 \text{GeV}$.

On the other hand, using Lohmann $dE/dx$, the relation between muon threshold energy and muon track length is no more agree with Oyama formula. Fig.5.10 shows that. The Lohmann’s relation curve is fitted by 2-dimensional polynomial function, and we got following expression with fitting condition $\chi^2/n = 0.0007165/195$,

$$E_{\text{th}}(x) = 0.0002164x^2 + 0.2258x - 0.02437.$$  \hspace{1cm} (5.19)

From this formula, the energy threshold corresponding to $7 \text{m}$ track length is reduced to $\sim 1.6 \text{GeV}$.

While muon track length is less than $20 \text{m}$, Oyama formula is almost agree with Lohmann and Bezrukov. This means Oyama formula is useful for energy threshold determination for Kamiokande-(II,III) detectors because maximum muon track length in Kamiokande-(II,III) detector is $19.5 \text{m}$. Nevertheless in Super-Kamiokande, maximum muon track length is $49.5 \text{m}$, so one must use Eq.(5.19) for determination of muon energy threshold.

Furthermore it should be noted that the stopping muons were rejected even if their range in the detector was longer than $7 \text{m}$ in this analysis. Accordingly, the effective minimum track length in the detector may be longer than $7 \text{m}$ and the threshold energy $E_{\text{th}}$ is dependent on muon track length. This effect should be taken into account, when one estimates the expected upward through going muon flux.

When the effective expected flux is calculated, the grid point method which was used for deriving the effective detection area is also applicable in this case. The effective expected flux, $d\Phi_{\mu}/d\Omega$, can be calculated from $d\phi(E_{\text{th}}(x_i), \cos \theta)/d\Omega$ as follows,

$$\frac{d\Phi_{\mu}(\cos \theta)}{d\Omega} = \frac{\sum_i \left( \frac{d\phi(E_{\text{th}}(x_i), \cos \theta)}{dx_i} \vartheta(x_i - x_0) \right)}{\sum_i \vartheta(x_i - x_0)}$$  \hspace{1cm} (5.20)

where $x_0$ is the minimum track length $7 \text{m}$, $x_i$ is the track length from the $i$-th grid point and $\vartheta(x)$ is step function given by Eq.(5.4).

Corresponding effective energy threshold is $\langle E_{\text{th}} \rangle \sim 2.8 \text{GeV}$ for Kamiokande-II,III and $\langle E_{\text{th}} \rangle \sim 6.0 \text{GeV}$ for Super-Kamiokande. Fig.5.11 shows zenith angle dependence of effective energy threshold.
**Figure 5.10:** The relation between muon threshold energy and muon track length. Dashed line is Oyama formula, solid line and dash line is calculated using Lohmann and Bezrukov $dE/dx$ respectively.

**Figure 5.11:** Zenith angle dependence of effective energy threshold. Dashed line is Kamiokande-II,III and solid line is Super-Kamiokande.
5.6 Calculated Upward Through Going Muon Fluxes

The expected total fluxes of upward through going muon is calculated using atmospheric muon neutrino flux – Butkevich, Bartol, Honda95, and parton distribution function – CTEQ3(M), MRS(A'), GRV94(DIS), and muon $dE/dx$ in the rock – Lohmann.

Results of calculation are summarized in Table 5.2 for Kamiokande-(II,III), and Table 5.3 for Super-Kamiokande. Where unit is $10^{-13}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$.

Average value of all is 2.51 $^{+0.18}_{-0.21}$ for Kamiokande-(II,III), and 2.05 $^{+0.13}_{-0.17}$ for Super-Kamiokande. The upper errors are estimated by the difference of maximum expected flux and averaged one and the lower errors are estimated by the difference of minimum expected flux and averaged one.

The flux distribution for zenith angle is shown in Fig.5.12 for Kamiokande-(II,III), and in Fig.5.13 for Super-Kamiokande.

<table>
<thead>
<tr>
<th></th>
<th>CTEQ3(M)</th>
<th>MRS(A')</th>
<th>GRV94(DIS)</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butkevich</td>
<td>2.62</td>
<td>2.69</td>
<td>2.53</td>
<td>2.61</td>
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<tr>
<td>Bartol</td>
<td>2.55</td>
<td>2.62</td>
<td>2.46</td>
<td>2.54</td>
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<tr>
<td>Honda95</td>
<td>2.38</td>
<td>2.44</td>
<td>2.30</td>
<td>2.37</td>
</tr>
<tr>
<td>average</td>
<td>2.52</td>
<td>2.58</td>
<td>2.43</td>
<td>2.51</td>
</tr>
</tbody>
</table>

Table 5.2: Expected total fluxes of upward through going muon in Kamiokande-III calculated by using various PDFs and $\nu_\mu$ fluxes.

<table>
<thead>
<tr>
<th></th>
<th>CTEQ3(M)</th>
<th>MRS(A')</th>
<th>GRV94(DIS)</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butkevich</td>
<td>2.13</td>
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</tr>
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<td>Bartol</td>
<td>2.10</td>
<td>2.16</td>
<td>2.02</td>
<td>2.09</td>
</tr>
<tr>
<td>Honda95</td>
<td>1.96</td>
<td>2.01</td>
<td>1.88</td>
<td>1.95</td>
</tr>
<tr>
<td>average</td>
<td>2.06</td>
<td>2.12</td>
<td>1.98</td>
<td>2.05</td>
</tr>
</tbody>
</table>

Table 5.3: Expected total fluxes of upward through going muon in Super-Kamiokande calculated by using various PDFs and $\nu_\mu$ fluxes.
CHAPTER 5. EXPECTED FLUX OF UPWARD THROUGH GOING MUONS

Figure 5.12: Angular distribution of expected upward through going muon fluxes in Kamiokande-III. The dashed line is maximum expectation and dotted line is minimum one.

Figure 5.13: Angular distribution of expected upward through going muon fluxes in Super-Kamiokande. The dashed line is maximum expectation and dotted line is minimum one.
Chapter 6

Results and Discussions

The observed upward through going muon flux by Kamiokande-II+III and Super-Kamiokade are presented in chapter 4 and various expected fluxes are presented in chapter 5. In this chapter, a possible neutrino oscillation hypothesis using upward through going muons will be tested. The difference between the observed flux and hypothetical ones in terms of neutrino oscillations will be quantitatively evaluated by the $\chi^2$ analysis.

6.1 Comparison between Expected and Observed Flux without Oscillations

From calculation of previous chapters, we got both observed and expected upward through going muon fluxes. Results of total flux are summarized in Table 6.1, where the units are $10^{-13}\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}$. The values of expected fluxes are the averaged ones which are calculated from various models of $\nu_{\mu}$ fluxes and PDFs (see Section 5.6). The errors of expected fluxes correspond to the differences of the models. Each model of expected flux has 20% uncertainty which mainly comes from the uncertainty of the primary muon neutrino fluxes.

Fig. 6.1 shows expected and observed total fluxes of upward through going muons in Kamiokande-II+III and Super-Kamiokade, where the inside errors of the expected fluxes are the differences of the models and the outside ones are (diff. of models) + (20% uncertainty of each model), where 20% uncertainty comes from the uncertainty of primary muon neutrino fluxes. Inside errors of the observed fluxes are statistical ones and the outside ones are (stat) + (syst) errors. For both Kamiokande-II+III and Super-Kamiokade, the average expected fluxes are larger than the observed fluxes, but taking into account the 20% uncertainty in the expected fluxes, the observed fluxes are consistent with expected ones.

Furthermore, we can compare the gradient of the zenith angle distribution of upward through going muon fluxes. Fig. 6.2 shows the zenith angle distribution of upward through going muon fluxes, where filled circles are observed data of Kamiokande-II,III (left figure) and blank circles are observed data of Super-Kamiokade (right figure). The solid, dashed histograms are maximum, minimum expectation, respectively.

In order to compare the gradient of the zenith angle distribution, the absolute values of expected fluxes are normalized by a scaling parameter, $\alpha$, and comparison is done by the $\chi^2$ fitting method where $\chi^2$ is defined by the following equation,
\[ \chi^2 = \sum_{i=1}^{10} \left\{ \frac{(d\Phi_E)^i_{obs} - \alpha (d\Phi_E)^i_{exp}}{\sigma_{obs}^i} \right\}^2 + \left\{ \frac{1 - \alpha}{\sigma_{\alpha}} \right\}^2, \tag{6.1} \]

where index \( i \) represents the \( i \)-th cosine zenith angle bin, \( (d\Phi_E)^i_{obs} \) is the observed flux, \( (d\Phi_E)^i_{exp} \) is the expected flux, \( \sigma_{obs}^i \) is \( \sqrt{(\text{stat})^2 + (\text{syst})^2} \) the error of the observed flux, \( \sigma_{\alpha} \) is the uncertainty of the absolute value of the flux \( \sigma_{\alpha} = 0.2 \). Fig.6.3 shows a comparison between the observed upward through going muon fluxes and the \( \chi^2 \) fitted expected fluxes in Kamiokande-II+III(left) and Super-Kamiokande(right). We can see that differences in the models are absorbed by a scaling parameter \( \alpha \). The results of the fitting conditions are \( \chi^2 \sim 27 \) for Kamiokande-II+III and \( \chi^2 \sim 13 \) for Super-Kamiokande.

<table>
<thead>
<tr>
<th></th>
<th>Kamiokande-II+III</th>
<th>Super-Kamiokande</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle E_{th} \rangle )</td>
<td>2.8 GeV</td>
<td>6.0 GeV</td>
</tr>
<tr>
<td>Expected (model)</td>
<td>( 2.51 \pm 0.18 )</td>
<td>( 2.05 \pm 0.17 )</td>
</tr>
<tr>
<td></td>
<td>(20%)</td>
<td>(20%)</td>
</tr>
<tr>
<td>Observed</td>
<td>( 1.99 \pm 0.10 )</td>
<td>( 1.72 \pm 0.08 )</td>
</tr>
<tr>
<td>(stat) (syst)</td>
<td>( 0.46 ) (20%)</td>
<td>( 0.08 ) (20%)</td>
</tr>
</tbody>
</table>

Table 6.1: Expected and observed total flux of upward through going muon.

Figure 6.1: Total flux of upward-through-going muons. Filled circles are the expected and the observed flux of Kamiokande-II+III (left). Blank circles are the expected and the observed flux of Super-Kamiokande(right). Inside errors of the expected flux are the differences of the models and the outside ones are (diff. of models) + (20% uncertainty of each model), where 20% uncertainty comes from the uncertainty of primary muon neutrino fluxes. The inside errors of the observed fluxes are statistical errors and the outside ones are statistical + systematic errors.
Figure 6.2: Zenith angle distributions of expected and observed upward through going muon fluxes in Kamiokande-II+III(left) and Super-Kamiokande(right). Solid histogram is maximum expectation and dashed one is minimum expectation.

Figure 6.3: Comparison between observed upward through going muon fluxes and $\chi^2$ fitted expected fluxes in Kamiokande-II+III(left) and Super-Kamiokande(right).
6.2 Method to Examine Neutrino Oscillations

For analysis assuming neutrino oscillations, a method to calculate the upward through going muon flux distorted by neutrino oscillations should be established. The $\nu_\mu \rightarrow \nu_\tau$ oscillation case will be discussed first, and the $\nu_e \rightarrow \nu_\mu$ case will be presented subsequently.

The probability that a neutrino born as $\nu_\mu$ remains $\nu_\mu$ at distance $L$ is given by

$$P_\nu(\nu_\mu \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu_\mu(t) | \nu_\mu \rangle|^2 = 1 - \sin^2(2\theta^\nu_\nu) \cdot \sin^2(\frac{\pi L}{L_v}) ,$$  \hspace{1cm} (6.2)

with

$$L_v = \frac{2.48 \text{ km} \times \frac{p_\nu \text{ GeV}}{\Delta m^2 \text{ eV}^2}}{\Delta m^2 \text{ eV}^2} ,$$  \hspace{1cm} (6.3)

where $\theta^\nu_\nu$ is mixing angle between $\nu_\mu$ and $\nu_\tau$ in vacuum oscillations, $p_\nu$ is the momentum of neutrino in unit of GeV and $\Delta m^2$ in unit of eV\(^2\). As we observe neutrinos produced on the opposite side of the earth, the neutrino propagation length, $L$ (km), can be written as

$$L = 2R \cos \theta$$  \hspace{1cm} (6.4)

where $R$ is radius of the earth (6371 km) and $\theta$ is zenith angle of the neutrino arrival direction. $P_\nu(\nu_\mu \rightarrow \nu_\mu)$ is calculated as a function of $\Delta m^2$, $\sin^2 2\theta^\nu_\nu$, $p_\nu$ and $\theta$.

If neutrinos actually oscillate, the muon neutrino flux from the opposite side of the earth will be given by $d\phi_\nu(E_{th}, \cos \theta)/dE_{\nu}d\Omega \cdot P_\nu(\nu_\mu \rightarrow \nu_\mu)$ instead of $d\phi_\nu(E_{th}, \cos \theta)/dE_{\nu}d\Omega$.

The upward through going muon flux without neutrino oscillations is given by Eq.(5.8). If neutrino oscillations take place, the flux will be modified as:

$$\frac{d\phi_\nu(E_{th}, \cos \theta)_{osc}}{d\Omega} = \int_{E_{th}}^{E_{th}} P(E_{\nu}, E_{th}) \cdot \frac{d^2 \phi_\nu(E_{\nu}, \cos \theta)}{dE_{\nu}d\Omega} \cdot P_\nu(\nu_\mu \rightarrow \nu_\mu) dE_{\nu}$$  \hspace{1cm} (6.5)

Finally, the upward through going muon flux adapted to the Kamiokande/Super-Kamiokande energy threshold, $d\phi_\mu(\cos \theta)_{osc}/d\Omega$, is obtained as:

$$\frac{d\Phi_\mu(\cos \theta)_{osc}}{d\Omega} = \sum_i \left( \frac{d\phi_\nu(E_{th}(x_i), \cos \theta)_{osc}}{d\Omega} \frac{\theta(x_i - x_0)}{\sum \theta(x_i - x_0)} \right)$$  \hspace{1cm} (6.6)

where $x_0$ is the minimum track length 7m, $x_i$ is the track length from the $i$-th grid point and $\theta(x)$ is step function given by Eq.(5.4).

In the case of $\nu_e \rightarrow \nu_\mu$ oscillations, the matter effect must be taken into account, as is discussed in Section 1.1.2. The upward through going muon flux is given by

$$\frac{d\phi_\mu(E_{th}, \cos \theta)}{d\Omega} = \int_{E_{th}}^{E_{th}} \left\{ P_{\nu_e}(E_{\nu}, E_{th}) \left( \frac{d^2 \phi_{\nu_e}(E_{\nu}, \cos \theta)}{dE_{\nu}d\Omega} \cdot P_m(\nu_\mu \rightarrow \nu_\mu) + \frac{d^2 \phi_{\nu_e}(E_{\nu}, \cos \theta)}{dE_{\nu}d\Omega} \cdot P_m(\nu_\mu \rightarrow \nu_\mu) \right) \right\} dE_{\nu}$$  \hspace{1cm} (6.7)

where $P_m(\nu_\mu \rightarrow \nu_\mu)$ is defined by

$$P_m(\nu_\mu \rightarrow \nu_\mu) = 1 - P_m(\nu_e \rightarrow \nu_\mu) = 1 - \sin^2(2\theta_m) \cdot \sin^2(\frac{\pi L}{L_m}) .$$  \hspace{1cm} (6.8)
In the case of $\nu_\mu \rightarrow \nu_\tau$ oscillations, $P_e(\nu_\mu \rightarrow \nu_\mu) = P_e(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$, while in the case of $\nu_e \rightarrow \nu_\mu$, $P_m(\nu_\mu \rightarrow \nu_\mu) \neq P_m(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ because matter effect act on neutrinos and anti-neutrinos in a different way. See Eq.(1.24), (1.25) and Eq.(1.38), (1.39).

To examine agreement between the expected upward through going muon flux with neutrino oscillations and observed one, $\chi^2$ is defined by the following equation,

$$\chi^2 = \sum_{i=1}^{10} \left\{ \left( \frac{d\Phi_e}{dT} \right)^i_{obs} - \alpha \left( \frac{d\Phi_e}{dT} \right)^i_{osc} \right\}^2 \frac{1 - \alpha}{\sigma^2_{\alpha}} ,$$

(6.9)

where index $i$ represents $i$-th cosine zenith angle bin, $\left( \frac{d\Phi_e}{dT} \right)^i_{obs}$ is observed flux, $\left( \frac{d\Phi_e}{dT} \right)^i_{osc}$ is the expected flux with neutrino oscillations, $\sigma^i_{obs}$ is $\sqrt{\text{stat}}^2 + \text{syst}$ error of observed flux, $\sigma_\alpha$ is the uncertainty of the absolute value of the flux ($\sigma_\alpha = 0.2$). The minimum $\chi^2$ on the $\Delta m^2$-$\sin^2 2\theta_e$ plane is searched for. Then, the 90\% and 95\% C.L.(confidence level) allowed parameter regions are defined as follows,

$$\Delta \chi^2 \equiv \chi^2(\Delta m^2, \sin^2 2\theta_e) - \chi^2_{\text{min}}$$

$$< \Delta \chi^2_{lh} ,$$

(6.10)

where $\Delta \chi^2_{lh}$ is set to 4.61(90\% C.L.) or 6.17(95\% C.L.) because $\Delta \chi^2$ defined by Eq.(6.10) depends on only two free parameters, $\Delta m^2$ and $\sin^2 2\theta_e$ (see [57]). Table 6.2 shows values of $\Delta \chi^2_{lh}$ for each C.L. and degree of freedom.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n = 1$</th>
<th>$n = 2$</th>
<th>$n = 3$</th>
<th>$n = 4$</th>
<th>$n = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>68% C.L.</td>
<td>1.00</td>
<td>2.30</td>
<td>3.53</td>
<td>4.72</td>
<td>5.89</td>
</tr>
<tr>
<td>90% C.L.</td>
<td>2.71</td>
<td>4.61</td>
<td>6.25</td>
<td>7.78</td>
<td>9.24</td>
</tr>
<tr>
<td>95% C.L.</td>
<td>4.00</td>
<td>6.17</td>
<td>8.02</td>
<td>9.70</td>
<td>11.3</td>
</tr>
</tbody>
</table>

Table 6.2: $\Delta \chi^2_{lh}$ for various degree of freedom($n$) and confidence level(C.L.).

Moreover, the combined $\chi^2$ of Kamiokande-II+III and Super-Kamiokande is calculated as follows,

$$\chi^2 = \sum_{i=1}^{10} \left\{ \left( \frac{d\Phi_e}{dT} \right)^i_{obs} - \alpha \left( \frac{d\Phi_e}{dT} \right)^i_{osc} \right\}^2 \frac{1 - \alpha}{\sigma^2_{\alpha}} \quad \text{Kam-II+III}$$

$$+ \left\{ \left( \frac{d\Phi_e}{dT} \right)^i_{obs} - \alpha \left( \frac{d\Phi_e}{dT} \right)^i_{osc} \right\}^2 \frac{1 - \alpha}{\sigma^2_{\alpha}} \quad \text{S-Kam} .$$

(6.11)
6.3 $\nu_\mu \leftrightarrow \nu_\tau$ Oscillations

In the case of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, it is sufficient to merely consider vacuum oscillations described in Section 1.1.1. We use the Bartol model for the primary $\nu$ flux and GRV94(DIS) for PDF and Lohmann's table for $dE/dx$ of muons in the rocks to calculate the expected flux. From the comparison between the observed flux and the expected flux with $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, we got the best fitted parameters calculated by Eq.(6.9), Eq.(6.11). The results are summarized in Table 6.3.

<table>
<thead>
<tr>
<th></th>
<th>Kam-II+III</th>
<th>S-Kam</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2_{\text{min}}$</td>
<td>15.9</td>
<td>9.59</td>
<td>26.6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.04</td>
<td>1.15</td>
<td>1.12</td>
</tr>
<tr>
<td>$\sin^2 2\theta^\nu$</td>
<td>1.00</td>
<td>0.78</td>
<td>1.00</td>
</tr>
<tr>
<td>$\Delta m^2$</td>
<td>$3.98 \times 10^{-3}$</td>
<td>$2.00 \times 10^{-2}$</td>
<td>$6.31 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 6.3: The fitted parameters of observed and expected upward through going muon fluxes with $\nu_\mu \leftrightarrow \nu_\tau$ oscillations in Kamiokande-II+III, Super-Kamiokande and both combined.

The zenith angle distributions of observed upward through going muon fluxes and expected ones with $\nu_\mu \leftrightarrow \nu_\tau$ oscillations are shown in Fig.6.4. The contour lines of the parameters of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations for the 90, 95 % C.L. are shown in Fig.6.5.

![Figure 6.4: Zenith angle distributions of upward through going muon fluxes. Filled circles, blank circles are observed fluxes in Kamiokande-II+III, Super-Kamiokande, respectively. The dashed line is the expected flux without oscillations, solid and dot-dashed ones are expected fluxes with $\nu_\mu \leftrightarrow \nu_\tau$ oscillations fitted to the Kam-II+III/S-Kam and the combined data, respectively.](image-url)
Figure 6.5: 90, 95% C.L. contour lines for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations from results of Kamiokande-II+III(dotted), Super-Kamiokande(dashed) and Combined (solid). The points marked by filled stars, blank stars and blank triangles represent $(\sin^2 2\theta^\prime, \Delta m'^2)$ of minimum $\chi^2$ for Kamiokande-II+III, Super-Kamiokande and Combined, respectively.
6.4 $\nu_e \leftrightarrow \nu_\mu$ Oscillations

In the case of $\nu_e \leftrightarrow \nu_\mu$ oscillations, it is taken into considerations of the MSW effect described in Section 1.1.2. We use the Bartol model for the primary $\nu$ flux and GRV94(DIS) for PDF and Lohmann's table for dE/dx of muons in the rock to calculate the expected flux. From the comparison between the observed flux and the expected flux with $\nu_e \leftrightarrow \nu_\mu$ oscillations, we got the best fitted parameters calculated by Eq.(6.9), Eq.(6.11). The results are summarized in Table 6.4.

<table>
<thead>
<tr>
<th></th>
<th>Kam-II+III</th>
<th>S-Kam</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2_{min}$</td>
<td>21.2</td>
<td>7.20</td>
<td>29.7</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.95</td>
<td>1.13</td>
<td>1.02</td>
</tr>
<tr>
<td>$\sin^2 2\theta_v$</td>
<td>0.62</td>
<td>0.72</td>
<td>0.63</td>
</tr>
<tr>
<td>$\Delta m^2$</td>
<td>$1.58 \times 10^{-2}$</td>
<td>$7.94 \times 10^{-2}$</td>
<td>$3.16 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 6.4: The fitted parameters of observed and expected upward through going muon fluxes with $\nu_e \leftrightarrow \nu_\mu$ oscillations in Kamiokande-II+III, Super-Kamiokande and both combined.

The zenith angle distributions of observed upward through going muon fluxes and expected ones with $\nu_e \leftrightarrow \nu_\mu$ oscillations are shown in Fig.6.6. The contour lines of the parameters of $\nu_e \leftrightarrow \nu_\mu$ oscillations for the 90, 95 % C.L. are shown in Fig.6.7.

![Upward-Through-Going Muon Flux (Kam-II+III)](image1)

![Upward-Through-Going Muon Flux (S-Kam)](image2)

Figure 6.6: Zenith angle distributions of upward through going muon fluxes. Filled circles, blank circles are observed fluxes in Kamiokande-II+III, Super-Kamiokande, respectively. The dashed line is the expected flux without oscillations, solid and dot-dashed ones are the expected fluxes with $\nu_e \leftrightarrow \nu_\mu$ oscillations fitted to the Kam-II+III,S-Kam and the combined data, respectively.
Figure 6.7: 90, 95% C.L. contour lines for $\nu_e \leftrightarrow \nu_\mu$ oscillations from results of Kamiokande-II+III(dotted) and Super-Kamiokande(dashed) and Combined (solid). The points marked by filled stars, blank stars and blank triangles represent $(\sin^2 2\theta_{\nu_e}, \Delta m^2)$ of minimum $\chi^2$ for Kamiokande-II+III, Super-Kamiokande and Combined, respectively.
### 6.5 Comparison with Other Experiments

Experimentally there are two type of detectors which observe upward through going muons. One is the water Cherenkov type detector: Kamiokande, Super-Kamiokande, IMB [12] and the other is the tracking type detector: Baksan [29], MACRO [30].

In general, the water Cherenkov type detector is able to observe not only vertically upward through going muons but also horizontally upward through going muons because the detector height is higher than the calorimeter type. On the other hand, the tracking type detector can detect more vertically upward through going muons than the water Cherenkov type because of the large bottom area of the detector.

The summary of upward going muon experiments is presented in Table 6.5, where live time, observed number of muon events, observed fluxes, expected fluxes, effective energy thresholds are presented.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Live Time (years)</th>
<th>Number of Events</th>
<th>Observed Flux ((\times))</th>
<th>Expected Flux ((\times))</th>
<th>Effective Energy Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baksan (^a)</td>
<td>11.94</td>
<td>558</td>
<td>3.25±0.19</td>
<td>2.85</td>
<td>1 GeV</td>
</tr>
<tr>
<td>MACRO (^b)</td>
<td>3.06</td>
<td>255</td>
<td>3.62±0.55</td>
<td>2.85</td>
<td>1 GeV</td>
</tr>
<tr>
<td>IMB-I,II (^c)</td>
<td>2.53</td>
<td>430</td>
<td>2.22±0.11</td>
<td>2.51</td>
<td>1.8 GeV</td>
</tr>
<tr>
<td>Kamiokande-II+III (^d)</td>
<td>6.73</td>
<td>373</td>
<td>1.99±0.12</td>
<td>2.46</td>
<td>2.8 GeV</td>
</tr>
<tr>
<td>Super-Kamiokande (^d)</td>
<td>0.99</td>
<td>413</td>
<td>1.72±0.08</td>
<td>2.02</td>
<td>6.0 GeV</td>
</tr>
</tbody>
</table>

Table 6.5: Summary of upward going muons. \(^a\): Ref.[29], \(^b\): Ref.[30], \(^c\): Ref.[12] and \(^d\): This work. The errors include systematic errors except IMB-I,II. Expected upward going muon flux is calculated from Bartol + GRV94(DIS) + Lohman in Kamiokande-II+III and Super-Kamiokande and Bartol + MRS(G) + Lohman in other experiments. (\(\times\)) unit is 10^{-13} cm^{-2} s^{-1} sr^{-1}.

The zenith angle distributions of upward through going muons in Baksan, MACRO, IMB-I,II, Kamiokande-II+III and Super-Kamiokande are shown in Fig.6.8, where the scale of Kamiokande-II+III is normalized to the scale of Super-Kamiokande.

The 90% C.L. contour lines of CDHSW, IMB(upward stop/through muon), Kam(contained) and Kam-II+III+S-Kam(upward through muon) for \(\nu_\mu \leftrightarrow \nu_\tau\) and Gösgen, Bugey, Chooz, Kam(contained) and Kam-II+III+S-Kam(upward through muon) for \(\nu_e \leftrightarrow \nu_\mu\) oscillations are shown in Fig.6.9. The 90% C.L. allowed regions of this work for both type set oscillations are consistent with those of Kamiokande(contained).

If we believe the result of Chooz [26], there are no allowed regions in \(\Delta m^2 > 10^{-3} \text{eV}^2\) for \(\nu_e \leftrightarrow \nu_\mu\) oscillations. It is natural to think that the \(\Delta m^2\) is allowed in lower regions which solar neutrino experiments indicate because the mass difference of the leptons, \(m_\mu - m_e\), is much lower than that of the leptons, \(m_\tau - m_\mu\).
Figure 6.8: Zenith angle distribution of upward through-going muons observed by (a) Baksan, (b) MACRO, (c) IMB-I+II, (d) Kamiokande-II+III and Super-Kamiokande, where the scale of Kamiokande-II+III is normalized to the scale of Super-Kamiokande. The histogram is expected upward through-going muon flux calculated from Bartol + GRV94(DIS) + Lohmann for Kamiokande-II+III and Super-Kamiokande and Bartol + MRS(G) + Lohmann for the other experiments.
Figure 6.9: 90% C.L. contour for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations (left) and $\nu_e \leftrightarrow \nu_\mu$ oscillations (right) for various neutrino experiments. Solid lines are the results of the combined $\chi^2$ of Kamiokande-II+III and Super-Kamiokande of this work and blank triangles are the minimum $\chi^2$ points of this work.
Chapter 7

Conclusion

Muon neutrino oscillations were searched for in Kamiokande-II,III and Super-Kamiokande detectors by using upward through going muons which are generated by atmospheric muon neutrinos.

The selected upward through going muons were restricted to those with a track length $> 7$ m, which corresponds to an effective muon energy threshold, $\langle E_{\mu} \rangle = 2.8$ GeV for Kamiokande-II,III and $\langle E_{\mu} \rangle = 6.0$ GeV for Super-Kamiokande. The mean energy of primary muon neutrinos is about $\sim 100$ GeV.

A total of 373 events were observed in Kamiokande-II+III during 2456 live days, and 413 events in Super-Kamiokande during 363 live days. The total flux of Kamiokande-II+III, $\phi_{\mu}^{k} \phi_{\mu}^{\nu}$, and Super-Kamiokande, $\phi_{\mu}^{sk}$, are

$$\phi_{\mu}^{k} = 1.99 \pm 0.10(\text{stat}) \pm 0.07(\text{syst}) \cdot$$

$$\phi_{\mu}^{sk} = 1.72 \pm 0.08(\text{stat}) \pm 0.04(\text{syst}) \cdot$$

On the other hand, the average of expected total fluxes are

$$\phi_{\mu}^{k} \phi_{\mu}^{\nu} = 2.51 \pm 0.21(\text{model}) \pm 0.54(20\%) \cdot$$

$$\phi_{\mu}^{sk} = 2.05 \pm 0.13(\text{model}) \pm 0.44(20\%) \cdot$$

(Units are $10^{-13}$ cm$^{-1}$ sec$^{-1}$ sr$^{-1}$)

Taking into account the 20% uncertainty of the expected total fluxes, the observed total fluxes are consistent with expected ones.

Furthermore, zenith angle distributions are compared between observed fluxes and expected ones with neutrino oscillations. $\nu_{\mu} \leftrightarrow \nu_{\tau}$ and $\nu_{e} \leftrightarrow \nu_{\mu}$ oscillations in the range of $10^{-3} \leq \Delta m^{2} \leq 1$ eV$^{2}$ are searched for. In the comparison, the Bartol neutrino flux, GRV94(DIS) parton distribution function and Lohmann $dE/dx$ in rock are used for the expected fluxes and the absolute values are normalized in the range of 20% uncertainty. In result, 90% C.L. allowed regions of $\sin^{2} 2\theta \Delta m^{2}$ plane in this work are consistent with that of Kamiokande Sub-GeV+Multi-GeV contained events for both $\nu_{\mu} \leftrightarrow \nu_{\tau}$ and $\nu_{e} \leftrightarrow \nu_{\mu}$ oscillations. In case of the $\nu_{e} \leftrightarrow \nu_{\mu}$ oscillations, the recent results of Chooz excluded $\Delta m^{2} > 10^{-2}$eV$^{2}$ regions. It is natural to think that $\Delta m^{2}$ is allowed in lower regions which solar neutrino experiments indicate because the mass difference of the leptons, $m_{\mu} - m_{e}$, is much lower than that of the leptons, $m_{\tau} - m_{\mu}$.
Appendix A

The Analysis by Another Energy Threshold

The energy threshold of the upward through going muons is one of the key parameters to determine their flux and it depends on the track length cut for muons. In this thesis, the track length cut is performed by the condition, track length $> 7m$ which corresponds to muon energy deposited in water, $E_{th} > 1.6$ GeV. Actually, we select only "through going muons", so stopping muons are not taken even if their track lengths are longer than 7m. Then the effective value of the energy threshold is higher than 1.6 GeV. It is dependent on the shape and size of detector. In case of Kamiokande-II+III, the effective energy threshold is $\langle E_{th} \rangle = 2.8$ GeV. In case of Super-Kamiokande, it is $\langle E_{th} \rangle = 6.0$ GeV (see Chapter 5.5).

The cut value of 7m is decided to get better angular resolution (see Fig.3.21, Fig.3.30), but in one sense, it is a conventional value. So it is important to adopt another track length cut and check the consistency of both of results.

Fig.A.1 shows track length distributions of upward through going muons in Kamiokande-II+III(left) and Super-Kamiokande(right). Plotted filled circles and blank circles are observed data of Kamiokande-II+III and Super-Kamiokande, respectively, histograms are expected distributions. The gradient of the distribution is almost flat near the cut length, 7m, so it is expected that the flux value will not change so much if the cut length is shifted slightly longer to 8m. The energy threshold of an 8m track length cut corresponds $E_{th} > 1.8$ GeV.

The effective areas becomes slightly smaller when the track length cut is changed from 7m to 8m. Fig.A.2 shows the effective areas of 7m track length cut(solid lines) and 8m track length cut(dashed lines). The effective area of the energy threshold becomes slightly larger when the track length cut is changed from 7m to 8m. Fig.A.3 shows The effective energy threshold of 7m track length cut(solid lines) and 8m track length cut(dashed lines).

Table A.1 shows the result of both of 7m and 8m track length cut. The model of expected fluxes are Bartol for $\nu_{\mu}$ flux, GRV94(DIS) for PDF and Lohman for $dE/dx$. The errors of expected fluxes are 20% uncertainty of $\nu_{\mu}$ flux. The errors of observed fluxes include both of statistical and systematic errors.
Figure A.1: Track length distributions of upward through going muons in Kamiokande-II+III (left, filled circles) and Super-Kamiokande (right, blank circles). Histograms are expected distributions, where absolute scales are normalized to data.

Figure A.2: The effective areas of the 7m track length cut (solid lines) and the 8m track length cut (dashed lines).
Figure A.3: The effective energy threshold of the 7m track length cut (solid lines) and the 8m track length cut (dashed lines).

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<th>track length cut</th>
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<th>8m</th>
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<tr>
<td>effective energy</td>
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<td>2.9 GeV (Kam23)</td>
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<td>threshold</td>
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<td>6.1 GeV (Skam)</td>
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<td>number of events</td>
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<td>346 (Kam23)</td>
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<td>413 (Skam)</td>
<td>404 (Skam)</td>
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<td>2.39±0.48 (Kam23)</td>
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<td>(10^{-13} cm^{-2} sec^{-1} sr^{-1})</td>
<td>2.02±0.40 (Skam)</td>
<td>1.98±0.40 (Skam)</td>
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<td>observed total flux</td>
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<td>(10^{-13} cm^{-2} sec^{-1} sr^{-1})</td>
<td>1.72±0.08 (Skam)</td>
<td>1.71±0.08 (Skam)</td>
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Table A.1: The result of both the 7m and 8m track length cuts. The model of expected fluxes are Bartol for $\nu_{\mu}$ flux, GRV94(DIS) for PDF and Lohmann for $dE/dx$. The errors of expected fluxes are 20% uncertainty of $\nu_{\mu}$ flux. The errors of observed fluxes include both of statistical and systematic errors.
APPENDIX A. THE ANALYSIS BY ANOTHER ENERGY THRESHOLD

Fig. A.4 shows zenith angle distributions of the upward through-going muon fluxes in Kamiokande-II+III(left) and Super-Kamiokande(right). Observed data for the 7m cut are plotted by circles and for the 8m cut are plotted by squares. Solid histograms are expected fluxes for the 7m cut and dashed ones are for the 8m cut.

Fig. A.5 shows 90% CL contour of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations(left) and $\nu_e \leftrightarrow \nu_\mu$ oscillations(right) from the results of Kamiokande-II+III(dotted) and Super-Kamiokande(dashed) and Combined (solid). Both the 7m and 8m track length cuts are shown.

It is found that results of 8m track length cut are consistent with results of 7m track length cut.

Figure A.4: Zenith angle distributions of the upward through-going muon flux in Kamiokande-II+III(left) and Super-Kamiokande(right). Observed data for the 7m cut are plotted by circles and for the 8m cut are plotted by squares. Solid histograms are the expected fluxes for the 7m cut and the dashed ones are for the 8m cut.
Figure A.5: 90% CL contour of \( \nu_\mu \leftrightarrow \nu_\tau \) oscillations (left) and \( \nu_e \leftrightarrow \nu_\mu \) oscillations (right) from results of Kamiokande-II+III (dotted) and Super-Kamiokande (dashed) and Combined (solid). Both the 7m and 8m track length cuts are shown.
Bibliography

BIBLIOGRAPHY


