Search for proton decay into three charged leptons in Super-Kamiokande

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Abstract

In nature, there are four interactions: strong, electromagnetic, weak and gravity and they are considered to be one unified interaction at the very beginning of the universe. One of the final goals in particle physics is explaining these four interactions by one unified theory. Now we are on the way of unifying the three interactions of strong, electromagnetic and weak. One of the candidates for this goal is grand unified theory (GUT). Basically the energy scale of GUT is considered to be $\sim 10^{15}$ GeV. In such significantly high energy scale, the direct search for GUT is impossible by any current high energy experiments. On the other hand, in GUT, the proton which is a stable particle in Standard Model (SM) is predicted to decay. A search for such proton decay events is a strong way to prove the GUT. Since the life time of proton is predicted to be much longer than the age of the universe, we need to prepare a tremendous number of proton source for the discovery. Water is often used as the proton source in pure water Cherenkov detectors like IMB, KAMIOKANDE and Super-Kamiokande (SK). Many decay modes of proton decay have been searched by these experiments but it has never been observed yet.

In this thesis, a search for proton decay into three charged leptons has been performed by using 0.37 Mton-years of data collected in Super-Kamiokande(SK). All combinations of electrons, muons and their anti-particles were considered as decay modes. The modes are complementary to decays to a lepton and a meson, which are already extensively searched for. The lifetime of proton is predicted to be $\sim 10^{33}$ years for these modes at the energy scale of 100 TeV according to the theory suggested by T. Hambye and J. Heeck[1]. This is a reachable lifetime scale in SK. Indeed, the last search performed for these modes was by IMB-3 detector 20 years ago, thus a huge improvement in sensitivity is expected with the exposure of SK, possibly leading to a discovery.

After dedicated selection criteria for each of six decay modes, the data are compared with background expectations derived from atmospheric neutrino Monte Carlo. Since no significant excess of events has been found over the background, lower limits on the proton lifetime divided by the branching ratio have been obtained for each mode. The limits range between $9.2 \times 10^{33}$ to $3.4 \times 10^{34}$ years at 90% confidence level, largely improving upon previous experiments. The improvement factors range between 15 to 1,800, and in one of the modes first limit has been set.
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Chapter 1
Introduction

There are four interactions in our universe: Strong, Electromagnetic, Weak and Gravity and these interactions are believed to be one unified interaction at the very beginning of the universe. One of the main goals in particle physics is understanding these four interactions by one unified explanation. Strong, Electromagnetic and Weak interactions are explained by the Standard Model (SM) and Electromagnetic and Weak interactions are already unified by Weinberg-Salam theory in 1961. Next step is unifying Strong, Electromagnetic and Weak interactions and many theories are suggested. One of the strong candidates is the Grand Unified Theory (GUT). In this section, SM and GUT are explained.

1.1 The Standard Model

1.1.1 Particles in the Standard Model

SM can explain almost all phenomena in particle physics. There are 6 quarks and 6 leptons called as fermion (Spin $S = \frac{1}{2}$), 4 gauge boson ($S = 1$) and Higgs boson ($S = 0$) in SM. Each quark and lepton has so called flavor: 6 quarks correspond to $u, d, c, s, t, b$ and 6 leptons correspond to $e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau$. The couplings of Weak interaction for $(u \leftrightarrow d), (c \leftrightarrow s), (b \leftrightarrow t)$ are stronger than other combinations. The mass scales are also different throughout these pairs. This categorization is called as the generation and lepton is also categorized by 3 generations. Quark has strong “charge” so called color and interacts with gluon by Strong interaction. There are 3 colors called as R(red), G(green) and B(blue). Fermion has 2 states depending on the direction of its spin. If the direction of the momentum and the spin of the particle is same, the particle is defined as helicity plus and minus for inverse situation. The chirality is a similar value with helicity, which is important in Weak interaction. The helicity and chirality are same under the relativistic situation. The chirality corresponding to helicity plus is called as right-handed, and helicity minus is called as left-handed. Charge and mass for each quark and leptons are summarized in Table 1.1. Each particle has its pair of so called anti-particle which has the same mass and spin but inverse charge with original particle.

There are 4 bosons of $S = 1$ (\(\gamma, W^\pm, Z^0\), gluon) which intermediate each interaction and Higgs boson ($H^0$) of $S = 0$ which gives the mass to other elemental particles. $\gamma, W^\pm, Z^0$, gluon are called gauge boson as these particles are introduced from gauge symmetry. The gauge boson carries energy and momentum by being created from one particle and absorbed by another particle. $\gamma$ is the massless vector particle which intermediates Electromagnetic interaction and
Table 1.1: Charge and mass for each quark and lepton. The mass for quark is so called current mass.[2]

<table>
<thead>
<tr>
<th>Generation</th>
<th>Quark</th>
<th>Charge ([e])</th>
<th>Mass ([\text{MeV}/c^2])</th>
<th>Lepton</th>
<th>Charge ([e])</th>
<th>Mass ([\text{MeV}/c^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(u)</td>
<td>(2/3)</td>
<td>(\sim 2.3)</td>
<td>(e^-)</td>
<td>-1</td>
<td>0.511</td>
</tr>
<tr>
<td></td>
<td>(d)</td>
<td>-1/3</td>
<td>(4.8)</td>
<td>(\nu_e)</td>
<td>0</td>
<td>(&lt; 2.2 \times 10^{-6})</td>
</tr>
<tr>
<td>2</td>
<td>(c)</td>
<td>2/3</td>
<td>(\sim 1,280)</td>
<td>(\mu^-)</td>
<td>-1</td>
<td>105.7</td>
</tr>
<tr>
<td></td>
<td>(s)</td>
<td>-1/3</td>
<td>(95)</td>
<td>(\nu_\mu)</td>
<td>0</td>
<td>(&lt;0.19)</td>
</tr>
<tr>
<td>3</td>
<td>(t)</td>
<td>2/3</td>
<td>(1.73 \times 10^5)</td>
<td>(\mu^-)</td>
<td>-1</td>
<td>1,777</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>-1/3</td>
<td>(4,200)</td>
<td>(\nu_\mu)</td>
<td>0</td>
<td>(&lt; 18)</td>
</tr>
</tbody>
</table>

Couples with charged particles. \(W^\pm\) and \(Z^0\) bosons are also vector particle with finite mass which intermediate Weak interaction and couples with all quarks and leptons. \(W^\pm\) boson can interact with only left-handed fermion or right-handed anti-fermion. \(Z^0\) boson can interact with both left- and right-handed particles but the strength of the coupling is different between left- and right-handed. Gluon is the massless vector particle which intermediates Strong interaction by being created and absorbed from quarks or other gluons. Gluons have color and anti-color and couples with color charge of quark or couple themselves. So called Higgs field interacts with other fermions or gauge bosons and they get the mass according to the Higgs mechanism. The strength of the coupling between Higgs boson and other particles is proportional to the mass of the particles. The interaction, charge and mass of each boson are summarized in Table 1.2.

Table 1.2: Interaction, charge and mass of each boson[2].

<table>
<thead>
<tr>
<th>Boson</th>
<th>Interaction</th>
<th>Charge ([e])</th>
<th>Mass ([\text{GeV}/c^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>Electromagnetic</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(W^\pm)</td>
<td>Weak</td>
<td>(\pm 1)</td>
<td>80.4</td>
</tr>
<tr>
<td>(Z^0)</td>
<td>Weak</td>
<td>0</td>
<td>91.2</td>
</tr>
<tr>
<td>Gluon</td>
<td>Strong</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(H^0)</td>
<td>-</td>
<td>0</td>
<td>125</td>
</tr>
</tbody>
</table>

1.1.2 Gauge symmetry

In the quantum mechanics, the probabilities of the physics events \(P\) are described by using the wave function \(\psi(x)\) as below.

\[
P = |\psi(x)|^2
\]

On the other hand, same wave function but with arbitrary complex phase \(\psi'(x)\) is considered as below.

\[
\psi'(x) = e^{if(x)}\psi(x)
\]

This is called as the gauge conversion. Since the probability \(P\) of both states are same, \(\psi(x)\) and \(\psi'(x)\) should describe the same physics phenomenon.

When \(\psi(x)\) is an electron, \(\psi(x)\) follows the Dirac equation as below.

\[
(i\gamma_\mu \partial^\mu - m)\psi(x) = 0
\]
1.1. THE STANDARD MODEL

But $\psi(x)$ does not follow the Dirac equation. This result is contradicted that $\psi(x)$ and $\psi'(x)$ describe the same physics phenomenon (electrons). Actually in case $\psi(x)$ is an electron, equation of motion for electromagnetic field $A_\mu$ also needs to be considered as below.

\[
\begin{aligned}
(i\gamma_\mu \partial^\mu - m)\psi &= e\gamma_\nu A^\nu \psi \\
\partial_\mu \partial^\mu A^\nu &= e[\bar{\psi}\gamma^\nu \psi]
\end{aligned}
\]  

(1.4)

The first equation describes the states of electron under the electromagnetic field $A^\nu$ and the second equation describes the electromagnetic field $A^\mu$ generated by 4-dimensional current. The electromagnetic field is also changed by gauge conversion as below.

\[
A^\mu \rightarrow A'^\mu = A^\mu - (\partial^\mu f)/e
\]  

(1.5)

Then equation (1.6) becomes as below after gauge conversion.

\[
\begin{aligned}
(i\gamma_\mu \partial^\mu - m)\psi' &= e\gamma_\nu A'^\nu \psi' \\
\partial_\mu \partial^\mu A'^\nu &= e[\bar{\psi}'\gamma^\nu \psi']
\end{aligned}
\]  

(1.6)

Therefore, this system is invariant under the gauge conversion and this is called as gauge symmetry. In the Standard Model, gauge bosons are introduced by the gauge symmetry. Generally in particle physics, certain phenomenon is described by the Lagrangian $L$ which is not changed under certain symmetry. The Lagrangian for the first function in equation (1.6) is described as below.

\[
L = \bar{\psi}(i\gamma_\mu D^\mu - m)\psi
\]  

(1.7)

\[
D^\mu = \partial^\mu + ieA^\mu
\]

This Lagrangian is invariant under the gauge conversion. The Lagrangian for the QED $L_{QED}$ is described as below by adding the factors for the photon motion.

\[
L_{QED} = \bar{\psi}(i\gamma_\mu D^\mu - m)\psi - \frac{1}{4} f^{\mu\nu} f_{\mu\nu}
\]  

(1.8)

\[
f^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu
\]

Strong interaction is described by the Quantum Chromodynamics (QCD) which is directly introduced by gauge symmetry. The gauge boson, gluon is introduced from color symmetry. Then the strong interaction occurs by the coupling between colors and gluons. The wave function of quark $\Psi$ is described by 3 color components as below.

\[
\Psi = \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix}
\]  

(1.9)

The strong interaction is invariant to color conversion as below.

\[
\Psi \rightarrow \Psi' = U_S \Psi
\]  

(1.10)

Here, $U_S$ is so called SU(3) conversion which is a kind of gauge conversion described as below.

\[
U_S = \exp\{-i[\theta_a(x)T_a]\}; \quad a = 1, 2, ..., 8
\]  

(1.11)
Here, $T_a$ is 8 Hermitian matrixes whose trace is 0. The Lagrangian for the strong interaction should be invariant for SU(3) gauge conversion and described as below.

\begin{align}
\mathcal{L}_{\text{QCD}} &= \bar{\Psi}(i\gamma_\mu D^\mu - m)\Psi - \frac{1}{4} F^\mu_\nu F^a_\mu F^a_\nu \\
D^\mu &= \partial^\mu + igS (G^\mu_a T^a) \\
F^a_\mu_\nu &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_S f_{abc} G^b_\mu G^c_\nu
\end{align}

Here, $G^\mu_a$ describes the gluon, $f_{abc}$ is the SU(3) structure constant which is completely antisymmetric for the swapping of $(a, b, c)$. Now $g_S f_{abc} G^b_\mu G^c_\nu$ appears because this gauge conversion is asymmetric for exchange ($U_1 U_2 \neq U_2 U_1$). This factor leads to the 3 gluon coupling or quark confinement effects which do not appear in electromagnetic interaction.

### 1.1.3 Spontaneous symmetry breaking

The Lagrangian for $Z^0$ boson and fermion should be almost the same as $\mathcal{L}_{\text{QED}}$ but with the factor for $Z^0$ mass.

\begin{align}
\mathcal{L}_{Zff} &= \bar{\psi}(i\gamma_\mu (\partial^\mu - gZ^\mu) - m)\psi - \frac{1}{4} F^\mu_\nu F^\nu_\mu + \frac{1}{2} M^2 Z^\mu Z_\mu \\
F^\mu_\nu &= \partial^\mu Z^\nu - \partial^\nu Z^\mu
\end{align}

Here, $Z^\mu$ is the wave function for $Z^0$ boson. But the factor of $Z^0$ mass $M^2 Z^\mu Z_\mu/2$ is not invariant for the gauge conversion $Z^\mu \rightarrow Z'^\mu = Z^\mu + \partial^\mu b$. This means the mass of $Z^0$ boson cannot be introduced by gauge symmetry. In order to solve this problem, the "spontaneous symmetry breaking" is introduced.

First, mass of $Z^0$ is assumed to be created by interaction with spin 0 boson $\phi$. Generally the Lagrangian for massless boson is described as below.

\begin{equation}
\mathcal{L} = \frac{1}{2} \partial^\mu \phi^* \partial_\mu \phi
\end{equation}

If the wave function of this boson $\phi$ is invariant for gauge conversion $\phi(x) \rightarrow \phi'(x) = e^{i\theta(x)} \phi(x)$, the Lagrangian for $\phi$ is described as below by adding the corresponding gauge field $B^\mu$ and potential $V(|\phi|^2)$.

\begin{align}
\mathcal{L}_{\phi B} &= \frac{1}{2} \left( (\partial^\mu + igB^\mu) \phi \right)^2 - V(|\phi|^2) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} \\
f_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu
\end{align}

This Lagrangian is invariant for gauge conversion. Then $V(\phi^2)$ is described with first two factors of $\phi^2$ as below.

\begin{equation}
V(\phi^2) = a\phi^2 + b\phi^4 \quad (a < 0, b > 0)
\end{equation}

The shape of this potential is shown in Fig.1.1. This potential has a local minimum value at

\begin{equation}
\phi = \pm \sqrt{-\frac{a}{2b}} = \pm v_0
\end{equation}

Here, $v_0$ is called as vacuum expectation value and measured as $v_0 = 246$ GeV. In this potential, $\phi = 0$ is not stable as the energy is higher but either of $\phi = \pm v_0$ is stable. In such kind of
symmetric potential with asymmetric stable states, either of stable states are selected and symmetry is broken. This idea is called as spontaneous symmetry breaking. The Lagrangian (1.19) can be redefined as below by using the gaps from stable states $\varphi = \phi \pm v_0$.

$$
\mathcal{L}_\varphi = \frac{1}{2} (\partial^\mu \varphi)^2 + 2a\varphi^2 + \frac{1}{2} g^2 v_0^2 B^2 - \frac{1}{4} f_{\mu
u} f^{\mu\nu} + \cdots
$$

Now $(\partial^\mu \varphi)^2/2 + 2a\varphi^2$ is the Lagrangian for boson $\varphi$ of mass $m_\varphi = 2\sqrt{-a}$, $g^2 v_0^2 B^2/2 - f_{\mu\nu} f^{\mu\nu}/4$ is the Lagrangian for boson $B$ of mass $m_B = g v_0$ and other factors are for various couplings. While the factors for mass themselves break the gauge symmetry, total Lagrangian is kept to be gauge invariant by spontaneous symmetry breaking. The mass of gauge boson is considered to be created like this.

### 1.1.4 Electroweak unification

The unification of the electromagnetic and weak interactions is described by the Glashow-Weinberg-Salam (GWS) theory. Since the weak interaction exchanges the $u$ and $d$ quarks, the wave function for weak interaction is the mixing states of $|u\rangle$ and $|d\rangle$ as below.

$$
\Psi(x) = u(x)|u\rangle + d(x)|d\rangle = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}
$$

(1.23)

The physics is not changed by exchanging the $u$ and $d$ quarks components, which is called as SU(2) flavor symmetry as below.

$$
\Psi \rightarrow \Psi' = U_W \Psi
$$

(1.24)

The conversion matrix $U_W$ is described with Pauli matrix $\vec{\sigma}$ as below.

$$
U_W = \exp[i\vec{\sigma}(x) \cdot \vec{\sigma}]
$$

(1.25)

Then the wave function after SU(2) conversion is described as below.

$$
\Psi \rightarrow \Psi' = e^{i\alpha(x) \cdot \vec{\sigma}} \Psi
$$

(1.26)
CHAPTER 1. INTRODUCTION

Assuming from the Lagrangian for QCD, the Lagrangian which is invariant for SU(2) conversion (1.26) is described as below.

\[ \mathcal{L}_{ffG} = \bar{\Psi} \gamma_\mu [i \partial^\mu - g_0 (\vec{W}^\mu \cdot \vec{\sigma})] \Psi \]

(1.27)

\[ \vec{W}^\mu = (W_1^\mu, W_2^\mu, W_3^\mu) \]

Considering the gauge symmetry, the simplest Lagrangian with both U(1) and SU(2) symmetry is described as below.

\[ \mathcal{L}_{ffG} = \bar{\Psi} \gamma_\mu [i \partial^\mu - g_1 B^\mu I - g_0 (\vec{W}^\mu \cdot \vec{\sigma})] \Psi \]

(1.28)

Here, \( g_0 \) and \( g_1 \) are different for the types of fermions which are determined by experimental results. In order to generate non-zero mass for \( W \) or \( Z^0 \) bosons by Higgs mechanism, more degrees of freedom of the Higgs field are needed. Therefore Higgs field is assumed to have 2 components and 4 degrees of freedom as below.

\[ \Phi = \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \phi_3^0 \\ \phi_4^0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \phi_1 + i \phi_2 \\ \phi_3 + i \phi_4 \end{pmatrix} \]

(1.29)

Then the Lagrangian for Higgs field with U(1) and SU(2) symmetry is described as below.

\[ \mathcal{L}_{fH} = \bar{\Psi} \gamma_\mu [i \partial^\mu - g_1 B^\mu I - g_0 (\vec{W}^\mu \cdot \vec{\sigma})] \Psi \]

(1.30)

The factor 1/2 is just for making later calculation simple. This potential has a local minimum value in the case below.

\[ |\Phi|^2 = \frac{1}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\alpha}{2b} \]

(1.31)

Now the case of \( \phi_1 - \phi_2 = \phi_4 = 0 \) is considered and the spontaneous symmetry breaking (SSB) is introduced by describing \( \Phi \) with Taylor series around \( \phi_3^2 = -\alpha/b \equiv v_0^2 \). This procedure corresponds to following conversion.

\[ \Phi \overset{\text{SSB}}{\rightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 + h(x) \end{pmatrix} \]

(1.32)

Then the Lagrangian for interaction between Higgs and gauge fields is converged as below.

\[ \mathcal{L}_{fH} = \frac{1}{4} \frac{1}{4} |g' B^\mu + g (\vec{W}^\mu \cdot \vec{\sigma})| \Phi|^2 \overset{\text{SSB}}{\rightarrow} \frac{(v_0 + h)^2}{8} \left( g^2 |W^+|^2 + |W^-|^2 \right) + g^2 W_3^2 + g^2 B^2 - gg' (W_3 B + BW_3) \]

(1.33)

Here \( W^\pm = (W_1 \mp iW_2)/\sqrt{2} \) describe the charged bosons. Considering the factor of boson mass in the Lagrangian is described as \( (M^2/2) W^2 \), the mass of \( W^\pm \) is described as below.

\[ M_W = \frac{1}{2} v_0 g \]

(1.34)
Remaining factors in (1.33) of $B$ and $W_3$ mean the exchange of $B$ and $W_3$ in Higgs field and their behavior is described by following equation.

$$\partial_{\mu}\partial^{\mu}\left(\begin{array}{c} B \\ W_3 \end{array}\right) = \frac{v_0^2}{8} \left( \begin{array}{cc} g'^2 & -gg' \\ -gg' & g^2 \end{array}\right) \left(\begin{array}{c} B \\ W_3 \end{array}\right)$$

(1.35)

By solving this equation, the mass eigenstates and the mass are obtained as below.

$$\begin{cases}
Z^0 = -\sin \theta_W B + \cos \theta_W W_3 : & M_Z = \frac{1}{2} v_0 \sqrt{g^2 + g'^2} \\
A = \cos \theta_W B + \sin \theta_W W_3 : & M_A = 0
\end{cases}$$

(1.36)

Here $\theta_W$ is the so called Weinberg angle and defined as $\tan \theta_W \equiv g'/g$. The massless boson $A$ corresponds to $\gamma$ and the $Z^0$ corresponds to the weak neutral boson. The relationship between $M_Z$, $M_W$ and $\theta_W$ is also introduced as below.

$$M_W = M_Z \cos \theta_W$$

(1.37)

Therefore, electromagnetic and weak interactions are described by one unified explanation, GWS theory. Most of experiments prove that the observed results are consistent with the expectation of GWS theory.

### 1.2 The Grand Unified Theory

After the success of electroweak theory, next step is unifying electroweak and strong interactions. The difference of the interactions for various particles at lower energy scale is mainly due to the mass of the gauge bosons which intermediate each interaction. When we focus on the interactions at higher energy scale, the differences among each interaction are mainly their coupling constants. Furthermore, coupling constants depend on the energy. Their values of electroweak and strong interactions become almost the same at the energy scale of $10^{15}$ GeV as shown in Fig.1.2. This is the main motivation for unification of electroweak and strong interactions. The Grand Unification Theory (GUT) tries to explain 3 interactions of strong, weak and electromagnetic with one gauge interaction by focusing that the coupling constants of each interaction become close to each other at higher energy scale.

#### 1.2.1 SU(5) model

SU(5) model is the minimum group which satisfy SU(3)×SU(2)×U(1) group. It has 24 parameters and can be separated to the expression of SU(3)×SU(2)×U(1) as below.

$$24 = (8,1) + (3^*,2) + (3,2^*) + (1,3) + (1,1)$$

(1.38)

Here $(8,1), (1,3)$ and $(1,1)$ correspond to SU(3),SU(2) and U(1) gauge field. Since the group of SU(3)×SU(2)×U(1) has only 12 parameters, new parameters described by $(3^*,2)$ and $(3,2^*)$ are introduced. Then the new gauge field for SU(5) is described as below.

$$V_{SU(5)} = \left(\begin{array}{cccccc}
G_{rr} - \frac{2}{\sqrt{30}} B & G_{rg} & G_{rb} & X_1 & Y_1 \\
G_{gr} & G_{gg} - \frac{2}{\sqrt{30}} B & G_{gb} & X_2 & Y_2 \\
G_{br} & G_{bg} & G_{bb} - \frac{2}{\sqrt{30}} B & X_3 & Y_3 \\
X_1 & X_2 & X_3 & \frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B & W^+ \\
Y_1 & Y_2 & Y_3 & \frac{1}{\sqrt{2}} W^3 - \frac{3}{\sqrt{30}} B & W^-
\end{array}\right)$$

(1.39)
CHAPTER 1. INTRODUCTION

Figure 1.2: Running coupling constants of SM as the function of energy scale with respect to each interaction[3].

Here $G$ is the gluon, $B, W^\pm, W^3$ are gauge bosons in electroweak theory, $X_i, Y_i$ are the new 12 gauge bosons of SU(3)$_C$-triplet and SU(2)$_L$-doublet, respectively, introduced in SU(5) model. Since these bosons can intermediate quarks and leptons, baryon and lepton numbers are not conserved under the SU(5) model. This feature predicts the nucleon decay which can never occur in the SM. The fermions in the SM are summarized by the following 5 and 10 expressions in SU(5) model.

\[
\begin{align*}
\bar{5} &= \begin{pmatrix}
\bar{d}_r \\
\bar{d}_g \\
\bar{d}_b \\
e^- \\
-\nu_e
\end{pmatrix}, \quad
10 = \begin{pmatrix}
0 & \bar{u}_b & -\bar{u}_g & -u_r & -d_r \\
-\bar{u}_b & 0 & \bar{u}_r & -u_g & -d_g \\
\bar{u}_g & -\bar{u}_r & 0 & -u_b & -d_b \\
u_r & u_g & u_b & 0 & e^+ \\
\bar{d}_r & \bar{d}_g & \bar{d}_b & -e^+ & 0
\end{pmatrix}
\end{align*}
\]

(1.40)

Here U(1)$_Y$ hypercharge can be automatically calculated by this SU(5) model, which means the charge of each particle in the SM can be naturally explained by this model. In order to break the electroweak symmetry at the energy of weak scale and obtain the mass of quarks and leptons, Higgs doublets are needed. In the SU(5) model, they are contained in either $\bar{5}_H$ or $5_H$. Three additional states are color-triplet Higgs scalars and their couplings also violate the baryon and lepton numbers, which leads to nucleon decay. The dominant decay mode of nucleon decay in SU(5) model is $p \rightarrow e^+\pi^0$ mode and the lifetime of this mode is predicted as $10^{31\pm1}$ years[4]. However this was already excluded by KAMIOKANDE experiment[5]. Furthermore, Weinberg angle is predicted as $\sin^2 \theta_W = 3/8$ in this model, but experimental results say $\sin^2 \theta_W = 0.23$. Therefore this SU(5) model is already ruled out.
1.2. SUSY SU(5)

Another idea is introducing so-called super-symmetry (SUSY) in SU(5) model. SUSY is the symmetry for the exchange of fermions and bosons. In order to cancel anomalies and get the mass for both up and down quarks, both Higgs multiplets $5_H$ or $5_H$ are required. One of interesting features of this SUSY SU(5) model is that running coupling constants can completely match at the energy around $2 \times 10^{16}$ GeV as shown in Fig. 1.3. The most dominant decay mode of proton decay in SUSY SU(5) is $p \rightarrow \bar{\nu}K^+$. The predicted upper limits of proton lifetime are, $< 2.9 \times 10^{30}$ years for $p \rightarrow \bar{\nu}K^+$ and $< 4.1 \times 10^{33}$ years for $p \rightarrow e^+\pi^0[9]$. The most stringent limit by proton decay search of SK is $5.9 \times 10^{33}$ years for $p \rightarrow \bar{\nu}K^+[11]$ and $1.6 \times 10^{34}$ years for $p \rightarrow e^+\pi^0[10]$. Comparing with these limits, this SUSY SU(5) model is also excluded as well.

1.2.3 SO(10) model

Since SU(5) model uses two independent representation $\bar{5}$ and 10, unification is not complete. In order to describe quarks and leptons by 1 unified representation, a right-handed neutrino is added to SU(5). While the right-handed neutrino is not contained in SM, it is introduced in the seesaw mechanism[12][13][14]. Then SM fermions can be described by one unified representation of SO(10) model, $16 = 10 + \bar{5} + 1$. The simplest decaying pattern of SO(10) is $SO(10) \rightarrow SU(5) \times U(1)$. The right-handed neutrino can give the explanation for too small left-handed neutrino mass or the asymmetry of matter and anti-matter. The dominant decay mode of proton decay in SO(10) model is also $p \rightarrow e^+\pi^0$ and the lifetime is predicted to be $10^{32} - 10^{39}$ years[15][16]. Comparing with most stringent limit of $1.6 \times 10^{34}$ years for $p \rightarrow e^+\pi^0$ by SK, this SO(10) model is still alive.
### 1.2.4 Proton decay into three charged leptons

Adopting not only baryon number \((B)\) and lepton number \((L)\) symmetries, but also adopting lepton flavor symmetries, many different decay modes are introduced. This model was introduced by T. Hambye and J. Heeck in 2018\[^1\]. They theoretically estimated the decay rate of the proton decay into three charged leptons and explained the physics advantages of these decay modes for the first time. In this theory, proton decay at the level of dimension \(d = 10\) operators is considered. There are two cases for \(d = 10\) operators with three leptons, 1) \(\Delta B = \Delta L\) which introduce six proton decay channels \(p \to l^+_\alpha l^+_\beta l^-\gamma\), 2) \(\Delta B = \Delta L = 3\) which introduce nucleon decay of four body decay such as \(n \to l^+ + l^-\). Since the former case has only charged leptons in final states, it is more interesting mode to be searched by Super-Kamiokande. Especially \(p \to e^+ e^- e^-\) and \(p \to e^- \mu^+ \mu^-\) modes can be dominant in all commonly discussed modes by a symmetry \(B_L \leq L + 2L_e + xL_\tau\) and \(2L_e + L_\mu + xL_\tau\) respectively (\(x\) is arbitrary value). Each symmetry value in each proton decay mode are summarized in Table 1.3. Thus it is possible to generate only \(d = 10\) proton decay modes assuming certain pattern of particles or symmetries in this world. Other four decay modes, \(p \to e^+ e^- e^-\), \(p \to \mu^+ e^- e^-\), \(p \to e^+ \mu^+ \mu^-\) and \(p \to \mu^+ \mu^+ \mu^-\), cannot be singled out from two-body decays as the \(e^+ e^-\) and \(\mu^+ \mu^-\) is replaced by a \(\pi^0\). The proton decay rate considering \(d = 10\) operators without a covariant derivative for the three charged leptons channel involving SM scalar double field \(H\) is calculated as follows.

\[
\Gamma(p \to l^+\alpha l^+\beta l^-\gamma) \sim \frac{\langle H \rangle^2 \beta_h^2 m_p^5}{6144\pi^3 \Lambda^{12}} \approx \frac{(100\text{TeV}/\Lambda)^{12}}{10^{35}\text{Yrs}} \quad (1.41)
\]

Here \(\beta_h \approx 0.014\) GeV\(^3\) is the hadronic matrix element, \(m_p\) is the mass of proton and \(\Lambda\) is the probing energy scale. Considering the probing energy scale of \(\Lambda \approx 100\text{ TeV}\), life time of such order is reachable in SK.

Proton decay into three charged leptons occurs by exchanging heavy particles along two different types of topologies. Topology \(A\) involves new heavy scalars and topology \(B\) involves a new heavy scalar and 2 new heavy fermions. The scalars in both topologies always couples to two SM fermions. The candidates of the scalars are SU(2)\(_L\) single di-quarks, di-leptons and leptoquarks (LQs)\[^17\][^18][^19\]. For the \(d = 10\) operators, there are various places where the SM double \(H\) can be inserted. In the minimal model for \(p \to e^- \mu^+ \mu^-\), two heavy LQs mediate the proton decay as shown in Fig.1.4.

Some of these proton decay modes were searched by some pure water Cherenkov detector experiments. The most stringent limit for following proton decay modes were set by IMB-3 experiment\[^20\] with the exposure of 7.6 kton-years.

- \(p \to e^+ e^- e^-\)
- \(p \to \mu^+ e^- e^-\)
- \(p \to e^+ \mu^+ \mu^-\)
- \(p \to \mu^+ \mu^+ \mu^-\)

And the most stringent limit for \(p \to e^- \mu^+ \mu^-\) mode was set by HPW experiment\[^21\] with the exposure of 0.14 kton-years. The observed data was consistent with expected background in these experiments and lower lifetime limit for each proton decay mode was set. The details...
1.3. Structure of this thesis

This thesis describes the search for proton decay into three charged leptons by using 0.37 Mton-years exposure of data measured in SK. In Chapter 2, SK detector and its calibration are described. In Chapter 3, Monte Carlo simulations for signal and background events are described. In Chapter 4, preselection criteria is described. In Chapter 5, event reconstruction algorithm is

Figure 1.4: Diagram for $p \rightarrow e^-\mu^+\mu^+$ mediated by two heavy LQs $\phi_1, \phi_2$.[1]

Table 1.3: $B - L$, $2L_e + L_\mu + xL_\tau$ and $L_e + 2L_\mu + xL_\tau$ values in each proton decay mode.

<table>
<thead>
<tr>
<th>Proton decay mode</th>
<th>$B - L$</th>
<th>$2L_e + L_\mu + xL_\tau$</th>
<th>$L_e + 2L_\mu + xL_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow e^+\pi^0$</td>
<td>1 → 1</td>
<td>0 → −2</td>
<td>0 → −1</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+\pi^0$</td>
<td>1 → 1</td>
<td>0 → −1</td>
<td>0 → −2</td>
</tr>
<tr>
<td>$p \rightarrow e^+e^-e^-$</td>
<td>1 → 1</td>
<td>0 → −2</td>
<td>0 → −1</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+e^-e^-$</td>
<td>1 → 1</td>
<td>0 → −1</td>
<td>0 → −2</td>
</tr>
<tr>
<td>$p \rightarrow \mu^-e^+e^+$</td>
<td>1 → 1</td>
<td>0 → −3</td>
<td>0 → −0</td>
</tr>
<tr>
<td>$p \rightarrow e^+\mu^-\mu^+$</td>
<td>1 → 1</td>
<td>0 → −2</td>
<td>0 → −1</td>
</tr>
<tr>
<td>$p \rightarrow e^-\mu^+\mu^+$</td>
<td>1 → 1</td>
<td>0 → 0</td>
<td>0 → −3</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+\mu^-\mu^-$</td>
<td>1 → 1</td>
<td>0 → −1</td>
<td>0 → −2</td>
</tr>
</tbody>
</table>

about these experiments are described in Appendix B. The latest results of these proton decay modes are summarized in Table 1.4. Compared with the predicted lower limit of $\sim 10^{33}$ years, these decay modes are not excluded yet. Furthermore, considering the most stringent limit $1.6 \times 10^{34}$ years for $p \rightarrow e^+\pi^0$ and $7.7 \times 10^{33}$ years for $p \rightarrow \mu^+\pi^0$ in SK, similar decay modes of proton decay into three charged leptons have potential for discovery in SK. The discovery of these proton decay modes can be one of the strong evidence of GUT theory.
Table 1.4: The latest results of the search for proton decay into three charged leptons.

<table>
<thead>
<tr>
<th>Proton decay mode</th>
<th>Experiment</th>
<th>Lower lifetime limit [years] (90% CL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow e^+ e^+ e^-$</td>
<td>IMB</td>
<td>$7.9 \times 10^{32}$</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+ e^+ e^-$</td>
<td>IMB</td>
<td>$5.3 \times 10^{32}$</td>
</tr>
<tr>
<td>$p \rightarrow \mu^- e^+ e^+$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$p \rightarrow e^+ \mu^+ \mu^-$</td>
<td>IMB</td>
<td>$3.6 \times 10^{32}$</td>
</tr>
<tr>
<td>$p \rightarrow e^- \mu^+ \mu^+$</td>
<td>HPW</td>
<td>$6 \times 10^{30}$</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+ \mu^+ \mu^-$</td>
<td>IMB</td>
<td>$6.8 \times 10^{32}$</td>
</tr>
</tbody>
</table>

described. In Chapter 6, selection criteria and the result are described. In Chapter 8, conclusion of this analysis is described.
Chapter 2

Super-Kamiokande Experiment

2.1 Overview

Super-Kamiokande (SK) is the largest pure water Cherenkov detector in the world, located in Kamioka township, Gifu prefecture, Japan. The scientific motivation of the SK is searching for proton decays and studies of neutrinos from various sources: the Sun, atmosphere, supernovae, gamma ray burst and other astrophysical sources or artificial neutrino beams. The SK detector mainly consists of a stainless tank (39.3 m diameter, 41.4 m height), 50 kton of ultra pure water (H\textsubscript{2}O) and the photomultiplier tubes (PMTs). The schematic view of the SK detector is shown in Figure 2.1. In order to reduce cosmic muon background, the detector is located 1,000 m under the peak of Mt. Ikenoyama (2,700 m water equivalent). Cosmic ray muons of energy less than 1.3 TeV cannot penetrate to the depth of 2,700 m.w.e. Observed muon flux is $6 \times 10^{-8} \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1}$ (2 kHz in SK detector) and this is 5 orders of magnitude smaller flux compared to that on the surface of the Earth. The rate of cosmic ray muons observed in the

Figure 2.1: Schematic view of the SK detector.[22]
SK detector is about 2 kHz.

A stainless-steel framework of 55 cm width, placed 2-2.5 m from the all top, bottom and barrel walls inside the SK tank, supports inward and outward facing PMTs. Inward facing PMTs and the water inside the volume is called as Inner Detector (ID). ID is a main detector of 36.2 m height and 33.8 m diameter which contains 32 kton of water. Outward facing PMTs and the water in the thickness of 2 m between the framework and wall is called as Outer Detector (OD) which is mainly used for the cosmic muon veto or preventing gamma rays from surrounding rock. ID and OD are optically separated by PET black sheets. On the ID surface 20-inch PMTs are uniformly mounted facing inward viewing direction and on the OD surface 8-inch PMTs are mounted facing outward viewing direction. There are twelve cable holes on top of the SK tank for signal and high voltage cables. Veto counters which consist of 2 m×2.5 m plastic scintillation counters are installed into four of these cable holes. The average geomagnetic field inside the SK tank is about 450 mG inclined by about 45° to the horizon at the detector site. Such a scale and uniform direction of geomagnetic field can make systematic bias against photoelectron trajectory and timing in the PMTs. In order to solve this problems, 26 sets of horizontal and vertical Helmholtz coils are arranged around the inner surface of the tank. As the result average magnetic field in the tank is reduced to about 50 mG.

The excavation started in December 1991 and the detector construction was completed in December 1995. Then more than 2 months were spent for filling up the water in the tank. Finally Super-Kamiokande successfully started operation on April 1st 1996 with 11,146 ID PMTs and 1,885 OD PMTs. This operation stopped in July 2001 to replace bad PMTs. This period is called as SK-I. Unfortunately, an accident happened during water filling after the replacement of PMTs in July 2001. One of the ID PMTs at the bottom collapsed and big shock wave was generated because the inside of the PMT is vacuum. The shock wave propagated via water and destroyed other PMTs continuously. Due to this accident 6,777 ID PMTs and 1,100 OD PMTs were destroyed. In order to avoid such accident, remained PMTs were protected by fiber reinforced plastics (FRPs) and acrylic cases and rearranged on the wall. Operation of so called SK-II period started with 5,182 ID PMTs and 1885 OD PMTs in October 2002 and stopped in October 2005. After the SK-II period, SK tank was opened again and PMTs were added. As the result, the number of PMTs was almost the same as SK-I period. In June 2006, operation of the SK-III period started and stopped in September 2008. In September 2008, read-out electronics and data acquisition system were upgraded. The SK-IV period started in that time and stopped in May 2018. This is the longest period in all SK periods. Photo coverage of ID is 40 % in all SK periods excepting for SK-II (19 %) Information of all SK periods are summarized in Table, 2.1

2.2 Principle of the Detector

When a charged particle passes through certain medium with greater speed than the speed of the light in the medium, electromagnetic shock wave is emitted radially. This phenomenon is known as Cherenkov-radiation and emitted light is called as Cherenkov-light or Cherenkov-ring. Schematic view of Cherenkov-light emission is shown in Fig. 2.2 and opening angle $\theta_c$ is described as below.

$$\cos \theta_c = \frac{1}{n\beta} \quad (2.1)$$
Table 2.1: Summary of all SK periods.

<table>
<thead>
<tr>
<th>Operation</th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Livetime (days)</td>
<td>1489.2</td>
<td>798.6</td>
<td>518.1</td>
<td>3244.4</td>
</tr>
<tr>
<td>Number of PMTs</td>
<td>ID</td>
<td>OD</td>
<td>ID</td>
<td>OD</td>
</tr>
<tr>
<td></td>
<td>11,146</td>
<td>1,885</td>
<td>5,182</td>
<td>1,885</td>
</tr>
<tr>
<td></td>
<td>OD</td>
<td>OD</td>
<td>ID</td>
<td>ID</td>
</tr>
<tr>
<td></td>
<td>1,885</td>
<td>1,885</td>
<td>1,885</td>
<td>1,885</td>
</tr>
<tr>
<td>Photo coverage</td>
<td>40%</td>
<td>19%</td>
<td>40%</td>
<td>40%</td>
</tr>
<tr>
<td>FRP &amp; Acrylic cases</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Electronics</td>
<td>ID</td>
<td>OD</td>
<td>ID</td>
<td>OD</td>
</tr>
<tr>
<td></td>
<td>ATM</td>
<td>QTC</td>
<td>ATM</td>
<td>QTC</td>
</tr>
<tr>
<td></td>
<td>OD QTC</td>
<td>OD QTC</td>
<td>OD QTC</td>
<td>QTC</td>
</tr>
<tr>
<td>Trigger</td>
<td>Hardware</td>
<td>Hardware</td>
<td>Hardware</td>
<td>Software</td>
</tr>
</tbody>
</table>

Figure 2.2: Schematic view of Cherenkov Radiation. Red and blue arrows show the direction of the charged particle and emitted Cherenkov light respectively.
Here \( n \) is the refractive index of the medium, \( \beta = v/c \), \( v \) and \( c \) is the speed of the charged particle and the light respectively. Since the speed of the charged particle \( \beta c \) has to be greater than the speed of the light in the medium \( c/n \), the minimum momentum of the charged particle to emit Cherenkov-light is described as below.

\[
\beta c > \frac{c}{n}
\]  

(2.2)

Here \( p, E \) and \( m \) is the momentum, energy and mass of the charged particle respectively. Using \( \beta = p/E \) by natural unit, above condition can be described with the mass \( m \), momentum \( p \) and energy \( E \) of charged particle as below.

\[
p > \frac{m}{\sqrt{n^2 - 1}}
\]  

(2.3)

Minimum momentum of each charged particle to make Cherenkov-light in water \((n = 1.34)\) is summarized in table 2.2. The opening angle of emitted Cherenkov ring in water as the function of the momentum of electron and muon is shown in Fig. 2.3. Charged particles gradually lose their energy by the interaction with the nucleus in water. When the speed of the particle becomes \( \beta < 1/n \), Cherenkov-light is not emitted anymore and the emitted light is observed as a ring shape on the detector wall.

<table>
<thead>
<tr>
<th>( e^- )</th>
<th>( \mu^- )</th>
<th>( \pi^- )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum momentum [MeV/c]</td>
<td>0.57</td>
<td>118</td>
<td>156</td>
</tr>
</tbody>
</table>

Table 2.2: Minimum momentum to emit Cherenkov-light in water.

![Diagram](https://via.placeholder.com/150)

Figure 2.3: The opening angle of emitted Cherenkov ring in water as the function of the momentum of electron and muon.
2.3. WATER PURIFICATION SYSTEMS

The number of emitted photons \( N \) per wavelength \( \lambda \) per unit travel distance \( x \) of a charged particle is described as follow.

\[
\frac{d^2N}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2} \left( 1 - \frac{1}{n^2\beta^2} \right)
\]  \hspace{1cm} (2.4)

Here \( \alpha \) is the fine structure constant. \( \sim 340 \) photons of wavelength between 300 nm and 600 nm are emitted per 1 cm by a charged particle with \( \beta \sim 1 \) in water.

2.3 Water Purification Systems

Since Cherenkov-light travels in water until it reaches the PMTs on the wall, high transparency of the water is important to achieve higher detection efficiency. In order to keep high transparency of the water, 50 ktons of water in the tank is continuously reprocessed in the water purification systems at a rate of about 30 tons/hour. Fig.2.4 shows the schematic view of the water purification systems of early 2002.

Figure 2.4: Schematic view of water purification systems.[22]

First, water passes through the 1 \( \mu \)m mesh filters to remove dusts and particles which reduce the transparency of the water or can be a possible radon source. The heat exchanger cools the water to reduce the dark noise level of PMTs and suppresses the growth of bacteria which can degrade the transparency of the water. Typical water temperature before 1st heat exchanger and after 2nd heat exchanger is 14.2 \( ^\circ \)C and 12.9 \( ^\circ \)C, respectively. Surviving bacteria are killed in UV sterilizer system. The Cartridge Polisher (CP) is used to remove heavy ions which can reduce the water transparency and include radioactive sources. 1,500 L of resin is used in this system. The Reverse Osmosis (RO) system and the tank to dissolve Rn-reduced air into the water were installed in 1999. The RO system removes additional particulates. Rn-reduced air into the water increases the removal efficiency of Rn in Vacuum Degasifier (VD)
system. The VD consists of a cylindrical stainless steel vessel and a vacuum pump. This system removes dissolved gas in the water like radon which is a serious background event source for solar neutrinos in the MeV energy scale, or oxygen which encourage the growth of bacteria in the water. The removal efficiency of radon gas is estimated to be about 96%. The Ultra Filter(UF), which consists of hollow fiber membrane filters, can remove molecules of size more than 10 nm diameter. Typical number of particles with a size more than 0.2 µm before the water purification system is 1,000 particles/cc and this is reduced to 6 particles/cc after reprocessing by all systems. The Membrane Degasifier (MD) consists of 30 hollow membrane modules and a vacuum pump can also remove radon dissolved in the water. Removal efficiency of radon is measured to be about 83%.

2.4 Air Purification Systems

Rn-reduced air is produced by the air purification systems and supplied to the gap between the water surface and the top of the SK tank. Rn-reduced air is kept at a slight over pressure to prevent radon-laden air from entering the SK tank. Schematic view of the air purification systems is shown in Fig. 2.5. It consists of a buffer tank, dryers, filters and activated charcoal filters. 8,000 L of activated charcoal is used in total and the last 50 L of it is cooled to -40°C to increase removal efficiency of radon. These systems are equipped near the entrance of Atotsu tunnel outside the mine. Fresh air produced by this system is continuously pumped at approximately 10 m³/min to the experimental area through an air duct along 1.8 km of Atotsu tunnel. Radon level of air in the mine tunnel changes dependent on the seasons. Fig. 2.6 shows the radon level of air inside(lower) and outside(upper) the experimental area. During the cool season, since the air flows into Atotsu tunnel entrance and path through the exposed rocks for relatively short distance, radon level is approximately 100-300 Bq m⁻³. On the other hand, during the warm season, since air flows from deep in the mine to outside and passes through exposed rock for a long distance, radon level near the experimental area is increased to approximately 2000-3000 Bq m⁻³. Thanks to this air purification system, radon level of air in the experimental area is kept to be 30-50 Bq m⁻³ throughout the year.

2.5 PMTs and associated structures

20-inch PMTs of 11-stage Venetian blind type (R3600) produced by Hamamatsu Photonics K.K. are used in the ID detector (Fig. 2.7). The bialkali (Sb-K-Cs) is used for the photocathode which
2.5. PMTS AND ASSOCIATED STRUCTURES

has a peak quantum efficiency of about 21\% at 360-400 nm wavelength as described in Fig. 2.8.

ID PMTs operate with high voltages from 1,700 V to 2,000 V. The collection efficiency of the first dynode, transit time spread and gain for single photoelectron are 70\%, 2.2 ns and $10^7$, respectively. Average dark noise rate at 0.25 pe threshold is about 3 kHz. Although the acrylic cover is attached to the PMTs in SK-II to SK-IV, transparent efficiency of photon with 350 nm wavelength is about 96\%. The basic unit for ID PMTs is a stainless frame which supports a $3 \times 4$ array of PMTs (super-module). Height, width and depth of the super-module is 2.1 m, 2.8 m, 0.55 m respectively. Super-modules are connected each other and stacked from bottom to the
top structure. Opaque PET black sheets cover the gaps between PMTs on the ID surface. These sheets can improve the optical separation between ID and OD and suppress low energy radio activity occurring behind ID PMTs. Schematic view of the support structure in ID detector is shown in Fig. 2.9.

8-inch Hamamatsu PMTs (R1408) are used in OD detector. These PMTs were used in IMB experiment and SK is reusing them. Each super-module has two OD PMTs. In order to improve the photon collection, reflective layers made from Type 1073B Tyvek sheets manufactured by DuPont cover the all surfaces of OD. Measured reflective efficiency of these sheets is order of 90% in excess of 400 nm wavelength and falling to 80% at about 340 nm. For further improvement of the photon collection efficiency, each PMT is coupled with wavelength-shifting plate of 1.3 cm thickness and 60×60 cm size as Fig. 2.10. Thanks to this structure, not only the light directly striking the photocathode of the PMT but also the light missing the PMT but striking the plate can be detected.

2.6 Electronics and data acquisition systems

New electronics and data acquisition systems were installed from SK-IV. The systems in SK-I to SK-III are described first, and then the systems in SK-IV are described in this section. These systems are contained in "quadrant-huts" and "Central hut" located on top of the SK tank. The quadrant-huts contain electronics racks and frontend DAQ computer serving ID and OD PMTs. The Central hut contains electronics and computers for triggering, housekeeping and Global Positioning System (GPS) time synchronization system.

2.6.1 Electronics and data acquisition systems in SK-I to SK-III

ID PMT signals are processed by custom built TRISTAN KEK Online (TKO) modules called ATM (Analog Timing Module). The ATM consists of Analog-to-Digital Converter (ADC) and
2.6. ELECTRONICS AND DATA ACQUISITION SYSTEMS

Figure 2.9: Schematic view of the support structure in ID detector[22].

Figure 2.10: Schematic view of the OD structure[23].
Time-to-Digital Converter (TDC), which records the integrated charge and arrival time of each PMT signal. There are 960 ATM boards, and each ATM reads the signals from twelve PMTs. Fig. 2.11 shows a schematic view of the analog input block of the ATM. Input PMT signals are divided into four lanes by Current splitter. One of them is fed to a discriminator with a threshold of $-1$ mV for each channel which corresponds to $1/4$ p.e. When the PMT signal is above the threshold, a HITSUM signal of 200 ns width and 15 mV pulse height is asserted. The HITSUM signals are summed up for all $12 \times 960$ PMTs signals and this "sum of HITSUM" is used to generate global trigger signal. At the same time, two of divided signals A, B are fed to Charge-to-Analog Converter (QAC) and Time-to-Analog Converter (TAC) starts integrating constant current. Once the global trigger is issued, integration of TAC is stopped. Then the information in the QAC/TAC is digitized by Analog-to-Digital Converter (ADC) and stored in internal memory buffers. If the global trigger is not issued within 1.3 $\mu$s, all information in QAC/TAC is cleared. A channel control chip generates start/stop signals for the TAC, the gate signal for QAC and clear signal for TAC/QAC. There are two TACs and QACs with respect to each channel to record events in rapid succession, such as muon with its decay electron, without dead time. Even though there are two TACs and QACs, actually the signal from +400 ns to +900 ns after the hit timing is neglected due to a cable reflection pulse caused by a impedance mismatch between ATM and PMT.

An ID trigger is issued when the sum of ID HITSUM signal exceeds the threshold. Overview of ID trigger in SK-I to SK-III is summarized in Figure 2.12. There are three types of ID event trigger, super low energy (SLE), low energy (LE) and high energy (HE) respectively. Threshold for sum of HITSUM with respect to each trigger in each period is summarized in Table 2.3.

In OD, signals from PMTs are processed and digitized in a similar way with ID systems. An OD trigger is issued when the OD HITSUM signal exceeds a threshold. The threshold of the OD trigger corresponds to 19 hits within 200 ns time window.
2.6. ELECTRONICS AND DATA ACQUISITION SYSTEMS

Figure 2.12: Overview of the ID trigger in SK-I to SK-III.[25]

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLE</td>
<td>186 mV</td>
<td>110 mV</td>
<td>186 mV</td>
</tr>
<tr>
<td>LE</td>
<td>320 mV</td>
<td>152 mV</td>
<td>302 mV</td>
</tr>
<tr>
<td>HE</td>
<td>340 mV</td>
<td>180 mV</td>
<td>320 mV</td>
</tr>
</tbody>
</table>
2.6.2 Electronics and data acquisition systems in SK-IV

In order to solve the dead time and cable reflection problems, new electronics QBEE (QTC Based Electronics with Ethernet) was introduced instead of the ATM from SK-IV. An application-specific integrated circuit (ASIC), called the high-speed QTC IWATSU CLC101, was designed in CMOS 0.35 µm technology and is build-in discriminators of QBEE. This new ASIC offers three charge dynamic ranges: Small, Medium and Large, each with a short cycle time (≤ 1 µs). Output signal from each range is digitized and one most appropriate range is chosen by field-programmable gate array (FPGA), then sent to data acquisition computers. The gain of each range can be adjusted to cover a wide dynamic range with reasonable resolution. For example, the overall charge dynamic range of the QTC is 0.2 - 2500 pC if the gain ratio of three ranges is set to 1 : 1/7 : 1/49. Block diagram of the QTC is shown in Fig.2.13.

![Block diagram of the QTC](image)

Figure 2.13: Block diagram of the QTC.[24]

A block diagram of the one QTC channel is shown in Fig.2.14. Input signals from PMTs are amplified by a low-noise-amplifier (LNA), delayed by a low-pass filter (LPF), processed by voltage-to-current (V/I) convertor, and integrated by capacitor. Timer block consists of three timers: a charging timer, a discharging timer and VETO timer. Once the amplified input signal exceeds the discriminator threshold, charging timer operates a ~400 ns charge gate and at the same time a trigger flag HIT is generated. The leading edge of the output signal corresponds to this timing. During the charge gate, input charge is integrated in the capacitor. Soon after the end of the charge gate, discharging timer operates a ~350 ns discharge gate. During the discharge gate, the integrated charge is discharged by a constant current. This discharging time which can be known by the width of output QTC signal is proportional to the integrated charge. Thus QTC output signals contain both charge and timing information. After the end of the discharge gate, reset and VETO signals are issued. Input signals during the VETO gate are ignored. Processing time for one input signal is ~900 ns. Timing chart of QTC operation in SK-IV is shown in Fig.2.15. This new system records all the hits and immediately integrates the charge and digitizes the timing and charge information. When the number of PMT hits within 200 ns sliding time window (N200) exceeds the threshold, a software trigger is issued. The digitized charge and timing are recorded when the trigger is issued. In this new system, there is no dead time and timing window of each event trigger is larger compared to previous systems in SK-I to SK-III because all hits can be recorded. SLE, LE and HE triggers are still used in this new systems with a larger timing window. In addition to these triggers, special high energy trigger (SHE) and after trigger (AFT) are introduced for using neutron tagging.
2.6. ELECTRONICS AND DATA ACQUISITION SYSTEMS

Figure 2.14: Block diagram of the one QTC channel.[24]

Figure 2.15: Timing chart of QTC operation in SK-IV.[24]
2.7 Detector Calibration

In order to keep good sensitivity of the physics analysis, it is important to understand the PMT response of charge and time in the SK detector precisely. We also need to understand how the Cherenkov photon propagates in the water to the PMTs. In this section, calibration methods of PMT gain or timing and water transparency are described.

2.7.1 High-voltage setting of the PMTs

All PMTs are required to give the same output charge for the same input light intensity by tuning the high-voltage (HV) of each PMT. This calibration is performed by an isotropic light source placed in the center of the SK tank. Since the SK tank is very large, amount of light reaching to each PMT from the source depends on the distance between the source and each PMT. Correcting only this geometrical difference is not sufficient because photon propagation in the water, which depends on the water quality, and reflection from the ID surface also needs to be considered. First 420 “standard PMTs” are determined to tune HV individually before the installation on the ID wall. Pre-calibration was performed to these standard PMTs and later mounted in ID as the reference for other PMTs. Fig. 2.16 shows the schematic view of the pre-calibration system. The light from a Xe flash lamp goes through a UV filter and injected into three optical fibers. One fiber is connected to “scintillator ball” in dark box made from \( \mu \)-metal to shield against geomagnetic field. The scintillator ball is used to diffuse the light emission uniformly. Other two fibers are connected to avalanche photodiode (APD) modules to monitor the light intensity of the Xe lamp. Two 2-inch PMTs are mounted in the box to monitor the light intensity of the ball. HV values of 420 standard PMTs are adjusted to give the same output charge by using this system.

The 420 standard PMTs after pre-calibration are placed in ID as the red points shown in the left figure of Fig. 2.17. These standard PMTs are used as references for other PMTs which can have similar geometrical relationships to the light source as shown in right figure of Fig 2.17. The same Xe lamp and scintillator ball are used, and the HV of PMTs in the same group are tuned to give the same output charge as the reference PMT.

<table>
<thead>
<tr>
<th>Trigger</th>
<th>( N_{200} ) threshold (PMT hits)</th>
<th>Time window (( \mu )s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLE</td>
<td>31</td>
<td>([-0.5, +1.0])</td>
</tr>
<tr>
<td>LE</td>
<td>47</td>
<td>([-5, +35])</td>
</tr>
<tr>
<td>HE</td>
<td>50</td>
<td>([-5, +35])</td>
</tr>
<tr>
<td>SHE</td>
<td>58</td>
<td>([-5, +35])</td>
</tr>
<tr>
<td>AFT</td>
<td>SHE without OD</td>
<td>([+35, +535])</td>
</tr>
<tr>
<td>OD</td>
<td>22</td>
<td>([-5, +35])</td>
</tr>
</tbody>
</table>

technique. The AFT trigger is only issued in case the SHE trigger is issued but the OD trigger is not issued and records additional 500 \( \mu \)s data. The information of each software trigger is summarized in Table 2.4.
2.7. DETECTOR CALIBRATION

Figure 2.16: Schematic view of the pre-calibration system.[26]

Figure 2.17: The location of the standard PMTs.[26]
2.7.2 Relative gain difference

In order to know the number of p.e.s from the output charge of PMTs, we need to determine the gain of each PMT. In the first step, relative gain difference of each PMT is measured. In this measurement, a stable light source emitting constant-intensity flashes is placed at a certain position in the tank. The light source is nitrogen-laser-driven dye laser fed into the diffuser ball. There are two measurements.

- High intensity flash is emitted so that every PMT gets a suitable number of photons. The average charge \( Q_{\text{obs}}(i) \) for each ID-PMT \( i \) is measured. \( Q_{\text{obs}}(i) \) is described by the following equation.
  \[
  Q_{\text{obs}}(i) \propto I_s \times a(i) \times \epsilon_{qe}(i) \times G(i) \quad (2.5)
  \]
  Here \( I_s \) is the average intensity of high intensity flash, \( a(i) \), \( \epsilon_{qe}(i) \), \( G(i) \) are the acceptance, quantum efficiency (QE) and the gain of \( i \)-th PMT respectively.

- Low intensity flash is emitted so that only a few PMTs are hit in each event to ensure each of hit is single p.e. The number of times \( N_{\text{obs}}(i) \) that \( i \)-th PMT records the charge which exceeds the threshold value. Now \( N_{\text{obs}}(i) \) is described as follow.
  \[
  N_{\text{obs}}(i) \propto I_w \times a(i) \times \epsilon_{qe}(i) \quad (2.6)
  \]
  Here \( I_w \) is the average intensity of low intensity flash.

Since the locations of light source are the same between two measurements, factors in \( Q_{\text{obs}}(i) \) and \( N_{\text{obs}}(i) \) are almost identical. Then \( G(i) \) is obtained by the ratio of (2.5) and (2.6).

\[
G(i) \propto \frac{Q_{\text{obs}}(i)}{N_{\text{obs}}(i)} \quad (2.7)
\]

Then relative gain of each PMT is obtained from \( G(i) \) normalized by the average gain of all PMTs. In order to get the mean value of the relative gain of each PMT, this measurement is repeated while changing the intensity of the flash. Fig.2.18 shows the normalized relative gain (2.7) of each PMT. Since the HV is tuned to make output charge of each PMT same for the same input light intensity, cause of deviation in Fig.2.18 is considered to be the difference of QE among PMTs. This factor is used for fine correction of charge to the number of p.e.s conversion.

2.7.3 Absolute gain conversion factor

In the second step, average gain over the whole detector is determined. In this measurement, a uniform and stable “nickel source” of single p.e level light is used. The source isotropically emits gamma rays of about 9 MeV from thermal neutron capture on nickel. It is placed at the center of the ID and delivers 0.004 p.e/PMT on average when the emitted \( \gamma \) hits one of PMTs. More than 99% of observed signal in each PMT are due to single p.e. The charge distribution obtained by this measurement after the relative gain correction and accumulating data from all PMTs is shown in Fig.2.19. In order to minimize the effect of dark hits, off-time (the timing window where the signal is not expected) data is subtracted from on-time (the timing window where the signal is expected) data. The value averaged over the whole pC region is defined as the conversion factor of single p.e. The conversion factors are measured to be 2.055, 2.293, 2.247 and 2.658 pC with respect to SK-I, II, III and IV respectively. This single p.e distribution is also implemented in MC simulations.
2.7. DETECTOR CALIBRATION

Figure 2.18: The normalized relative gain of each PMT.[26]

Figure 2.19: Charge distribution of nickel source calibration after the relative gain correction and accumulating data from all PMTs.[26]
2.7.4 Relative difference of QE

As described in equation (2.6), observed hit probability for each PMT is proportional to the value of QE if the intensity of the light source is low enough. While the number of hits in each PMT can be measured, it is difficult to know how many photons exactly reach the PMT. MC simulation is used to know the number of photons arriving at the PMT. In this measurement, a nickel source measurement described in section 2.7.3 is used. The hit probability in each PMT is corrected by the following function.

\[
N_{\text{obs}}(i) \times \frac{R(i)^2}{a(\theta(i))}
\]

Here \( i \) is the index of ID PMTs, \( R(i) \) is the distance between the nickel source and \( i \)-th PMT position, \( a(\theta(i)) \) is acceptance as a function of photon incident angle \( \theta(i) \). The hit probability after this correction as a function of PMT position is shown in Fig. 2.20. Even after the correction (2.8), position dependence still remains due to the reflection from neighboring surface or scattering and absorption by the water. Considering these position dependence, remaining differences are due to QE differences of individual PMTs. The quantities of QE differences are tabulated and used in MC simulation.

2.7.5 Light absorption and scattering in water

In order to extract light absorption and scattering coefficients as a function of wavelength, collimated laser beam is injected into the tank and the observed timing and spatial distributions are compared with MC. Schematic view of the calibration setup is shown in Fig. 2.21. Fig. 2.22 shows the data and MC results for typical PMT hit timing distribution after time-of-flight is subtracted. TOF is now based on the distance between the PMT position and target of the laser at the bottom. The left region consists of scattered photons and used to tune the symmetric and asymmetric scattering parameters and absorption parameter. The right peak represents
2.7. DETECTOR CALIBRATION

Figure 2.21: Schematic view of the calibration setup for light absorption and scattering. [26]

the photons reflected by the bottom PMT and black sheets, which is used to tune the reflection parameter (Section 2.7.6). Total number of scattered photons and the shape of time distribution in the left region are compared between data and MC. Various MC samples with different parameters are generated and the one which minimizes the $\chi^2$ value for difference between data and MC is selected finally. The number of photons in the water is described as follow.

$$I(\lambda) = I_0(\lambda) \exp\left(-\frac{l}{L(\lambda)}\right)$$

(2.9)

Here $\lambda$, $l$ are the wavelength and travel length of the light, $I_0(\lambda)$ is the initial intensity, $I(\lambda)$ is the intensity at $l$ and $L(\lambda)$ is total attenuation length caused by absorption and scattering. Now $L(\lambda)$ is called as "water transparency" and defined as follow.

$$L(\lambda) = \frac{1}{\alpha_{\text{abs}}(\lambda) + \alpha_{\text{sym}}(\lambda) + \alpha_{\text{asym}}(\lambda)}$$

(2.10)

Here $\alpha_{\text{abs}}(\lambda), \alpha_{\text{sym}}(\lambda)$ and $\alpha_{\text{asym}}(\lambda)$ is the absorption, symmetric and asymmetric scattering parameters respectively. The absorption parameter $\alpha_{\text{abs}}$ (m$^{-1}$) as the function of wavelength $\lambda$ (nm) is determined empirically using the laser beam data as follow.

$$\alpha_{\text{abs}}(\lambda) = P_0 \times \frac{P_1}{\lambda^4} + C$$

(2.11)

$$C = P_0 \times P_2 \times (\lambda/500)^{P_3}$$

(2.12)

Here $C$ is the amplitude based on the experimental data and $P_0 - P_3$ are the fitting parameters.
Figure 2.22: Typical TOF subtracted timing distribution in the measurement for the calibration of light absorption, scattering and reflection. Black circle is the data and red line is the MC of best tuned. Both distributions are normalized by total observed p.e.s. The top plot is for the SK top wall and the second to bottom plots corresponds to the SK barrel wall regions shown in Fig. 2.21 [26].
2.7. DETECTOR CALIBRATION

The angular distribution of the scattered photons is divided into two components, "symmetric" and "asymmetric". The symmetric component consists of Rayleigh and symmetric Mie scattering described by $1 + \cos \theta^2$ where $\theta$ is the scattered angle of photon. The asymmetric component consists of forward Mie scattering and the scattering probability increases linearly from $\cos \theta = 0$ to 1, and no scattering occurs for $\cos \theta < 0$. The Mie scattering has a forward peaked distribution for the particle size greater than wavelength of the light. The asymmetric component is simply approximated because the particle sizes are unknown. The symmetric and asymmetric factors $\alpha_{\text{sym}}$ and $\alpha_{\text{asym}}$ are empirically determined as follow.

$$\alpha_{\text{sym}}(\lambda) = \frac{P_4}{\lambda^4} \times \left( 1.0 + \frac{P_5}{\lambda^2} \right)$$ (2.13)

$$\alpha_{\text{asym}}(\lambda) = P_6 \times \left( 1.0 + \frac{P_7}{\lambda^4} \times (\lambda - P_8)^2 \right)$$ (2.14)

Here $P_4 - P_8$ are the fitting parameters. Typical fitting results for $\alpha_{\text{abs}}(\lambda), \alpha_{\text{sym}}(\lambda), \alpha_{\text{asym}}(\lambda)$ and the total attenuation length $L(\lambda)$ is shown in Fig. 2.23.

![Plot](image.png)

Figure 2.23: Typical fitting results for attenuation length in water. The points show the data and the line is the fit function of the points[26]. Each data point is obtained by fitting MC to data in left region of Fig. 2.22.
2.7.6 Light reflection at PMTs and black sheets

Light reflection at PMT surface is tuned by using data and MC in right peak of Fig. 2.22. Four layers of material (refractive index) are taken into account from the surface to the inside of the PMT: water (1.33), glass (1.472 + 3670/λ²), bialkali (n_{real} + i · n_{img})[32] and vacuum (1.0). n_{real} and n_{img} are the real and imaginary parts of complex refractive index. Best fit value of n_{real} and n_{img} are obtained finally.

Cherenkov photons are also reflected and absorbed on the black sheets. The reflectivity of the black sheets are measured by using laser light injected to black sheets in the tank. Schematic view of this measurement is shown in Fig. 2.24. The reflected charge (Q_{scat}) is measured at three incident angles (30°, 45° and 60°) with three wavelengths (337 nm, 400 nm and 420 nm). As the reference of the Q_{scat}, the direct charge (Q_{direct}) without black sheets is also measured and the ratio \( R = \frac{Q_{scat}}{Q_{direct}} \) is used for the tuning of the reflectivity.

2.7.7 PMT timing

Understanding the timing response of each readout channel including PMTs and readout electronics is important for a precise reconstruction of the vertex and track direction. The response time of readout channel depends on the transit time of the PMTs, length of PMT signal cables and processing time of readout electronics. In addition to these issues, the response time of readout channel (the timing of discriminator output) also depends on the pulse height of PMTs, because the rise time of larger pulse is faster than smaller one. This is known as “time-walk” effect. The total process time and time-walk effect are calibrated by injecting fast light pulse into PMTs and changing the intensity of the light. Schematic view of the timing calibration system is shown in Fig. 2.25. Nitrogen laser is used as the light source of the calibration which emits fast pulsing light of 0.4 ns FWHM and 337 nm wavelength. The laser output is monitored.
Figure 2.25: Schematic view of the timing calibration system. [26]
by a 2-inch PMT to know the time of laser light injection. The wavelength of the laser light is shifted to 398 nm by a dye, where the response with Cherenkov spectrum of PMTs is almost maximum. The pulse width of the dye is 0.2 ns FWHM. The filtered light is injected into a diffuser ball located in the center of the tank by optical fiber to emit isotropic light. Timing calibration is conducted based on a two dimensional map of timing and pulse height (charge) so called “TQ distributions”. Fig. 2.26 shows a typical TQ distribution for one readout channel. The calibration constants so called ”TQ map” is obtained by fitting the TQ distribution with a polynomial function with respect to each channel. The TQ map is used to correct the hit timing.

### 2.7.8 OD PMTs

Precision of the OD calibration is not as important as ID calibration. Accuracy of 10-20% for charge reconstruction and 5-10 ns for timing is sufficient for physics analysis. In order to determine the charge in pC corresponding to single p.e, OD hits outside the trigger window are used. Since the hits preceding the trigger time have a high probability to be single p.e hits, such events are accumulated and the mean value is taken as pC per single p.e. The charge response per single p.e is validated by nitrogen dye laser flushed at very low light levels. OD timing information is only used for selection of OD hits within the time window around the trigger time. The goal of OD time calibration is to confirm that timing offset between ID and OD and the relative timing offset of each OD PMT are sufficiently small. These are measured with a laser system and cosmic ray muons.

### 2.7.9 Energy Scale

Understanding the detector energy scale precisely is important for proton decay analysis. The Michel electrons, stopping muons and neutral pion samples are used for this.
Since the energy spectrum of Michel electrons is already well known, it is useful for energy scale check. Michel electrons produced by stopping muon are used to determine the absolute energy scale in the range of a few tens of MeV.

The Cherenkov angle is described by a function of the momentum as equation (2.1). The stopping muon sample is useful for checking this correlation between the Cherenkov angle and momentum of muons. However, since the Cherenkov angle soon reaches the limit ($\sim 42^\circ$) as the momentum increases, this check is useful for lower energy muons. Therefore, stopping muons with a momentum below 400 MeV are collected for this check. Then measured momentum and calculated momentum expected from measured Cherenkov angle are compared.

For high energy muons, the correlation between the stopping range of muon track in the detector and the momentum is used for energy scale check. The range of muon track is obtained by the distance between the muon entering point and the vertex of Michel electron. Neutral current interaction of atmospheric neutrinos with the nucleus often generates neutral pions $\pi^0$. Such events can be identified easily and used for absolute energy scale calibration. The mass of $\pi^0$ is reconstructed by 2 e-like rings assuming both of rings are created from $\gamma$.

In order to estimate the systematic uncertainty for the energy scale, the ratio of values between data and MC in each calibration method is used as shown in Fig. 2.27. The largest difference in all calibration methods is used for the error value for energy scale with respect to each SK period. The error values are estimated to be 3.3%, 2.8%, 2.4% and 2.1% for SK-I, II, III and IV, respectively.

![Figure 2.27: Absolute energy scale measurements for each SK period](image)

The detector uniformity of the energy scale is also estimated by using decay electron samples, because the vertex and momentum distribution of the decay electron are almost uniform. In order to consider the muon polarization, the electron with the direction perpendicular to the parent muon direction ($|\cos \Theta_{e\mu}| < 0.25$) is only used for this measurement. The difference of the electron momentum between data and MC as a function of the electron direction is checked. Then the largest deviation is used as the detector uniformity error with respect to each SK period. The error values correspond to 0.6%, 0.6%, 1.3% and 0.5% for SK-I, II, III and IV, respectively.
Chapter 3
Monte Carlo Simulations

Signal and background samples are created by using Monte Carlo method. For the signal samples, each mode of proton decay into three charged leptons is created. For the background sample, only atmospheric neutrino events are considered as other backgrounds become negligible by applying the event reduction described in Chapter 4. Finally, Geant3[105] based detector simulation is performed to both signal and background samples.

3.1 Signal

The source of proton decay is protons contained in H$_2$O molecules. One H$_2$O molecule has 10 protons: 2 protons from hydrogen and 8 protons from oxygen. The protons in hydrogen so called "free proton" do not interact with other nuclei. In case of free proton, mass of free proton is exactly 938.27 MeV and momentum is 0. On the other hand, the protons in oxygen so called "bound proton" interact with other nucleus. In case of bound proton, the mass of proton is smaller than its exact mass. Also momentum is not 0 due to some effects described later. In the simulation, phase space is only considered. Any correlations between charged leptons are not considered. Mass and momentum of the initial protons are distributed to decayed three leptons. The true momentum of 3 electrons for free protons in $p \rightarrow e^+e^+e^-$ is shown in Fig.3.1. Since the signal is three body decay, momentum of each charged lepton distributes widely. In case of free proton, maximum limit of the charged lepton momentum corresponds to the half of proton mass (469 MeV).

In case of bound proton, mainly three effects are considered: nuclear binding energy in $^{16}$O, Fermi motion and the correlated decay. For the nuclear binding energy, two bound states p-state and s-state are considered. These two types of binding energy are simulated as Gaussian distribution. Each mean and $\sigma$ of the binding energy is (39.0 MeV, 10.2 MeV) for s-state and (15.5 MeV, 3.82 MeV) for p-state. The ratio of s-state is 22.5%, p-state is 67.5% (45% for P(3/2) and 22.5% for P(1/2)) and the correlated decay is 10%. The binding energy is subtracted from proton mass in the simulation. The proton momentum from Fermi motion is simulated based on electron-$^{12}$C scattering data[36]. The invariant mass and momentum distribution of each bound state are shown in Fig.3.2.

The bound proton sometimes correlates with surrounding nucleus during its decay and this effect is called as correlated decay[37]. Due to this effect, the kinematics of the proton is changed because of the momentum transfer to the surrounding nucleons. The correlated decay is considered to happen with 10% probability and produce broad pseudo peak at lower mass in
CHAPTER 3. MONTE CARLO SIMULATIONS

Figure 3.1: True momentum of charged leptons for free proton in $p \rightarrow e^+e^+e^-$. Each histogram corresponds to the lepton with minimum(red), middle(green) and maximum(blue) momentum. The error bars correspond to the statistics error. There is kinematic edge of half of proton mass in maximum momentum lepton.

proton mass distribution. The invariant mass and momentum distribution of the proton with correlated decay are also shown in Fig.3.2.

In this analysis, all patterns of electron, muon and their anti-particles are considered as signal. Any correlations between charged leptons are not considered in the simulation. Therefore following six types of signal samples are created.

\begin{itemize}
  \item $p \rightarrow e^+e^+e^-$
  \item $p \rightarrow \mu^+e^+e^-$
  \item $p \rightarrow \mu^-e^+e^+$
  \item $p \rightarrow \mu^+\mu^+e^-$
  \item $p \rightarrow \mu^+\mu^-e^+$
  \item $p \rightarrow \mu^+\mu^+\mu^-$
\end{itemize}

3.2 Background

Only atmospheric neutrino events are only considered for the background in this analysis since other non-neutrino backgrounds are negligibly small. This sample is created by Monte Carlo simulation composed of three parts: neutrino flux, neutrino interaction with water and particle tracking in the detector.
Figure 3.2: True mass(left) and momentum(right) distribution of proton with respect to each state.

3.2. BACKGROUND

3.2.1 Neutrino flux

The flux of atmospheric neutrino at the Super-Kamiokande detector is calculated by the model of M.Honda et.al.[38][39].

In order to reproduce real hadronic interactions accurately, DPMJET-III[40] is used for the simulation. The original DPMJET-III is modified by changing so called Z-factor to reproduce atmospheric muon and neutrino flux at high energy better[41]. This modified DMPJET-III can reproduce observed atmospheric neutrino flux above 1 GeV accurately at sea level and mountain altitude[42]. However, since neutrino flux below 1 GeV cannot be reproduced well by DPMJET-III, another interaction model called JAM which is used in PHITS (Particle and Heavy-Ion Transport code System)[43] is used. Calculated results of DPMJET-III and JAM are compared with experimental results of HARP collaboration which released detailed studies of proton hadronic interactions on thin N\textsubscript{2} and O\textsubscript{2} targets at 12 GeV/\textit{c}[44] as shown in Fig.3.3. As the JAM has an upper limit on projective energy, for the energy below 32 GeV JAM is used and for the energy above 32 GeV DPMJET-III is used in the simulation.

The primary flux model is based on the data of cosmic ray flux measurements by the BESS[99][100] and AMS[97][98] experiments. For the model of the atmosphere and the geomagnetic field, US-standard '76[101] and IGRF2005[102] are used respectively. The combination of 3-dimensional and 1-dimensional calculation is used to get the atmospheric neutrino flux averaged over all azimuthal angles. For the 3-dimensional calculation, the Earth is assumed to be sphere with the radius \( R_e = 6378.180 \) km. In addition to this sphere, 3 more spheres: injection, simulation and escape spheres are considered. The injection sphere is defined as \( R_{inj} = R_e + 100 \) km, the simulation sphere and the escape sphere are defined as \( R_{sim} = R_{esc} = 10 \times R_e \). First, cosmic rays are generated on the surface of injection sphere uniformly toward the inward direction followed by given primary cosmic ray spectra. Before the simulation in the atmosphere, generated cosmic rays are tested whether they could pass the geomagnetic barrier by checking the motion in the inverse time direction. If the cosmic ray reaches the escape sphere without touching injection sphere again in the inverse time direction, it’s determined to pass the geomagnetic barrier in the normal time direction. The propagation of the cosmic ray in the atmosphere is simulated in the space between the surface of the Earth and the simulation sphere. When the cosmic rays enter the Earth, the events are discarded. Since the detector is very small compared to the size of Earth, finite size "virtual
Figure 3.3: Comparison of interaction models and experimental results of HARP (data points). The solid line is JAM and the dashed line is DPMJET-III. [39]
3.2. **BACKGROUND**

detector” is used in 3-dimensional calculation with some corrections. 3-dimensional calculation is used for the neutrino energy below 32 GeV, and 1-dimensional calculation is used above 32 GeV. Calculated atmospheric neutrino flux averaged over all directions as a function of neutrino energy at Kamioka compared with other flux models is shown in Fig. 3.4. The zenith angle dependences averaged over all azimuthal angles for Kamioka at 3 ranges of neutrino energy are also shown in Fig. 3.5.

![Figure 3.4: The atmospheric neutrino flux for Kamioka averaged over all directions (left) and the flux ratio (right) compared with other flux model, HKKM06 (previous calculation of this model), model by Bartol group[45][46] and model by FLUCA group[47].](image)

**3.2.2 Neutrino interaction**

The interaction of atmospheric neutrino with hydrogen or oxygen nucleus in water is simulated by NEUT program[48]. This program covers a wide energy range of neutrino from several tens of MeV to hundreds of TeV. In NEUT, following neutrino interactions are considered.

- CC/NC (quasi-) elastic scattering ($\nu N \rightarrow lN'$)
- CC/NC single $\pi$ production ($\nu N \rightarrow lN'\pi$)
- CC/NS single $\gamma$ production ($\nu N \rightarrow lN'\gamma$)
- CC/NC single $K$ production ($\nu N \rightarrow l\Lambda K$)
- CC/NC single $\eta$ production ($\nu N \rightarrow lN'\eta$)
- CC/NC deep inelastic scattering ($\nu N \rightarrow lN'\text{hadrons}$)
- CC/NC coherent $\pi$ production ($\nu^{16}\text{O} \rightarrow l\pi X$)

Here $N, N'$ are proton or neutron, $l$ is the lepton and $X$ is the remaining nucleon.
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Figure 3.5: The atmospheric neutrino flux by Honda model as a function of zenith angle for Kamioka averaged over all azimuthal angles. For the neutrino energy of 0.32 GeV (left), 1.0 GeV(center) and 3.2 GeV(right). $\theta$ is the arrival direction of neutrinos. $\cos \theta = 1$ corresponds to vertically downward going neutrinos, and $\cos \theta = -1$ corresponds to vertically upward going neutrinos. Neutrino oscillation effect is not considered.

**Elastic and quasi-elastic scattering**

The cross section for the neutrino nucleon CC quasi-elastic (CCQE) scattering is evaluated based on the Llewellyn-Smith model\[^{50}\]. In order to calculate the cross section for nucleons in the nucleus, the relativistic Fermi gas model by Smith and Monitz \[^{51}\] is used in NEUT. Both models use the vector and axial-vector form factors of nucleon. The axial-vector coupling constant is set to $1.2 \text{GeV/}c^2$ to have good agreement with the recent neutrino experiments \[^{52}\][\(^{53}\)]. Comparison of calculated CCQE cross section and observed cross section in some experiments is shown in Fig.3.6.

Figure 3.6: Cross section of neutrino CCQE scattering as the function of neutrino energy \[^{48}\]. Solid line is calculated cross section for free proton and dashed line is for proton in Oxygen. Data points corresponds to ANL\[^{54}\], Gargamelle \[^{55}\][\(^{56}\)], BNL\[^{57}\], Serpukhov\[^{58}\] and SKAT\[^{59}\].
The cross section for NC elastic scattering is estimated by using following relations [60].

\begin{align}
\sigma(\nu p \rightarrow \nu p) &= 0.153 \times \sigma(\nu n \rightarrow e^- p) \\
\sigma(\bar{\nu} p \rightarrow \bar{\nu} p) &= 0.218 \times \sigma(\bar{\nu} p \rightarrow e^+ n) \\
\sigma(\nu n \rightarrow \nu n) &= 1.5 \times \sigma(\nu p \rightarrow \nu p) \\
\sigma(\bar{\nu} n \rightarrow \bar{\nu} n) &= 1.0 \times \sigma(\bar{\nu} p \rightarrow \bar{\nu} p)
\end{align}

(3.1) to (3.4)

In this process, \( \pi^0 \) or \( \pi^\pm \) are sometimes additionally generated. \( \pi^0 \) decays to \( 2 \gamma \) and emitted Cherenkov ring from \( \gamma \) is basically identified as electron ring. Then CCQE events with electron (muon) created from neutrino by charged interaction and \( 2 \gamma \) from \( \pi^0 \) can have similar final states as \( p \rightarrow e^+ e^- e^- (p \rightarrow \mu^+ e^- e^-) \) signals. On the other hand, Cherenkov ring from \( \pi^\pm \) is often mis-identified as a muon ring. Then CCQE events with electron or muon from neutrino and some \( \pi^\pm \) can have similar final states as \( p \rightarrow e^+ \mu^- \) or \( p \rightarrow \mu^+ \mu^- e^- \) signals.

**Single \( \pi, \gamma, K \) and \( \eta \) production**

The process of single \( \pi \) production is separated to following 2 steps in the simulation by using Rein and Sehgal’s method [61].

\[
\nu + N \rightarrow l + N^* \rightarrow \pi + N'
\]

(3.5)

Here \( N, N' \) are nucleons and \( N^* \) is a baryon resonance. In order to calculate cross section, amplitude of each resonance production and the decay probability of each resonance into one \( \pi \) and one nucleon are taken into account. 18 resonances below 2 GeV/\( c^2 \) and axial-vector mass of 1.2 GeV/\( c^2 \) are considered. Comparison of calculated cross section and experimental results are shown in Fig.3.7. The cross section of single \( \gamma, K \) and \( \eta \) productions can be calculated by changing the decay probability of the resonances.

Single \( \pi \) production can have similar final states as each signal process due to the same reasons as CCQE events associated with pion. Single \( \eta \) production can also have similar final states as \( p \rightarrow e^+ e^- e^- \) or \( p \rightarrow \mu^+ e^- e^- \) signals because \( \eta \) often decays to \( 2 \gamma \).

**Deep inelastic scattering**

In order to calculate the cross section, following function is integrated in the range of \( W > 1.3 \text{ GeV}/c^2 \), here \( W \) is the invariant mass of the hadronic system.

\[
\frac{d^2 \sigma}{dx dy} = \frac{G_F^2 M_N E_\nu}{\pi} \left[(1 - y + \frac{1}{2} y^2 + C_1) F_2(x, q^2) \pm y (1 - \frac{1}{2} y + C_2) x F_3(x, q^2) \right] \\
C_1 = \frac{y M_l^2}{4 M_N E_\nu x} - \frac{x y M_N}{2 E_\nu} - \frac{M_l^2}{4 E_\nu^2} - \frac{M_l^2}{2 M_N E_\nu x} \\
C_2 = -\frac{M_l^2}{4 M_N E_\nu x}
\]

(3.6) to (3.8)

Now \( x = -q^2/(2M(E_\nu - E_i)) \), \( y = (E_\nu - E_i)/E_\nu \), \( M_N \) is the mass of nucleon, \( M_l \) is the mass of lepton, \( E_\nu \) and \( E_i \) are the energy of incoming neutrino and outgoing lepton in the laboratory frame, respectively. Here \( F_2 \) and \( x F_3 \) are nucleon structure functions taken from parton distribution function GRV98[62] with corrections proposed by Bodek and Yang [63]. Comparison of
CHAPTER 3. MONTE CARLO SIMULATIONS

Figure 3.7: Comparison of calculated cross section (solid line) and experimental results (black points)\cite{48}.

Figure 3.8: Calculated cross sections and experimental results for total CC cross sections\cite{48}. Upper 4 lines corresponds to neutrino and lower 4 lines corresponds to anti-neutrino. Solid line is calculated cross section with each PDF and data points shows some experimental results as below. CCFR90\cite{64}, CDHSW87\cite{65}, GGMPS79\cite{66}, GHARM88\cite{67}, BNL80\cite{68}, CRS80\cite{69}, BEBCWBB79\cite{70}, IHEP-JINR96\cite{71}, IHEP-TEP79\cite{72}, CCFRR84\cite{73}, SKAT\cite{74}.
calculated cross sections and experimental results for total CC cross sections including CCQE scattering, single meson productions and deep inelastic interactions are shown in Fig. 3.8.

In this process, multiple pions are often generated and the events can have similar background states as each signal. Furthermore, proton is also generated or scattered by DIS. Such events can have similar final states as signal with muon because the Cherenkov ring from proton is often identified as a muon ring.

In order to calculate the cross section for NC multi π production, following relations are used which are estimated by the experimental results[75][76].

\[
\begin{align*}
\frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\nu N \rightarrow \mu^- X)} &= 0.26 \quad \text{(for } E_\nu < 3 \text{ GeV)} \\
\frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\nu N \rightarrow \mu^- X)} &= 0.26 + 0.04 \times \frac{E_\nu - 3}{3} \quad \text{(for } 3 < E_\nu < 6 \text{ GeV)} \\
\frac{\sigma(\nu N \rightarrow \mu^- X)}{\sigma(\nu N \rightarrow \mu^- X)} &= 0.30 \quad \text{(for } E_\nu > 6 \text{ GeV)} \\
\frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} X)}{\sigma(\nu N \rightarrow \mu^- X)} &= 0.39 \quad \text{(for } E_\nu < 3 \text{ GeV)} \\
\frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} X)}{\sigma(\bar{\nu} N \rightarrow \mu^- X)} &= 0.39 - 0.02 \times \frac{E_\nu - 3}{3} \quad \text{(for } 3 < E_\nu < 6 \text{ GeV)} \\
\frac{\sigma(\bar{\nu} N \rightarrow \mu^- X)}{\sigma(\bar{\nu} N \rightarrow \mu^- X)} &= 0.37 \quad \text{(for } E_\nu > 6 \text{ GeV)}
\end{align*}
\]

Coherent π production

The differential cross section for NC coherent π production is described as below. [77]

\[
\begin{align*}
\frac{d^3\sigma}{dQ^2 dy dt} &= \frac{G^2 M_N}{2\pi^2} f_\pi^2 A^2 E_\nu (1 - y) \frac{1}{16\pi} [\sigma_{\text{tot}}]_N^2 \times (1 + r^2) \left( \frac{m_A^2}{m_\pi^2 + Q^2} \right) e^{-b|t|} F_{\text{abs}}(3.15) \\
r &= \text{Re} f_{\pi N}(0)/\text{Im} f_{\pi N}(0) \\
F_{\text{abs}} &= e^{-(x)/\lambda} \\
\lambda^{-1} &= \sigma_{\text{inel}}^N \rho 
\end{align*}
\]

Here \(Q^2\) is the square of 4-momentum transfer of lepton, \(t\) is the square of 4-momentum transfer to the nucleus, \(m_A\) is the axial vector mass, \(f_\pi = 0.93 m_\pi\), \(b = 80\text{ GeV}^{-2}\), \(G\) is the weak coupling constant, \(M_N\) is the mass of nucleon, \(y = (E_\nu - E_l)/E_\nu\) is the fraction of lepton energy loss, \(E_\nu, E_l\) are energy of neutrino and outgoing lepton and \(A\) is the atomic number of target nucleus. And \(F_{\text{abs}}\) is the factor for pion absorption in nucleus, \(\langle x \rangle\) is the mean path length of pion in Oxygen, \(\rho = \left( \frac{4\pi R^3}{3} \right)^{-1}\) is the nuclear density, \(R\) is the radius of the nuclear, \(\sigma_{\text{tot}}^N, \sigma_{\text{inel}}^N\) are averaged total and inelastic pion-nucleon cross sections obtained from the experimental results and fitted results of Rein and Sehgal [61].

In order to calculate the cross section for CC coherent π production, the correlation factor of the lepton mass (COR) as below is additionally taken into account [78].

\[
\begin{align*}
\text{COR} &= 2 \left( 1 - \frac{1}{2} \frac{Q_{\min}^2}{Q^2 + m_\pi^2} \right)^2 + \frac{1}{4} y \frac{Q_{\min}^2 (Q^2 - Q_{\min}^2)}{(Q^2 + m_\pi^2)^2} \\
Q_{\min}^2 &= m_\pi^2 \frac{y}{1 - y}
\end{align*}
\]
CHAPTER 3. MONTE CARLO SIMULATIONS

Figure 3.9: The cross section of CC (left) and NC(right) coherent $\pi$ production as the function of neutrino energy\[48\]. Solid, dotted and dash-dotted lines correspond to the calculated cross section with the model by Rein and Sehgal\[78\], Kartavsev et al. \[79\] and Alvarez-Russo et al. \[80\]. The filled box and circle shows the upper limit from the measurements by the K2K \[81\] and SciBooNE \[82\] experiments respectively. The open box, circle and cross correspond to the observed results by MiniBooNE \[83\], Aachen-Padova \[84\] and Gargamelle \[85\] experiments respectively.

Comparison of calculated cross section and experimental results is shown in Fig.3.9.

3.2.3 Nuclear effects

Since the hadrons generated by neutrino interactions in nucleus can interact within the nucleus, observed particles are not always same as the ones initially generated. In order to take into account this effect, the interactions of $\pi$, $K$, $\eta$ and nucleon in the target nucleus are simulated in NEUT by using the cascade model \[48\][\[49\]. In the cascade model, first particle is moved by a unit length starting from generated point of hadrons. Then the interaction types (including no interaction) are determined by following the probabilities of each interaction. This procedure continues until one of the interactions happens or the particle exits the nucleus.

The pion production is one of the dominant modes in neutrino interactions and the cross section of pion interaction in the nucleus is also quite large. In NEUT, inelastic scattering, charge exchange and absorption are considered for pion interactions. Particle production is also considered for high energy pions. The mean free path (MFP) of various internuclear mechanisms are calculated by Delta-hole model\[91\] for low energy pion ($p_\pi < 500$ MeV/$c$). In this model, Fermi motion of the nucleon in the nucleus and Pauli blocking effect are taken into account. Additionally MFPs for absorption and scattering are tuned so that calculated cross sections agree with $^{12}$C scattering data as shown in Fig.3.10.

For high energy pions ($p_\pi > 500$ MeV/$c$), experimental data of free $\pi^\pm$ cross section is used. The cross section for quasi-elastic scattering $\sigma_{QE}^{\text{free}}$, for hadron production $\sigma_{\text{had}}^{\text{free}}$ and for single charge exchange $\sigma_{\text{SCX}}^{\text{free}}$ are calculated by SAID partial wave analysis fit\[92\] shown in Fig.3.11. Then the cross sections for $\pi$-nucleus scattering are calculated as $\sigma_{QE} = \sigma_{QE}^{\text{free}}$, $\sigma_{\text{had}} = \sigma_{\text{had}}^{\text{free}}$ and $\sigma_{\text{SCX}} = \sigma_{\text{SCX}}^{\text{free}}$. Finally both $\sigma_{QE}$ and $\sigma_{\text{SCX}}$ are scaled by factor 1.8 which was obtained by $^{12}$C QE scattering\[86\] and SCX\[87\] data. Simulated results show a good agreement with data.

The interaction of $K$ in nucleus is also simulated by using similar method as pion interac-
3.2. BACKGROUND

Figure 3.10: The cross section of $\pi^+^{12}\text{C}$ (left) and $\pi^-^{12}\text{C}$ scattering as a function of pion energy\cite{49}. Solid lines are simulated cross section and data points are experimental results\cite{86,87,88,89,90}.

Figure 3.11: The cross section of free $\pi^\pm p$ scattering\cite{49}. Data from PDG and fit by SAID.
tion but refers to experimental results of $KN$ and $\bar{K}N$ scattering[93]. For the interaction of $\eta$, absorption of $\eta (\eta N \rightarrow N^* \rightarrow \pi (\pi) N' )$ is considered. The main resonances related to this process are $N(1540)$, $N(1650)$ and the cross section for $\eta N \rightarrow N^* \rightarrow \pi (\pi) N'$ is calculated as follows[94].

$$\sigma = \frac{\pi}{k^2} \left( J + \frac{1}{2} \right) \frac{\Gamma_{\eta N} \Gamma_{\pi (\pi) N}}{(W - M^*)^2 + \Gamma_{\text{tot}}/4}$$

(3.21)

Here $k$ is the momentum of $\eta$ in center of mass system, $J$, $\Gamma$ and $M^*$ are spin, decay width and mass of the resonance, $W$ is the invariant mass of the $\eta N$ system. The nucleon re-scattering in the nucleus is simulated by almost the same way as pion interactions, considering elastic scattering and a single or two delta(s) production for the pion production. The interaction probabilities are extracted from nucleon-nucleon scattering experiments[95]. The delta production is simulated based on the isobar production model[96].

### 3.2.4 Neutron generation

Neutrons are generated in primary interactions (10%), hadron and meson interactions in the nucleus (17%), and interactions of hadrons in water (73%). For hadron-water interactions at energies below 3.5 GeV, Nucleon-Meson-Transport-Code (NMTC) is used, which is based on the Bertini internuclear hadronic cascade model[103]. For low energy ($<20$ MeV) neutron propagation, the Monte Carlo Ionization Chamber Analysis Package (MICAP) code[104] is used. Thermalized neutrons are simulated until they captured by hydrogen and 2.2 MeV gamma ray is emitted.

### 3.3 Detector

The particle propagation, Cherenkov radiation, propagation of Cherenkov photons in water and PMTs or electronics response are simulated by Geant3[105] based package SKDETSIM.

Cherenkov photons are generated by following the equations (2.1) and (2.4). The group velocity $v_g$ of photons propagated in water is calculated as follows.

$$v_g = \frac{c}{n(\lambda) - \lambda \frac{\partial n(\lambda)}{\partial \lambda}}$$

(3.22)

Here $c$ is the speed of light in vacuum, $\lambda$ is light wave length and $n(\lambda)$ is the refractive index as a function of wave length. The process of absorption and scattering is taken into account for photon propagation in water. For the photon with shorter wavelength ($\lambda \lesssim 450$ nm), Rayleigh scattering is the dominant interaction which is caused by small particles ($r \ll \lambda$, where $r$ is the radius of the particles). For longer wavelength ($\lambda \gtrsim 450$ nm) absorption by $\text{H}_2\text{O}$ molecule is the dominant process. The probability of absorption as a function of wavelength is obtained by experimental data (2.7.5). In addition to these processes, Mie scattering which is caused by relatively large particle ($r \gg \lambda$) is considered in the simulation. The attenuation coefficients of Rayleigh and Mie scattering are tuned to be consistent with laser measurements, and the parameters are confirmed by cosmic ray muon events. The reflection or absorption at the black sheets or the reflection on the PMTs surface are also simulated by using measured results. The PMT timing is smeared by the measured timing resolution.

For the hadronic interaction, GCALOR[106] and NEUT pion cascade model are used for pion momentum above and below 500 MeV/c respectively.
Chapter 4

Event Preselection

In this analysis, reconstructed vertex and all visible particles are required to be inside the Inner Fiducial Volume (FV). FV is defined as the area 2 m inside the top, bottom and barrel wall of the ID, which corresponds to the mass of 22.5 kton. These events are called as fully contained (FC) events. Most of FC events consist of atmospheric neutrino events. In SK, there are three types of atmospheric neutrino events: FC, partially contained (PC) and upward going muon (UPMU) events. PC events require that reconstructed vertex is inside the Inner FV but any of charged particles go outside. UPMU events consist of high energy muons generated by upward going neutrinos interacted with the rock around the detector. Schematic view for each type of atmospheric neutrino in SK is shown in Fig. 4.1. In order to make data set of FC events,

![Schematic view of atmospheric neutrino types in SK.](image)

Figure 4.1: Schematic view of atmospheric neutrino types in SK.

we need to remove background events caused by cosmic ray muons, radioactivities, electronic noise and so on. This event preselection consists of following 5 steps.

- Low energy event rejection
- Electric noise event rejection
- Specific background rejection
- Flasher event rejection
• Specific background rejection II

In addition to these reduction steps, real time data check by human eyes is performed after the specific background rejection I.

4.1 Low Energy Event Rejection

Trigger rate in SK is $10^6$ events a day mainly fired by low energy events from radioactive background or cosmic ray muon events. In order to reject such low energy events we require following 2 cuts.

$$PE_{300} \geq 200 \text{ p.e.s (100 p.e.s for SK-II)}$$  \hspace{1cm} (4.1)

Here $PE_{300}$ is the maximum number of photoelectrons (p.e.s) observed by ID PMT in sliding 300 ns time window. The threshold roughly equals to an electron momentum of 22 MeV/c. Most of radioactive background, low energy Michel electron from cosmic ray muons and solar neutrino events are rejected after this cut. Since the number of PMT in SK-II is half compared to other periods, the threshold is also half in SK-II.

$$NHITA_{800} \leq 50 \text{ (55 for SK-IV) or OD trigger is off}$$ \hspace{1cm} (4.2)

Here $NHITA_{800}$ is the number of OD PMT hits in the time window of [-400 ns, +400 ns]. This cut rejects cosmic ray muon, PC and UPMU events. The threshold is different in SK-IV because OD electronics and PMT gains are changed in the period. After these cuts, event rate is decreased to $10^3$ events a day.

4.2 Electric Noise Event Rejection

Events satisfying the following two cuts are kept in this step.

$$PE_{\text{max}} / PE_{300} \leq 0.5$$ \hspace{1cm} (4.3)

Here $PE_{\text{max}}$ is the highest observed p.e.s of single PMT in all ID PMTs. This cut can reject electrical noise events which tend to have one larger PMT hit.

$$NHITA_{800} \leq 25 \text{ (30 for SK-IV) or } PE_{t\text{ot}} \geq 100,000 \text{(50,000 for SK-II)}$$ \hspace{1cm} (4.4)

Here $PE_{t\text{ot}}$ is the total observed p.e.s in the ID. This is a more stringent cut for OD PMT hit than First Reduction. The cut for total p.e.s can keep very high energy FC events which have light leakage in OD area. After these cuts, event rate is decreased to the order of 100 events a day.

4.3 Specific Background Rejection I

This step rejects some specific background like hard muons, through going muons, stopping muons, cable hole muons, coincidence muons, flasher and low energy events. After this step, event rate is decreased to about 40 events a day.
4.3. SPECIFIC BACKGROUND REJECTION I

4.3.1 Hard muon
Cosmic ray muons of energy $\geq 1$ TeV is defined as hard muons. Such hard muons are rejected by the following cut.

$$NHITA_{500} \leq 40$$  \hspace{1cm} (4.5)

Here $NHITA_{500}$ is the number of observed OD p.e.s in sliding 500 ns time window.

4.3.2 Through going muon
The cosmic ray muons which penetrate the SK tank are called as the through going muons. Through going muons deposit a lot of energy in ID so we require more than 1,000 PMTs have 230 p.e.s for each. Then through going muon fitter is applied for such events to find muons entering and exiting point. The goodness of the through going muon fit is defined as follow.

$$\text{Goodness} = \frac{1}{\sum_i \frac{1}{\sigma_i^2}} \times \sum_i \frac{1}{\sigma_i^2} \exp \left\{ -\frac{(t_i - T_i)^2}{2(1.5 \times \sigma_i^2)} \right\}$$  \hspace{1cm} (4.6)

Here $t_i$, $T_i$, $\sigma_i$ are the observed hit time, estimated hit time from the muon entering time and its track, time resolution of $i$-th PMT. Events satisfying following two requirements are classified as the through going muon and rejected.

$$\text{Goodness} \geq 0.75$$  \hspace{1cm} (4.7)

$$NHITA_{in} \geq 10 \text{ or } NHITA_{out} \geq 10$$  \hspace{1cm} (4.8)

Here $NHITA_{in}$ ($NHITA_{out}$) is the number of OD PMT hits located in 8 m from the entrance (exit) point within fixed 800 ns time window. Events which pass this cut are rejected.

4.3.3 Stopping muon
The cosmic ray muons which enter the SK tank but stopped inside the ID region are called as a stopping muon. Stopping muon fitter is applied to find muon entering point by using similar way with through going muon fitter. The goodness of the fitter is defined by equation (4.6). Events which satisfy following requirement are rejected as stopping muon.

$$NHITA_{in} \geq 10$$  \hspace{1cm} (4.9)

Additional rejection criterion is defined only for SK-I as follow.

$$NHITA_{in} \geq 5 \text{ and } \text{goodness} > 0.5$$  \hspace{1cm} (4.10)

4.3.4 Cable hole muon
There are twelve cable holes on top of the SK tank for signal and high voltage cables. Since no OD PMTs are installed in the holes, cosmic ray muons can enter through these holes into ID region without OD PMT hits. Such events can be misidentified as FC or PC neutrino events. In order to avoid such misidentified events, veto counters consisting of 2 m $\times$ 2.5 m plastic
scintillation counters were installed into four out of the twelve cable holes in April 1997. We require one veto counter hit and following rejection criteria:

\[ L_{\text{veto}} \leq 4 \text{ m} \]  

Here \( L_{\text{veto}} \) is the distance between the cable hole and reconstructed vertex. Additional cut for the holes without veto counter are introduced in SK-IV as follows.

\[ \text{Goodness of stopping muon fit} \geq 0.4 \]  

\[ \text{PE} \text{total} \geq 4,000 \]  

\[ \text{Traveled in downward direction } (\cos \theta \leq -0.6) \]  

\[ L_{\text{veto}} \leq 2.5 \text{ m} \]

Events which satisfy these rejection criteria are rejected. The probability of rejecting true neutrino events is estimated to be less than 0.01%.

### 4.3.5 Coincidence muon

Cosmic ray muons sometimes enter the tank just after the low energy trigger by coincidence. Such events are called as coincidence muon events and can pass former selection criteria. Following cuts are applied to reject coincidence muon events.

\[ \text{NHITA}_{\text{off}} \leq 20 \]  

Here \( \text{NHITA}_{\text{off}} \) is the number of OD PMT hits in fixed 500 ns timing window from +400 ns to +900 ns after the trigger timing.

\[ \text{PE}_{\text{off}} \leq 5,000 \text{ p.e.s} (2,500 \text{ p.e.s for SK-II}) \]  

Here \( \text{PE}_{\text{off}} \) is the number of p.e.s observed by ID PMTs in fixed 500 ns timing window from +400 ns to +900 ns after the trigger timing.

### 4.3.6 Flasher event

Light is sometimes emitted from PMT itself due to electrical discharge. Such events are called as flasher events and sometimes misidentified as FC neutrino events. Usually hit timing distribution of the flasher events is wider than that of particle induced events. In order to reject such events, following cut is applied.

\[ \text{NMIN}_{100} \leq 20 \text{ (15 for SK-I)} \]  

Here \( \text{NMIN}_{100} \) is the minimum number of ID PMT hit in sliding 100 ns time window from +200 ns to +700 ns after trigger timing.
4.3.7 Low energy event

Remaining low energy events like radioactive background or electrical noise are rejected by the following cut.

\[ \text{NHIT}_{50} \geq 50 \text{ (25 for SK-II)} \]  
(4.19)

Here NHIT\(_{50}\) is the maximum number of ID PMT hits in sliding 50 ns time window. Hits are counted after subtracting the time of flight of observed photons assuming all photons come from one vertex point. The vertex is defined as the position where the time of flight subtracted PMT timing distribution makes sharpest peak. Now \(\text{NHIT}_{50} = 50\) equals to visible energy of 9 MeV and this cut is low enough to keep efficiency.

4.4 Flasher Event Rejection

In this step, additional selection criteria is applied to reduce remaining flasher events. Flasher events are caused by electrical discharge in PMTs, so the events usually have patterns repeated in a cycle of a few hours or days. Pattern matching algorithm is used to find such events as follows.

- ID wall is divided into 1,450 patches of 2 m \(\times\) 2 m.
- Correlation factor \(r\) is calculated as follow.

\[ r = \frac{1}{N} \sum_i \frac{(Q_i^A - \langle Q^A \rangle) \times (Q_i^B - \langle Q^B \rangle)}{\sigma_A \times \sigma_B} \]  
(4.20)

Here \(N\) is the number of the patches, \(Q_i^{A(B)}\) is total observed p.e.s in \(i\)-th patch for event \(A(B)\), \(\langle Q^{A(B)} \rangle\) and \(\sigma_{A(B)}\) are the averaged observed p.e.s of all patches and its standard deviation for event \(A(B)\), respectively.

- Then \(\text{DIST}_{\text{max}}\) is calculated as the distance between PMTs with maximum pulse height in event A and B.
- If \(\text{DIST}_{\text{max}} \leq 75\) cm, offset value is added as \(r = r + 0.15\)
- If \(r\) exceeds the threshold \(r_{\text{th}}\), event A and B are determined to be matched events. \(r_{\text{th}}\) is defined as follows.

\[ r_{\text{th}} = 0.168 \times \log_{10} \left\{ (\text{PE}_{\text{tot}}^A + \text{PE}_{\text{tot}}^B)/2 \right\} + 0.130 \]  
(4.21)

- Above calculations are repeated over 10,000 events around the target event and the number of matched events are counted. Target events with large \(r\) or large number of matched events are rejected finally.

After this step, event rate is decreased to about 18 events a day.

4.5 Specific Background Rejection II

In this step, remaining cosmic ray muons, flasher events, electronic noise and calibration related events are rejected by applying some additional selection criteria.
CHAPTER 4. EVENT PRESELECTION

4.5.1 Stopping muon

Following cuts are applied to reduce remaining stopping muon events.

\[ \text{NHITA}_{\text{in}} \leq 10 \] (4.22)

or

\[ \text{NHITA}_{\text{in}} \leq 5 \text{ if goodness of stopping muon fit } \geq 0.5 \] (4.23)

Definition of the goodness is same as (4.6).

4.5.2 Cable hole muon

In order to reject remaining events of cosmic ray muon which enter the cable holes without veto counter, tighter cut by more precise event reconstruction is necessary. Following rejection criteria is applied only for SK-IV in addition to the selection (4.12), (4.13), (4.14),(4.15).

\[ \text{NHITA}_{\text{APfit}} \geq 4 \] (4.24)

Here \( \text{NHITA}_{\text{APfit}} \) is the maximum number of OD PMT hits within 8 m from the muon entering point fitted by an event reconstruction algorithm APfit (chapter 5), in a sliding 200 ns time window from \(-400 \) ns to \(+400 \) ns. Events which satisfy these rejection criteria are rejected.

4.5.3 Coincidence muon

In addition to the coincidence muon cut in the third reduction, tighter rejection criteria is applied as follows.

\[ \text{PE}_{500} \leq 300 \text{ p.e.s (150 p.e.s for SK-II)} \] (4.25)

\( \text{PE}_{500} \) is the total number of observed ID p.e.s in a fixed time window from \(-100 \) ns to \(+400 \) ns.

\[ \text{PE}_{\text{late}} \geq 20 \text{ p.e.s} \] (4.26)

Here \( \text{PE}_{\text{late}} \) is the maximum number of the OD PMT hits in a sliding 200 ns time window from \(+400 \) ns to \(+1600 \) ns. Events which satisfy these rejection criteria are rejected.

4.5.4 Invisible muon

Cosmic ray muons below the Cherenkov threshold can enter ID region without OD PMT hits and decay into Michel electron. Cherenkov light emitted from the Michel electron can be detected by ID PMT and then such events can be misidentified as FC events. Such invisible muon events which satisfy the following rejection criteria are rejected.

\[ \text{PE}_{\text{tot}} \leq 1,000 \text{ p.e.s (500 p.e.s for SK-II)} \] (4.27)

\[ \text{NHITAC}_{\text{early}} \geq 5 \] (4.28)

Here \( \text{NHITAC}_{\text{early}} \) is the maximum number of OD PMT hits in a sliding 200 ns time window from \(-800 \) to \(-100 \) ns.

\[ \text{NHITAC}_{\text{early}} + \text{NHITAC}_{500} \geq 10 \text{ (if } \text{DIST}_{\text{clust}} \leq 500 \text{ cm)} \] (4.29)

Here \( \text{NHITAC}_{500} \) is the number of OD PMT hits in a fixed time window from \(-100 \) ns to \(+400 \) ns and \( \text{DIST}_{\text{clust}} \) is a distance between two OD hit clusters used for \( \text{NHITAC}_{\text{early}} \) and \( \text{NHITAC}_{500} \).
4.5.5 Long tail flasher event

In addition to the flasher event cut in the third reduction, tighter rejection criteria is applied as follows.

\[ N_{MIN}^{100} \geq 6 \text{ if goodness of TOF fitter } \leq 0.4 \quad (4.30) \]

From SK-II to SK-IV, the rejection criteria is changed as follows.

\[ N_{MIN}^{100} \geq 5 \text{ if goodness of TOF fitter } \leq 0.3 \quad (4.31) \]

Events which pass above criteria are rejected.

4.6 Rejection for ID/OD cross talk

The new rejection step for ID/OD cross talk was implemented recently by H.Tanaka et al. only in SK-IV [109]. OD PMT hits are induced just after the ID PMT hits by ID/OD cross talk. Basically such OD cross talk hits have very low charge (<0.2 p.e) and appear after ~50 ns after ID hits. Therefore OD PMT hits satisfying the following requirements are excluded in all rejection steps.

\[ Q_{OD} < 0.2 \text{ p.e} \quad (4.32) \]
\[ \Delta T_{ID-OD} > 50 \text{ ns} \quad (4.33) \]

Here \( Q_{OD} \) is the charge of the OD PMT hits and \( \Delta T_{ID-OD} \) is the difference of OD PMT hit time and the time of most recent ID PMT hits from 4 closest ID PMTs.
Chapter 5

Event Reconstruction

In the physics analysis, charge and time information of PMTs need to be converted to the information of physics quantities: the number of Cherenkov rings, momentum of the ring, particle type and so on. A reconstruction scheme APfit is used in this analysis. APfit is applied to the events which pass the preselection (chapter 4). APfit reconstructs events step by step as follow.

- Vertex fitting
- Ring counting
- Particle identification
- Momentum reconstruction
- Michel electron search
- Neutron tagging (SK-IV only)

Typical hit maps and timing distributions for single electron and single muon MC events are shown in Fig.5.1.

5.1 Vertex Fitting

Once the speed of charged particle exceeds the threshold, it starts to emit Cherenkov light. This start point is defined as the vertex, and the precise vertex reconstruction is important to estimate Cherenkov angle precisely. Event reconstruction starts from vertex fitting which consists of three steps as follows.

- Point fit
- Ring edge search
- TDC fit

First, the vertex position is roughly estimated assuming the Cherenkov light is emitted from one point (Point fit). Then, the most energetic Cherenkov ring and its ring edge are searched (Ring edge search). Actually, the electrons (muon) with a few hundreds MeV, which is the
Figure 5.1: Typical hit maps (top) and timing distributions [ns] (bottom) for single electron (left) and single muon (right) of MC events. The hits around 1,250 ns in the single muon event correspond to the decay electron.
5.1 VERTEX FITTING

The typical energy scale for the decayed charged leptons in the proton decay, run for a few tens of cm (a few m) in water until they stop by losing their energy. Therefore, Cherenkov light is continuously emitted along the track of charged particles. Finally the vertex position is precisely reconstructed by considering this feature (TDC fit).

5.1.1 Point fit

The point fit assumes all Cherenkov light comes from one point at the same time and does not consider the track length of charged particles. The principle of this algorithm is that timing residual of (photon arrival time at each PMT) — (time of flight) should be peaked at correct vertex position. Fig. 5.2 shows the residual distribution with true vertex position and 4 m off from the true vertex position. The residual with true vertex position clearly shows a sharper peak. In order to find the vertex position, Goodness of the point fit is defined as follow.

\[
\text{Goodness(Point fit)} = \frac{1}{N} \sum_i \exp \left[ -\frac{(t_i(x) - t_0)^2}{2 \times (1.5\sigma)^2} \right]
\]  

(5.1)

Here \(N\) is the number of PMT hits, \(t_i\) is time of flight (TOF) subtracted timing of the \(i\)-th PMT, \(t_0\) is a free parameter chosen to maximize the Goodness(Point fit), \(\sigma\) is the PMT timing resolution of 2.5 ns and factor 1.5 is chosen to optimize the performance of the fit. Finally, the

![Figure 5.2: The timing residual of (photon arrival time at each PMT) — (time of flight) with correct vertex position (top) and 4 m off from true vertex position (bottom) by using \(p \rightarrow e^+\pi^0\) MC[28].](image)
position which maximizes the Goodness(Point fit) is chosen as the reconstructed vertex by Point fit.

5.1.2 Ring edge search

Next step is finding the ring edge and the direction of most energetic Cherenkov ring. First, the number of p.e.s is calculated as a function of opening angle $\theta$ of the ring with respect to the vertex position determined by the point fit and assumed test direction ($\text{PE}(\theta)$). Example for the distribution of $\text{PE}(\theta)$ with certain vertex and direction in a single ring event is shown in Fig. 5.3. Then the opening angle of the ring edge $\theta_{\text{edge}}$ is defined which satisfies following equation. And $\theta_{\text{edge}}$ is also required to be more than $\theta_{\text{peak}}$, where the $\theta_{\text{peak}}$ is the opening angle at the maximum point of $\text{PE}(\theta)$.

$$\frac{d^2\text{PE}(\theta)}{d^2\theta} \bigg|_{\theta=\theta_{\text{edge}}} = 0 \hspace{1cm} (5.2)$$

Finally, in order to determine the ring direction, the estimator $Q(\theta_{\text{edge}})$ is defined as follow.

$$Q(\theta_{\text{edge}}) = \int_0^{\theta_{\text{edge}}} \frac{\text{PE}(\theta)d\theta}{\sin \theta_{\text{edge}}} \times \left( \frac{d\text{PE}(\theta)}{d\theta} \bigg|_{\theta=\theta_{\text{edge}}} \right)^2 \times \exp \left[ -\frac{(\theta_{\text{edge}} - \theta_{\text{exp}})^2}{2\sigma^2_\theta} \right] \hspace{1cm} (5.3)$$
5.1. VERTEX FITTING

Here $\theta_{\text{exp}}$ is expected opening angle calculated from total charge inside $\theta_{\text{edge}}$ assuming the Cherenkov ring is emitted from electron and $\sigma_0$ is its resolution. The $\theta_{\text{edge}}$ and $Q(\theta_{\text{edge}})$ is calculated changing the direction of the charged particle. Then the direction which maximizes $Q(\theta_{\text{edge}})$ is chosen and the $\theta_{\text{edge}}$ in that case is determined as the opening angle of the ring edge. The motivation of $Q(\theta_{\text{edge}})$ is selecting the direction with larger amount of p.e.s inside $\theta_{\text{edge}}$, larger slope of PE($\theta_{\text{edge}}$) and smaller difference between $\theta_{\text{edge}}$ and $\theta_{\text{exp}}$. In case of $\theta_{\text{edge}} > 43$ degree, definition of $Q(\theta_{\text{edge}})$ is changed as follows.

$$Q(\theta_{\text{edge}}) = \int_0^{\theta_{\text{edge}}} \frac{\text{PE}(\theta)}{\sin \theta_{\text{edge}}} d\theta \times \exp \left[-\frac{(\theta_{\text{edge}} - \theta_{\text{exp}})^2}{2\sigma^2} \right]$$ (5.4)

PMTs on the ring edge determined by $\theta_{\text{edge}}$ are tagged and used in the next step, precise vertex fitting.

5.1.3 TDC fit

Finally, vertex is reconstructed more precisely by TDC fit. TDC fit assumes that photons are emitted along the track of the charged particle. First, the track length between the vertex and photon emitting point, which is detected in $i$-th PMT, is calculated as below.

$$l_i' = \frac{\int_0^{l_i} Q(l)dl}{\int_0^{L_t} Q(l)dl} \times L_t$$ (5.5)

Here $l_i$ is the position on the track of charged particle extrapolated from $i$-th PMT by using the opening angle $\theta_c$, where $\theta_c$ is averaged opening angle of the PMTs tagged by previous step. And $Q(l)$ is the charge distribution of PMTs projected on the track of the charged particle. Now $l_i'$ is replaced by $l_i$ in case $l_i' > l_i$ to avoid to calculate too long track length for the electron ring. Schematic view for the definition of the parameters is shown in Fig.5.4.

Then the residual of observed and expected photon arrival timing of $i$-th PMT $t_i$ is considered as follows.

$$t_i = \begin{cases} t_i^0 - \frac{\nu}{c} - \frac{n}{c} \times |\vec{P}_i - \vec{X}_i'| & \text{for PMTs inside ring edge} \\ t_i^0 - \frac{1}{c} \times |\vec{P}_i - \vec{O}| & \text{for PMTs outside ring edge} \end{cases}$$ (5.6)

Here $t_i^0$ is the hit timing of $i$-th PMT, $n$ is the refractive index of water and $c$ is the speed of light in vacuum. And $\vec{X}_i'$ is the photon emitting position, which is detected by $i$-th PMT estimated by equation (5.5), $\vec{P}_i$ is the position of $i$-th PMT and $\vec{O}$ is the vertex position. PMT hits inside the ring edge are mainly from direct photons and outside the ring are mainly from scattered or reflected light on the detector wall. Different fitting estimators are used for PMT hits inside and outside the ring edge. The estimator $G_I$ for the PMT hits inside the ring edge is defined as follow.

$$G_I = \sum_i \frac{1}{\sigma_i} G_{\text{direct}}(t_i, t_0)$$ (5.7)

$$G_{\text{direct}}(t_i, t_0) = \exp \left[-\frac{(t_i - t_0)^2}{2(1.5 \times \langle \sigma \rangle)^2} \right]$$ (5.8)
Figure 5.4: Schematic view of the parameters used in TDC fit[29].
5.2 Ring Counting

Here $\sigma_i$ is the timing resolution of $i$-th PMT as a function of detected p.e.s, $\langle \sigma \rangle$ is averaged timing resolution for detected p.e.s over hit PMTs, $t_i$ is the same value as (5.6) and $t_0$ is a free parameter chosen to maximize the $G_I$. There are two estimators $G_{O1}, G_{O2}$ for the PMT hits outside the ring edge. In case $t_i$ is greater than $t_0$ contribution from scattered photon is considered. Then $G_{O1}$ and $G_{O2}$ are defined as follows.

$$G_{O1} = \sum_i \frac{1}{\sigma_i^2} \left( G_{\text{direct}}(t_i, t_0) \times 2 - 1 \right) \text{ for } t_i \leq t_0$$

(5.9)

$$G_{O2} = \sum_i \frac{1}{\sigma_i^2} \left( \max[G_{\text{direct}}(t_i, t_0), G_{\text{scatt}}(t_i, t_0)] \times 2 - 1 \right) \text{ for } t_i > t_0$$

(5.10)

$$G_{\text{scatt}}(t_i, t_0) = \frac{R_q}{1.5^2} \times G_{\text{direct}}(t_i, t_0) + \left( 1 - \frac{R_q}{1.5^2} \right) \times \exp \left( -\frac{t_i - t_0}{60 \text{ ns}} \right)$$

(5.11)

Here $R_q$ is the fraction of the number of p.e.s inside the ring edge over inside the opening angle 70 degree. Total estimator of TDC fit is defined as follow.

$$G_T = \frac{G_I + G_{O1} + G_{O2}}{\sum_i 1/\sigma_i^2}$$

(5.12)

The vertex position which maximizes $G_T$ is chosen finally. The distance between true and reconstructed vertex for $p \rightarrow e^+e^-\nu$ signal is shown in Fig. 5.5. The accuracy of the vertex reconstruction is a few tens of cm.

![Figure 5.5: The distance between true and reconstructed vertex for $p \rightarrow e^+e^-\nu$ signal.](image)

5.2 Ring Counting

After vertex and angle of most energetic Cherenkov ring are determined, additional Cherenkov rings are searched. Ring counting consists of two steps as follows.

- Ring candidate selection
- Ring candidate test
5.2.1 Ring candidate selection

The pattern recognition algorithm known as Hough transformation technique [31] is used for ring candidate selection. Fig. 5.6 shows the concept of Hough transformation technique. For example, four hit PMTs on unknown ring with radius \( r \) are considered. By using Hough transformation, circles with radius \( r \) whose center are each hit PMT are drawn. All circles are crossed at the center of unknown ring. Thus the center of unknown ring is determined by finding peak accumulated by each circle. Before drawing the circles, PMTs which belong to the Cherenkov ring already found are removed. Hough space is defined by 36 bins of Polar angle \( \Theta \) and 72 bins of azimuthal angle \( \Phi \). Now \( \Theta \) and \( \Phi \) are measured from the position of reconstructed vertex. Detected p.e.s in each PMT which are corrected for attenuation length and acceptance are mapped on \((\Theta, \Phi)\) Hough space. Actually, the opening angle of 42\(^\circ\) towards each hit PMT is assumed and the circle is projected on Hough space around each PMT hit. Then the peak accumulated by each circle is searched and it is determined as the center of the ring candidate. An example of p.e. distribution on \((\Theta, \Phi)\) Hough space is shown in Fig. 5.7. In order to avoid to find fake rings, any ring candidates which are very close to the ring (angle < 15\(^\circ\)) already found are discarded. In order to take into account the different Cherenkov angle for non-electron particles, Hough transformation on \( Q_e \) space is performed. Here \( Q_e(\theta) \) is expected p.e. distributions with respect to the vertex and determined ring center as the function of opening angle \( \theta \).

5.2.2 Ring candidate test

Ring candidates are checked by likelihood method to know if the ring is true or fake. In case the number of rings is \( N \), \((N+1)\)-th ring candidate is newly assumed. Then p.e.s inside the \((N+1)\)-th ring is expected by considering the contribution from other rings and \((N+1)\)-th ring itself. Actually p.e.s from crossed Cherenkov rings are overlapped in some PMTs, so we have to know the fraction of p.e.s in one PMT contributed from other rings. The ring separation method is used for this (section 5.3.2). Expected charge inside the \((N+1)\)-th ring is compared with observed charge and the likelihood is defined as follows.

\[
L_{N+1} = \sum_i \log \left\{ \text{prob} \left( \frac{q_i^{\text{obs}}}{\sum_{n=1}^{N+1} \alpha_n \cdot q_{i,n}^{\text{exp}}} \right) \right\}
\]

(5.13)
5.2. RING COUNTING

Figure 5.7: An example of p.e. distribution on ($\Theta, \Phi$) Hough space[29]. Two clear peaks are seen, which are identified as centers of Cherenkov rings.

The sum of $i$ extends over PMT hits inside the $(N + 1)$-th Cherenkov ring. $q_{i}^{\text{obs}}$ is observed p.e.s in $i$-th PMT, $q_{i;n}^{\text{exp}}$ is expected p.e.s in the $i$-th PMT from $n$-th ring and $\alpha_{n}$ is a changing scale which maximize the $L_{N+1}$. The probability $\text{prob}(q_{i}^{\text{obs}}, q_{i}^{\text{exp}})$ is defined as follows.

$$\text{prob}(q_{i}^{\text{obs}}, q_{i}^{\text{exp}}) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{(q_{i}^{\text{obs}} - q_{i}^{\text{exp}})^2}{2\sigma^2} \right) & (\text{for } q_{i}^{\text{exp}} > 20 \text{ p.e.}) \\ \text{Convolution of single p.e and Poisson distribution} & (\text{for } q_{i}^{\text{exp}} < 20 \text{ p.e.}) \end{cases}$$

(5.14)

Here $\sigma$ is the resolution for $q_{i}^{\text{exp}}$. If there are no ring candidates which satisfy $L_{N+1} \geq L_{N}$, the number of ring is determined to be $N$. If any of ring candidates satisfy $L_{N+1} \geq L_{N}$, following functions are calculated.

$F_{1}$: $L_{N+1} - L_{N}$ corrected for the total p.e.s.

$F_{2}$: Average of expected p.e.s on the ring edge of the $(N + 1)$-th ring candidate ($Q_{\text{edge}}$).

$F_{3}$: Average of expected p.e.s outside the $(N + 1)$-th ring candidate ($Q_{\text{out}}$) is calculated. Then the difference $Q_{\text{edge}} - Q_{\text{out}}$ is corrected for total p.e.s.

$F_{4}$: The residual of p.e.s detected and expected in $i$-th PMT.

As the value of each function is larger, the $(N + 1)$-th ring candidate becomes more reliable. Then the four functions are used for the final evaluator as follows.

$$F = \sum_{i}^{4} \alpha_{i} F_{i} \quad (5.15)$$
Here $\alpha_i$ are optimized weights for each function. In case $F$ is positive, $(N + 1)$-th ring is determined to be true ring and process of ring candidate selection is repeated again.

The ring reconstruction efficiency as a function of the particle momentum for single electron and single muon MC is shown in Fig. 5.8. The turn on in both efficiency plots are basically due to the Cherenkov threshold of the particle or lack of p.e.s. At the plateau region of the efficiency, almost all Cherenkov rings can be reconstructed both for single electron and muon cases. For the multi Cherenkov rings can be reconstructed both for single electron and muon cases. For the multi ring case, reconstruction efficiency basically becomes lower due to the overlap of the rings.

Figure 5.8: The ring reconstruction efficiency as a function of the particle momentum for single electron (left) and single muon (right) events. The vertical error bars correspond to the statistical uncertainty of each bin. If the angle between the true particle and reconstructed ring is below 20°, the Cherenkov ring is defined to be reconstructed.

5.3 Particle Identification

Particle identification (PID) algorithm can classify each Cherenkov ring into showering ($e$-like) and non-showering ($\mu$-like) types. Showering particles such as electrons or gamma-rays are multiple scattered and make electromagnetic showers in water. As a result Cherenkov rings with diffuse edge are detected on ID wall. On the other hand, non-showering particles such as muons or pions make Cherenkov rings with clear edge. Such differences of detected ring pattern are used in PID algorithm. Another important information for PID is opening angle of each Cherenkov ring. Since electron is highly relativistic ($\beta \sim 1$) as their mass is very small, opening angle of Cherenkov rings from electron or gamma-rays are 42°. On the other hand, muons or pions can make Cherenkov angle with smaller opening angle when they are not highly relativistic.

PID algorithm starts from ring separation in order to know exact observed p.e.s contributed from each ring. Then observed p.e.s distribution of each ring is compared with expected p.e.s distributions of electron or muon. One of expected p.e.s distributions which can reproduce the observed one better is determined as the particle type of the ring.
5.3. PARTICLE IDENTIFICATION

5.3.1 Expected p.e.s distribution

Expected p.e.s distribution of electron is made by using Monte Carlo simulation. First expected p.e.s distribution on hypothetical spherical surface with $R_{\text{sph}} = 16.9$ m as a function of opening angle $\theta$ is made. Actually distributions of expected p.e.s $Q^{\exp}(\theta, p)$ are tabulated for electron momentum $p = 100$ MeV, $300$ MeV and $1,000$ MeV. Then expected p.e.s of $i$-th PMT for electron is calculated as follows.

$$q_i^{\exp}(e) = \alpha_n(e) \times Q^{\exp}(\theta_{i,n}, p_n) \times \left(\frac{R_{\text{sph}}}{r_i}\right)^{1.5} \times \frac{1}{\exp(r_i/L)} \times f(\Theta_i) + q_i^{\text{scatt}} \quad (5.16)$$

$q_i^{\exp}(e)$: expected p.e.s for $i$-th PMT from $n$-th ring assuming particle is electron.

$\alpha_n(e)$: normalization factor for electron.

$\theta_{i,n}$: opening angle between $i$-th PMT and direction of $n$-th ring.

$p_n$: momentum of $n$-th ring.

$r_i$: distance between vertex and $i$-th PMT.

$L$: attenuation length of light in water.

$f(\Theta_i)$: correction factor for the acceptance to $i$-th PMT as a function of the photon incident angle $\Theta_i$ shown in Fig. 5.9.

$q_i^{\text{scatt}}$: expected p.e.s for $i$-th PMT from scattered photon.

![Figure 5.9: Correction factor $f(\Theta)$ for the acceptance to PMT as the function of the photon incident angle $\Theta$ [29].](image)

Expected p.e.s for muon is analytically calculated as follows.

$$q_i^{\exp}(\mu) = \left(\alpha_n(\mu) \times \left[r_i \left(\sin \theta_{x_i,n} + r_i \cdot \frac{d\theta}{dx}_{x=x_i,n}\right) \times \sin^2 \theta_{x_i,n}\right]^{-1} + q_i^{\text{knock}}\right) \times \frac{1}{\exp(r_i/L)} \times f(\Theta_i) + q_i^{\text{scatt}} \quad (5.17)$$
\( q_i^{\text{exp}}(e\mu) \): expected p.e.s for \( i \)-th PMT from \( n \)-th ring assuming particle is muon.

\( \alpha_n(\mu) \): normalization factor for muon.

\( x \): distance between vertex and muon.

\( x_{i,n} \): distance between vertex and Cherenkov light emission point of \( n \)-th ring estimated from \( i \)-th PMT.

\( \theta \): opening angle of muon traversing at \( x \). Energy loss of the muon is taken into account.

\( \theta_{x_{i,n}} \): opening angle of muon traversing at \( x = x_{i,n} \).

\( q_i^{\text{knock}} \): expected p.e.s of knock-on electrons estimated by Monte Carlo simulation.

Here \( \sin^2 \theta \) comes from photon intensity of Cherenkov light described in (2.4). And \( r(\sin \theta + r \cdot (d\theta/dx)) \) comes from the cross section of Cherenkov photon \( 2\pi r \sin \theta (dx \sin \theta + r d\theta) \). It considers that a muon traverses for \( dx \) changing the opening angle \( \theta \) as a function of the muon energy loss. Schematic view of the cross section is shown in Fig. 5.10

![Schematic view of the cross section](image)

**Figure 5.10:** The cross section of Cherenkov photon \( 2\pi r \sin \theta (dx \sin \theta + r d\theta) \). A muon traverses for \( dx \) changing the opening angle \( \theta \) as a function of the muon energy loss. [29]

### 5.3.2 Ring separation I

In this step, contribution of each ring to the detected p.e.s in each PMT is determined assuming the charged particle is electron. First observed p.e.s in \( i \)-th PMT \( q_i^{\text{obs}} \) and sum of expected p.e.s from \( n \)-th ring in \( i \)-th PMT \( q_i^{\text{exp}}_{i,n} \) are compared with in \( 70^\circ \) opening angle of \( n \)-th ring. \( q_i^{\text{exp}}_{i,n} \) is calculated by (5.16). Initial p.e distributions are determined by minimizing following \( \chi^2 \).

\[
\chi^2_n = \sum_{\theta_{\ell,n}<70^\circ} \left( \frac{q_{i,n}^{\text{obs}} - \sum_{n'} q_{i',n'}^{\text{exp}}}{\sqrt{q_i^{\ell}}} \right)^2
\]  
(5.18)
5.3. PARTICLE IDENTIFICATION

Summation is performed for all \( i' \)-th PMTs within 70° opening angle of \( n \)-th ring. Calculation of \( \chi^2_n \) continues changing the momentum of \( n \)-th ring until minimum \( \chi^2_n \) value is found. This procedure is performed continuously ring by ring until all \( \chi^2_n \) values are converged. As a result, we get initial separated p.e.s \( q_{i,n} \) from \( n \)-th ring in \( i \)-th PMT by using determined \( q_{i,n}^{\text{exp}} \) and following function.

\[
q_{i,n} = q_{i}^{\text{obs}} \times \frac{q_{i,n}^{\text{exp}}}{\sum_{n'} q_{i,n'}^{\text{exp}}} \quad (5.19)
\]

Using this initial separated p.e.s \( q_{i,n} \), expected p.e distribution \( Q_{n}^{\text{exp}}(\theta) \) as the function of opening angle \( \theta \) towards \( n \)-th ring direction is calculated. \( Q_{n}^{\text{exp}}(\theta) \) is expected p.e distribution projected on hypothetical spherical surface whose center is the reconstructed vertex. In order to make \( Q_{n}^{\text{exp}}(\theta) \), \( q_{i,n} \) needs to be corrected by considering the distance from the vertex, the attenuation length and PMT acceptance as follows.

\[
q_{i,n}' = q_{i,n} \times \exp \left( -\frac{R_{\text{ sph}}}{L_{\text{ abs}}} \right) \times \frac{1}{\exp(-r_{i}/L)} \times f(\Theta) \quad (5.20)
\]

Here \( q_{i,n}' \) is corrected p.e distribution and \( L_{\text{ abs}} \) is a parameter for absorption length in Monte Carlo simulation set to be 55 m. And \( Q_{n}^{\text{exp}}(\theta) \) is made by summing up \( q_{i,n}' \) within opening angle \( \theta \) of \( n \)-th ring. Then \( Q_{n}^{\text{exp}}(\theta) \) is optimized by considering non-overlap region of the ring, by applying proper normalization and smoothing and by using the likelihood.

In the next step, \( q_{i,n}^{\text{exp}} \) is recalculated again by using optimized \( Q_{n}^{\text{exp}}(\theta) \) as follows.

\[
q_{i,n}^{\text{exp}} = Q_{n}^{\text{exp}}(\theta_{i,n}) \times \exp \left( -\frac{R_{\text{ sph}}}{L_{\text{ abs}}} \right) \times \frac{1}{\exp(-r_{i}/L)} \times f(\Theta) \quad (5.21)
\]

Optimization of charge separation is performed by using following likelihood function.

\[
L_{n} = \sum_{\theta_{i',n}<70^\circ} \left( \text{prob}(q_{i'}^{\text{obs}}, \sum_{n'} \alpha_{n'} \cdot q_{i',n'}^{\text{exp}}) \right) \times \sqrt{Q_{n}^{\text{exp}}(\theta_{i',n})} \times \min[1, \sqrt{\theta_{c}^{\circ}/\theta_{i',n}}] \quad (5.22)
\]

\( \text{prob} \) is the same function as \((5.14)\) to calculate the probability of how the observed and expected p.e.s in each PMT are same. \( \theta_{c}^{\circ} \) is the reconstructed opening angle of \( n \)-th ring. \( \alpha_{n} \) is the normalization parameter of \( n \)-th ring for the optimization. The factors of \( \sqrt{Q_{n}^{\text{exp}}(\theta_{i',n})} \times \min[1, \sqrt{\theta_{c}^{\circ}/\theta_{i',n}}] \) enhance the contribution from the PMTs around and inside the Cherenkov edge. \( \alpha_{n} \) which maximizes the \( L_{n} \) is taken and this optimization is iterated until all \( \alpha_{n} \) converge. Finally, observed p.e.s are separated again by using optimized \( \alpha_{n} \) as follows.

\[
q_{i,n} = q_{i}^{\text{obs}} \times \frac{\alpha_{n} \cdot q_{i,n}^{\text{exp}}}{\sum_{n'} \alpha_{n'} \cdot q_{i,n'}^{\text{exp}}} \quad (5.23)
\]

5.3.3 Determination of particle type

In order to determine the particle type of the ring, following likelihood function is used.

\[
L_{n}(e \text{ or } \mu) = \prod_{\theta_{i}<1.5 \times \theta_{c}} \text{prob} \left( q_{i}^{\text{obs}}, q_{i,n}^{\text{exp}}(e \text{ or } \mu) + \sum_{n' \neq n} q_{i,n'}^{\text{exp}} \right) \quad (5.24)
\]
Here prob is the same function as (5.14), $q_n^{\text{exp}}$ is expected p.e from $n$-th ring assuming the particle is electron (5.16) or muon (5.17). And $q_n^{\text{exp}}$ is the contribution from other rings determined by ring separation method described in section 5.3.2. Now $L_n$ is optimized to get the maximum value by changing the direction and opening angle of $n$-th ring. Then $L_n$ is converted to the probability to determine the particle type from the pattern of p.e.s. as follows.

$$
P_n^{\text{pattern}}(e \text{ or } \mu) = \exp \left( -\frac{\{\chi_n^2(e \text{ or } \mu) - \min[\chi_n^2(e), \chi_n^2(\mu)]\}^2}{2\sigma_n^2} \right)$$  \hspace{1cm} (5.25)

$$\chi_n^2(e \text{ or } \mu) = -2 \log L_n(e \text{ or } \mu) + \text{constant}$$  \hspace{1cm} (5.26)

Now $\sigma_n^2 = \sqrt{2N}$ and $N$ is the number of PMTs used in the calculation. The probability from the opening angle of Cherenkov ring is defined as follow.

$$P_n^{\text{angle}}(e \text{ or } \mu) = \exp \left( -\frac{\{\theta_{\text{obs}}^n - \theta_n^{\text{exp}}(e \text{ or } \mu)\}^2}{2(\delta\theta_n)^2} \right)$$  \hspace{1cm} (5.27)

Here $\theta_{\text{obs}}^n$ and $\delta\theta_n$ are reconstructed opening angle and its fitting error respectively. And $\theta_n^{\text{exp}}$ is expected opening angle of $n$-th ring from the reconstructed momentum assuming electron or muon. Then the combined probability is calculated as follows.

$$P_n(e \text{ or } \mu) = P_n^{\text{pattern}}(e \text{ or } \mu) \times P_n^{\text{angle}}(e \text{ or } \mu)$$  \hspace{1cm} (5.28)

Finally following probability is calculated and used to determine the particle type.

$$P_{\text{PID}} = \sqrt{-\log P(\mu)} - \sqrt{-\log P(e)}$$  \hspace{1cm} (5.29)

In case $P_{\text{PID}} < 0$ the ring is determined to be $e$-like and in case $P_{\text{PID}} > 0$ the ring is determined to be $\mu$-like.

Now $P_{\text{PID}}$ values for true electrons and muons in are shown in Fig.5.11. The PID efficiency plots for electrons and muons as a function of the particle momentum are also shown in Fig.5.12. The inefficiency for lower momentum electron and muon is mainly due to the lack of PMT hits. For higher momentum electrons and muons, almost all rings from electron or muon can be identified correctly in case the ring is well independent. For the multi ring case, the PID efficiency basically becomes lower due to the overlap of the rings.

### 5.3.4 Ring separation II

In this step, contribution of p.e.s from each ring in each PMT is calculated again by assuming the particle type determined in PID process. In order to get expected p.e in each PMT, tabulated p.e distribution made by MC simulation is used. Scattered and reflected light on the detector wall is also taken into account. Ring separation is optimized by using the following likelihood function.

$$L_n = \sum_{\theta_{\text{obs}}^n < 70^\circ} \left( \text{prob}(q_{\text{obs}}^n, \sum_{n'} \alpha_{n'} \cdot q_{\text{exp}}^{n'}) \right)$$  \hspace{1cm} (5.30)

prob is the same function as (5.14). $\alpha_n$ is changed repeatedly until finding the maximum $L_n$. Finally separated p.e.s is calculated by using the function (5.19).

For the multi ring case, additional correction is applied as below.
Figure 5.11: $P_{\text{PID}}$ values for true electrons (black) and true muons (red) in single electron and muon MC, respectively.

Figure 5.12: The PID efficiency as a function of the particle momentum for single electron (left) and single muon (right) events. The vertical error bars correspond to the statistical uncertainty of each bin. The definition of the PID efficiency is that the number of $e$-like ($\mu$-like) rings with respect to the number of rings matched (angle below 20°) to the true electron (muon).
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• 2 close rings $a$ and $b$ with the energy $E_a$ and $E_b$, respectively are considered ($E_b > E_a$). If the angle between 2 rings $\theta$ is less than $30^\circ$ and $E_a \sin \theta < 60$ MeV, ring $a$ is rejected.

• If $E_a / E_{\text{all}} < 0.05$ and $E_a < 40$ MeV, ring $a$ is rejected. $E_{\text{all}}$ is the total energy of all rings.

5.4 Momentum Reconstruction

The momentum of each ring is reconstructed from the total number of p.e.s within a $70^\circ$ half opening angle towards the ring direction. Contribution of exact p.e in each PMT from other rings are calculated by ring separation process. In order to obtain the total charge, p.e.s in each PMT from $n$-th ring is summed up and corrected for light attenuation in water as the following equation.

$$RT_{TOT, n} = \text{constant} \times \frac{G_{\text{data}}}{G_{\text{MC}}} \times \sum_{\theta_{i,n} < 70^\circ} q_{i,n} \times \exp \left( \frac{r_{i,n}}{L} \right) \times \frac{1}{f(\Theta_i)} \times \cos \Theta_i \quad (5.31)$$

Here $G$ is PMT gain parameter deduced from cosmic ray muon sample. The ratio of $G$ corrects for the difference of PMT gain between data and MC. $RT_{TOT}$ is converted to the proper momentum depending on the particle type by using conversion table made from MC.

5.5 Michel Electron Search

Tagging the decayed electron from muon so called Michel electron is a good indicator for the existence of muons. In this tagging method, delayed hit clusters from primary timing are searched. Observed Michel electron is classified to three types as follow.

**sub-event type ($\Delta t > 0.9 \mu s$)**

Michel electron observed in separated event (sub-event).

**primary-event type ($\Delta t < 0.9 \mu s$)**

Michel electron observed in primary event.

**split type ($\Delta t \sim 0.9 \mu s$)**

Michel electron observed around the border timing of event window. Observed PMTs from Michel electrons are separated into both primary-event and sub-event.

Here $\Delta t$ is the timing difference between primary event and following Michel electron event. Following selection criteria are required for sub-event type.

• $\Delta t < 30 \mu s$

• NHIT > 50. NHIT is the number of ID PMT hits.

• Reconstructed vertex is well fitted.

• NHIT(50 ns) > 30. NHIT(50 ns) is the maximum number of ID PMT hits in a 50 ns sliding timing window after subtracting TOF.
5.6. **NEUTRON TAGGING**

- Total number of p.e.s is less than 2,000.

For primary-event type, second peak from Michel electron is searched. Following selection criteria are required for primary-event type.

- NHIT(30 ns) > 20. NHIT(30 ns) is the maximum number of ID PMT hits in a 30 ns sliding timing window after primary peak.

In addition to the above selection criteria, additional selections are applied as follows for proton decay analysis only in SK-I to SK-III.

1. NHIT > 60 (30 for SK-II) for sub-event type.
2. NHIT > 40 (20 for SK-II) for primary-event type.
3. \(0.1 \mu s < \Delta t < 0.8 \mu s\) or \(1.2 \mu s < \Delta t < 30 \mu s\)

The selections 1 and 2 reject the gamma emission from \(\mu^-\) captured in \(^{16}\text{O}\) nuclei. The selection 3 rejects the dead time due to the impedance mismatch of ATM boards between 800 ns and 1,200 ns from the primary event timing. This dead time region is solved in SK-IV by using new electronics QBEE.

Since about 20% of \(\mu^-\) are captured by nucleus and does not emit a decay electron, the tagging efficiency of decay electron for \(\mu^-\) is lower than that of \(\mu^+\). The tagging efficiency is \(\sim 95\%\) for \(\mu^+\) and \(\sim 80\%\) for \(\mu^-\) in SK-IV. Furthermore, due to the electronics dead time, the tagging efficiency in SK-I-III are lower than that in SK-IV. The tagging efficiency in SK-I-III is \(\sim 80\%\) for \(\mu^+\) and \(\sim 65\%\) for \(\mu^-\).

### 5.6 Neutron Tagging

Neutrons are often produced by atmospheric neutrino interaction. On the other hand, the probability of the neutron emission by the de-excitation of nucleus after proton decay is small and no neutrons are emitted from hydrogen. So the neutron tagging technique is important for the sensitivity of proton decay analysis[33]. Recent improvement of neutron tagging by T. Mochizuki is also included in this analysis.[110]

Free neutrons traveling in water is quickly thermalized and captured by oxygen or hydrogen nucleus. Neutrons are mainly captured by the following interaction and its probability is \(\sim 100\%\).

\[
n + p \rightarrow d + \gamma \ (2.2 \text{ MeV})
\]  

(5.32)

This 2.2 MeV \(\gamma\) is the key of the neutron tagging technique. The mean traveling time of neutron until capturing is measured to be 204.87 \(\mu s\)[34]. In order to access such a long time range, additional trigger "AFT" is introduced from the beginning of SK-IV. AFT is issued when SHE trigger is issued without OD trigger and saves additional 500 \(\mu s\) data following 35 \(\mu s\) data saved by SHE trigger. \(\sim 92\%\) of neutron capture data can be saved in the data. There are many background sources like radioactive decays from the surrounding rocks and detector material, radon in water, PMT dark noise and so on. In order to distinguish 2.2 MeV \(\gamma\) signal from neutron capture, two steps of neutron tagging techniques are applied.

First, dark noise cut is applied which is recently implemented by T. Mochizuki. The dark noise sometimes appears in PMTs by burst dark noise or just randomly. It is important for the
algorithm with small number of PMT hits like neutron tagging to reject such dark noise hits. Main cause of burst dark noise is considered to be the scintillation light of glass induced by RI in the glass. The timing constant of burst dark noise is found to be about 10 $\mu$s. In order to reject burst dark noise events, PMT hits which have other hits in 10 $\mu$s in same PMT are rejected.

Then, in order to select 2.2 MeV $\gamma$ candidate, following selection is applied.

\[ N_{10} \geq 5 \]  \hspace{1cm} (5.33)

Here $N_{10}$ is the number of PMT hits in a 10 ns sliding time window within 18 $\mu$s to 535 $\mu$s. Once first window of $N_{10} \geq 7$ is found, the time is defined as $t_0$. In order to avoid multiple counting of one neutron capture event, a peak value within 20 ns after $t_0$ is defined as the same 2.2 MeV $\gamma$ candidate. Peaks after $t_0 + 20$ ns are defined as different candidates. Then additional two selections are applied as follows.

\[ N_{10} \leq 50 \]  \hspace{1cm} (5.34)

\[ N_{200} \leq 200 \]  \hspace{1cm} (5.35)

Here $N_{200}$ is the number of PMT hits in 200 ns time window around $t_0$. These selections are applied for removing high energy events like cosmic ray muons.

Finally, neural network based selections are applied. Following 16 variables are used as inputs of the neural network in the default neutron tagging algorithm.

$N_{10}$: Background increase exponentially for smaller $N_{10}$ values.

$N_c$: Each of $N_{10}$ hit vector is calculated and if the angle between two vectors is less than 14.1°, the pair is defined as a cluster. Calculations are repeated over all PMT hits in $N_{10}$. Total number of hits in the cluster is defined as $N_c$. Background events tend to occur near the PMTs so $N_c$ is relatively larger. On the other hand, it is rare that neutron capture events occur near the PMTs so $N_c$ tends to be smaller.

$N_{low}$: Most neutron captures occur within 200 cm from vertex point of neutrino interaction. Therefore PMTs close to the neutrino interaction point have higher probability of detecting $\gamma$ from neutron capture. First, probability of detecting hits in $i$-th PMT is calculated and area of high probability PMTs is defined. Then the number of PMT hits outside the area is counted as $N_{low}$. Since 2.2 MeV $\gamma$ signals are concentrated on higher probability area, $N_{low}$ in neutron capture events tends to be smaller. On the other hand, since background events are originated from anywhere in the tank, $N_{low}$ in background events does not show any biases.

$N_{300}$: $N_{300}$ is the number of hits in 300 ns time window around 10 ns time window. Some backgrounds have widely distributed hits. This parameter is sensitive to such kind of backgrounds.

$\phi_{rms}$: $\phi_{rms}$ is the root mean square of the hit vectors in $N_{10}$. 2.2 MeV $\gamma$ from neutron capture deposits most of energy in a single direction. On the other hand, background events such as PMT dark noise make relatively random hit pattern.
\( \theta_{\text{mean}} \): \( \theta_{\text{rms}} \) is the average of the angles between the direction of charged particles and each PMT hits. 2.2 MeV \( \gamma \) from neutron capture tends to make clustered hit patterns.

\( t_{\text{rms}} \): 2.2 MeV \( \gamma \) signal has narrow hit timing distribution. \( t_{\text{rms}} \) is the root mean square of all \( N_{10} \) hits.

\( t_{\text{rms}, \text{Min}} \): Many candidates with a few hits from 2.2 MeV \( \gamma \) and rest of hits from dark noise or other background can be selected only by using \( t_{\text{rms}} \). In order to reject such events, minimum value of \( t_{\text{rms}} \) calculated from all the combinations of hits in \( N_{10} \) is used.

\( BS_{\text{wall}}, BS_{\text{energy}} \): The vertex and energy for low energy events in SK are reconstructed by special fitting tool Bonsai[35]. \( BS_{\text{wall}} \) is the distance between reconstructed vertex and the nearest ID wall. Many background sources tend to be generated near the wall. \( BS_{\text{energy}} \) is the reconstructed energy of candidate events. 2.2 MeV \( \gamma \) is mono-energetic.

\( NF_{\text{wall}}, \Delta N_{10}, \Delta t_{\text{rms}} \): \( NF_{\text{wall}} \) is the distance between reconstructed vertex by Neut-fit and nearest wall. Neut-fit is another vertex fitter for small number of PMT hits. Neut-fit only uses the hits in \( N_{10} \) so the reconstructed values are biased in the region. \( \Delta t_{\text{rms}} \) and \( \Delta N_{10} \) are the differences between \( t_{\text{rms}} \), \( N_{10} \), respectively, calculated with APfit and Neut-fit. Since Neut-fit focuses on 2.2 MeV \( \gamma \) related region, difference of reconstructed values with APfit and Neut-fit can be good values for separating signal and background.

\( (NF - BS)_{\text{dis}}, (NF - AP)_{\text{dis}} \): \( (NF - BS)_{\text{dis}} \) and \( (NF - AP)_{\text{dis}} \) are the distances between reconstructed vertexes with Neut-fit and Bonsai, Neut-fit and APfit, respectively.

In addition to these variables, new variables: Cherenkov angle, \( D_{\text{wall}} \) along hit direction, acceptance and spread of hits are added recently by T. Mochizuki. Efficiency of neutron tagging including recent improvement is 25.2\% and the fake rate is 0.018. Systematic uncertainty for neutron tagging is estimated to be 6\%.
Chapter 6

Proton Decay Search

A search for proton decay into three charged leptons is performed in this chapter using the SKI-IV data corresponding to 6050.3 days of live time. First, selection criteria are defined for each signal mode by comparing signal and background MC. Then systematic uncertainty is estimated for both signal and background. Events in background MC are reweighed by considering solar activity and 3 flavor neutrino oscillation based on the recent neutrino oscillation analysis in SK$^{108}$. Before opening the data in signal box, data and MC agreement in control region is checked. Finally data is opened in the signal box and checked if there are any proton decay candidates. The candidates are carefully checked and judged if they are true proton decay events or not. Even if no proton decay events are discovered, lower lifetime limit for proton is calculated finally with respect to each decay mode.

6.1 Event selection

FC data set is used in this analysis. First, following selection criteria is applied to the events which pass the preselection (chapter 4).

\[ D_{\text{wall}} \geq 2.0 \text{ m} \]  \hspace{1cm} (6.1)

Here \( D_{\text{wall}} \) is the distance between the ID wall and reconstructed vertex by APfit.

\[ \text{NHITAC} \leq 15 \text{ (9 for SK-II)} \]  \hspace{1cm} (6.2)

Here NHITAC is the number of OD PMT hits in the largest charge cluster.

\[ E_{\text{vis}} \geq 30 \text{ MeV} \]  \hspace{1cm} (6.3)

Here \( E_{\text{vis}} \) is the total energy of all reconstructed rings assuming all rings are from electrons. Final event rate is about 8 events a day.

Most of remained background is atmospheric neutrino events and other backgrounds are negligible. In order to further reduce background events, additional kinematic selections are applied. Selection criteria are defined for each signal mode using reconstructed variables as follows.
6.1.1 Number of reconstructed rings

Since signal has three charged leptons in final states, basically three Cherenkov rings are emitted. We require that exactly three rings are reconstructed. Number of reconstructed rings for \( p \rightarrow e^+e^+e^- \) and background is shown in Fig. 6.1. \( \sim 80\% \) of events can reconstruct 3 rings in \( p \rightarrow e^+e^+e^- \) mode. The main cause of 2 ring events in \( p \rightarrow e^+e^+e^- \) mode is two leptons decayed close to each other and detected rings are overlapped on the wall. There is another cause of 2 ring events in other muon including signal modes. Since Cherenkov threshold for muon is much larger than that of electron, some of low momentum muons cannot emit Cherenkov light in muon including modes. As more muons contained in the signal, larger phase space is taken by created muon mass. This feature decreases the momentum scale of muons and makes more muons below the Cherenkov threshold. Number of reconstructed rings with respect to each signal mode is also shown in Fig. 6.1. After this requirement, about 90% of background is decreased and signal is kept to be \( \sim 80\%, \sim 70\%, \sim 50\% \) and \( \sim 40\% \) for \( p \rightarrow e^+e^+e^- \), \( p \rightarrow \mu^+e^+e^- \), \( p \rightarrow e^+\mu^+\mu^- \) and \( p \rightarrow \mu^+\mu^+\mu^- \) respectively. Performance of ring counting is almost the same between \( p \rightarrow \mu^+e^+e^- \) and \( p \rightarrow \mu^-e^+e^+ \) or \( p \rightarrow e^+\mu^+\mu^- \) and \( p \rightarrow e^-\mu^+\mu^+ \). Using 2 ring events can increase the signal efficiency but background events are increased as well. Hence only 3 ring events are used in this analysis.

![Figure 6.1: Number of ring distributions for background and signal in \( p \rightarrow e^+e^+e^- \) mode after FC & FV cut (left). Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary. Right figure show the comparison of \( p \rightarrow \mu^+e^+e^- \), \( p \rightarrow e^+\mu^+\mu^- \) and \( p \rightarrow \mu^+\mu^+\mu^- \) modes.](image)

6.1.2 Number of \( e^- \)-like or \( \mu^- \)-like rings

Each detected ring is classified as electron like or muon like by PID technique described in 5.3. Not only the information of charge distribution on the wall but also opening angle information is used for the PID, because the Cherenkov light from charged leptons should be emitted from exact vertex position. Requirements for the number of \( e^- \)-like or \( \mu^- \)-like rings are set depending on the exact number of electrons and muons in each signal mode. For example, 3 \( e^- \)-like rings are required for \( p \rightarrow e^+e^+e^- \) mode or 2 \( e^- \)-like and 1 \( \mu^- \)-like rings are required for \( p \rightarrow \mu^+e^+e^- \) mode. Number of \( e^- \)-like rings for \( p \rightarrow e^+e^+e^- \) and background or number of \( \mu^- \)-like rings for \( p \rightarrow \mu^+e^+e^- \) and background after 3 rings cut are shown in Fig. 6.2. Same distributions for
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$p \rightarrow e^+\mu^+\mu^-$ and $p \rightarrow \mu^+e^+e^-$ after 3 rings are also shown in Fig.6.3. More than 90% of signal events can be kept after this cut for all modes. There are almost no period dependence. Performance of PID is almost the same between $p \rightarrow \mu^+e^+e^-$ and $p \rightarrow \mu^-e^+e^+$ or $p \rightarrow e^+\mu^+\mu^-$ and $p \rightarrow e^-\mu^+\mu^+$. 

![Figure 6.2: Number of e-like rings for background and signal in $p \rightarrow e^+e^+e^-$ mode after FC & FV cut and number of ring cut (left). Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary. Right figure shows the Number of $\mu$-like rings for background and signal in $p \rightarrow \mu^+e^+e^-$ mode.]

![Figure 6.3: Number of $\mu$-like rings for background and signal in $p \rightarrow e^+\mu^+\mu^-$ (left) and $p \rightarrow \mu^+\mu^-\mu^-$ (right) modes after FC & FV cut and number of ring cut (left). Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary.]

6.1.3 Number of decay electrons

Decayed electrons from muons can be tagged by searching for the delayed signal after primary signal which is described in 5.5. Number of tagged decay electrons can be used for the signal background separation. Since there is no muon and decay electron in $p \rightarrow e^+e^+e^-$ mode, the number of decay electrons is required to be 0. On the other hand, $p \rightarrow \mu^+e^+e^-$ and
$p \rightarrow e^+\mu^+\mu^-$ modes require 1 and 2 decay electrons, respectively. $p \rightarrow \mu^+\mu^+\mu^-$ mode require 2 or 3 decay electrons to keep higher signal efficiency. Number of decay electrons for each signal mode and background after all previous all cuts is shown in Fig. 6.4 and Fig. 6.5.

Since $\mu^-$ is sometimes captured by nucleus before the decay with the probability of $\sim 20\%$ and does not emit decay electron, there are some difference in the number of decay electrons between $p \rightarrow \mu^+e^+e^-$ and $p \rightarrow \mu^-e^+e^+$ or $p \rightarrow e^+\mu^+\mu^-$ and $p \rightarrow e^-\mu^+\mu^-$. Comparison of these modes are shown in Fig. 6.6. Furthermore, there are period dependences on the number of decay electrons. Since SK-I-III has dead time between 800 ns and 1,200 ns due to the impedance mismatch of ATM boards, efficiency of decay electron tagging is worse than that of SK-IV. Number of decay electrons for $p \rightarrow \mu^+e^+e^-$ with respect to SK-I-IV is shown in Fig. 6.7.

![Figure 6.4](image1.png)

**Figure 6.4:** Number of decay electrons for background and signal in $p \rightarrow e^+e^+e^-$ (left) and $p \rightarrow \mu^+e^+e^-$ (right) modes after all previous cuts. Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary.

![Figure 6.5](image2.png)

**Figure 6.5:** Number of decay electrons for background and signal in $p \rightarrow e^+\mu^+\mu^-$ (left) and $p \rightarrow \mu^+\mu^+\mu^-$ (right) modes after all previous cuts. Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary.
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Figure 6.6: Comparison of number of decay electrons between $p \rightarrow \mu^+ e^+ e^-$ and $p \rightarrow \mu^- e^+ e^+$ (left) or $p \rightarrow e^+ e^+ \mu^-$ and $p \rightarrow e^- \mu^+ \mu^+$ (right) modes after all previous cuts.

Figure 6.7: Comparison of number of decay electrons in $p \rightarrow \mu^+ e^+ e^-$ for each period after all previous cuts.
6.1.4 \(\pi^0\) veto

Main background for \(p \rightarrow e^+e^-e^-\) and \(p \rightarrow \mu^+e^-e^-\) modes are charged current \(\pi^0\) production events which decays to 2 \(\gamma\). Mass of 2 \(e\)-like rings can be a good parameter for the separation of signal and such backgrounds. But in case of \(p \rightarrow e^+e^-e^-\) mode, background rejection is not so good compared to the loss of the signal. Therefore this requirement is only applied for \(p \rightarrow \mu^+e^-e^-\) mode. In order to reduce \(\pi^0\) production events, events with mass of 2 \(e\)-like rings below 185 MeV are rejected. Mass of 2 \(e\)-like rings for \(p \rightarrow e^+e^-e^-\) and background is shown in Fig. 6.8. \(p \rightarrow \mu^-e^+e^+\) shows almost the same distribution.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{mass_of_2_e_like_rings.png}
\caption{Mass of 2 \(e\)-like rings after all previous cut for background and signal in \(p \rightarrow e^+e^-e^-\) (left) and \(p \rightarrow \mu^+e^-e^-\) (right) modes. Arrows and dotted lines show selected events by this selection. Signal of free proton is also shown by dotted line. Scale of signal is just arbitral. In \(p \rightarrow e^+e^-e^-\) mode, a pair with closest mass to true \(\pi^0\) mass (135 MeV) is chosen. \(\pi^0\) mass is not used in the selection of \(p \rightarrow e^+e^-e^-\) mode.}
\end{figure}

6.1.5 Neutron veto

The probability of neutron generated by deexcitation of a nucleus after proton decay is rather small, and neutrons are not generated from free proton decay. Furthermore, proton decay events do not produce neutrons by secondary interactions because there are only charged leptons in final states. On the other hand, neutron is often emitted by the interaction of the atmospheric neutrino and the nucleus in water. Emitted neutron is captured by nucleus in water and delayed \(\gamma\) signal is emitted. Searching for this \(\gamma\) signal, neutron can be tagged. Detail of this neutron tagging technique is described in section 5.6. Information of this tagged neutron can be a good parameter for the separation of signal and background. The number of tagged neutron is required to be 0 for all modes only in SK-IV because the neutron tagging technique is only available in SK-IV. Number of tagged neutron for \(p \rightarrow e^+e^-e^-\) and background after all previous cuts is shown in Fig. 6.9. About 5% of signal events have tagged neutron because neutron is sometimes emitted in signal by correlated decay. Basically more than 90% of signal can be kept and about 50% of background events in SK-IV are rejected after this requirement for all modes.
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Number of tagged neutron

Figure 6.9: The number of tagged neutrons after all previous cuts for background and signal in \( p \rightarrow e^+e^+e^- \) mode. Arrows and dotted lines show selected events by this selection. Signal of free proton is also shown by dotted line. Scale of signal is arbitrary.

6.1.6 Total mass and momentum

Invariant mass (total mass) and momentum (total momentum) reconstructed from 3 rings are the most important parameters in selection criteria. Basically these values correspond to the initial proton mass and momentum in signal mode. Total mass and momentum for signal and background after all previous cuts without the requirement of the number of tagged neutron with respect to each mode are shown in Fig.6.10-6.13. Total mass makes a peak around the true proton mass at 938 MeV in signal events. Hence total mass is required to be between 800 MeV and 1,050 MeV. Total momentum for free proton distributes at lower momentum region as the true momentum of free proton is almost 0. Total momentum for bound proton distributes at relatively higher momentum region as the true bound proton has finite initial momentum due to Fermi motion. Total momentum is required to be below 250 MeV. In order to make almost background free selection criteria, requirement for total momentum is further separated to 2 categories: Low signal box (total momentum below 100 MeV) and High signal box (total momentum between 100 MeV and 250 MeV). The effect of the correlated decay makes the lower and higher tail of total mass and total momentum, respectively.

6.1.7 Selection summary

Selection criteria for each mode is summarized in Table6.1. The signal efficiency and expected background events after all selections are summarized in Table6.2. The signal efficiency is defined as the number of signal events after all selection criteria divided by the initial number of signal events with true \( D_{\text{wall}} \) above 2 m. Cut flow for the signal and background are summarized in Table6.3-6.8 and Table6.9-6.12, respectively. The interaction modes of background events remained in signal box for 2,000 years MC are also summarized in Table6.13. Event displays for signal and typical background events in each mode are shown in Fig.6.14 - 6.17 and Fig.6.18 - 6.21, respectively.
Figure 6.10: Total mass (left) and total momentum (right) for background and signal in $p \rightarrow e^+e^+e^-$ (left) mode after all previous cuts without neutron tagging. Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary.

Figure 6.11: Total mass (left) and total momentum (right) for background and signal in $p \rightarrow \mu^+e^+e^-$ (left) mode after all previous cuts without neutron tagging. Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary.
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Figure 6.12: Total mass (left) and total momentum (right) for background and signal in $p \rightarrow e^+\mu^+\mu^-$ (left) mode after all previous cuts without neutron tagging. Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary.

Figure 6.13: Total mass (left) and total momentum (right) for background and signal in $p \rightarrow \mu^+\mu^+\mu^-$ (left) mode after all previous cuts without neutron tagging. Arrows and dotted lines show selected events by this selection. Signal of free protons is also shown by dotted line. Scale of signal is arbitrary.
Table 6.1: Summary of selection criteria for each mode.

<table>
<thead>
<tr>
<th></th>
<th>( p \rightarrow e^+e^- )</th>
<th>( p \rightarrow \mu^+e^- ) ((p \rightarrow \mu^-e^+))</th>
<th>( p \rightarrow e^+\mu^+\mu^- ) ((p \rightarrow e^-\mu^+\mu^-))</th>
<th>( p \rightarrow \mu^+\mu^+\mu^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC &amp; FV</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Number of rings</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>PID</td>
<td>3 (e)-like</td>
<td>2 (e)-like &amp; 1 (\mu)-like</td>
<td>1 (e)-like &amp; 2 (\mu)-like</td>
<td>3 (\mu)-like</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2 or 3</td>
</tr>
<tr>
<td>(\pi^0) veto ((M &gt; 185\ MeV))</td>
<td>-</td>
<td>○</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Number of neutron (\text{(SK-IV)})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total mass ((800 &lt; M &lt; 1,050\ MeV))</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Total momentum Low ((P &lt; 100\ MeV))</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Total momentum High ((100 &lt; P &lt; 250\ MeV))</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

Table 6.2: Summary of signal efficiency, expected background events with respect to each mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Efficiency (%)</th>
<th>Background (events)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \rightarrow e^+e^- ) (\text{(Low)})</td>
<td>I 22.7 II 19.8 III 23.1 IV 22.4</td>
<td>I &lt;0.01 II &lt;0.01 III &lt;0.01 IV &lt;0.01</td>
</tr>
<tr>
<td>( p \rightarrow e^+e^- ) (\text{(High)})</td>
<td>I 43.9 II 40.4 III 44.3 IV 41.1</td>
<td>I 0.19±0.04 II 0.10±0.02 III 0.05±0.01 IV 0.24±0.07</td>
</tr>
<tr>
<td>( p \rightarrow \mu^+e^- ) (\text{(Low)})</td>
<td>I 15.0 II 13.5 III 16.3 IV 17.6</td>
<td>I 0.02±0.01 II 0.02±0.01 III 0.01±0.00 IV &lt;0.01</td>
</tr>
<tr>
<td>( p \rightarrow \mu^+e^- ) (\text{(High)})</td>
<td>I 27.1 II 26.0 III 27.3 IV 30.3</td>
<td>I 0.13±0.03 II 0.10±0.02 III 0.05±0.01 IV 0.17±0.05</td>
</tr>
<tr>
<td>( p \rightarrow \mu^-e^+ ) (\text{(Low)})</td>
<td>I 11.9 II 11.1 III 12.6 IV 14.9</td>
<td>I 0.02±0.01 II 0.02±0.01 III 0.01±0.00 IV &lt;0.01</td>
</tr>
<tr>
<td>( p \rightarrow \mu^-e^+ ) (\text{(High)})</td>
<td>I 20.8 II 19.8 III 22.3 IV 25.9</td>
<td>I 0.13±0.03 II 0.10±0.02 III 0.05±0.01 IV 0.17±0.05</td>
</tr>
<tr>
<td>( p \rightarrow e^+\mu^+\mu^- ) (\text{(Low)})</td>
<td>I 9.2 II 8.1 III 9.1 IV 11.7</td>
<td>I &lt;0.01 II &lt;0.01 III &lt;0.01 IV &lt;0.01</td>
</tr>
<tr>
<td>( p \rightarrow e^+\mu^+\mu^- ) (\text{(High)})</td>
<td>I 15.8 II 14.1 III 16.2 IV 20.9</td>
<td>I 0.09±0.02 II 0.07±0.02 III 0.03±0.01 IV 0.08±0.03</td>
</tr>
<tr>
<td>( p \rightarrow e^-\mu^+\mu^- ) (\text{(Low)})</td>
<td>I 11.1 II 10.9 III 11.9 IV 14.4</td>
<td>I &lt;0.01 II &lt;0.01 III &lt;0.01 IV &lt;0.01</td>
</tr>
<tr>
<td>( p \rightarrow e^-\mu^+\mu^- ) (\text{(High)})</td>
<td>I 19.9 II 18.2 III 20.0 IV 24.2</td>
<td>I 0.09±0.02 II 0.07±0.02 III 0.03±0.01 IV 0.08±0.03</td>
</tr>
<tr>
<td>( p \rightarrow \mu^-\mu^+\mu^- ) (\text{(Low)})</td>
<td>I 10.8 II 10.4 III 12.0 IV 12.2</td>
<td>I &lt;0.01 II &lt;0.01 III &lt;0.01 IV &lt;0.01</td>
</tr>
<tr>
<td>( p \rightarrow \mu^-\mu^+\mu^- ) (\text{(High)})</td>
<td>I 19.9 II 17.2 III 20.4 IV 20.4</td>
<td>I 0.10±0.03 II 0.05±0.01 III 0.03±0.01 IV 0.22±0.06</td>
</tr>
</tbody>
</table>
### 6.1. EVENT SELECTION

Table 6.3: Signal efficiency [%] at each step of selection criteria for $p \to e^+e^-e^-$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>98.7±1.5</td>
<td>98.7±1.5</td>
<td>98.8±1.5</td>
<td>98.9±1.5</td>
</tr>
<tr>
<td>Number of rings</td>
<td>80.4±1.3</td>
<td>76.7±1.3</td>
<td>81.0±1.3</td>
<td>81.1±1.3</td>
</tr>
<tr>
<td>PID</td>
<td>75.1±1.3</td>
<td>68.6±1.2</td>
<td>75.8±1.3</td>
<td>75.5±1.3</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>75.1±1.3</td>
<td>68.4±1.2</td>
<td>75.8±1.3</td>
<td>75.5±1.3</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>66.6±1.1</td>
<td>60.2±1.0</td>
<td>67.4±1.1</td>
<td>66.5±1.0</td>
</tr>
<tr>
<td>Neutron veto</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>63.5±1.0</td>
</tr>
<tr>
<td>Low signal box</td>
<td>22.7±0.6</td>
<td>19.8±0.5</td>
<td>23.1±0.6</td>
<td>22.4±0.6</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(14.3±0.4)</td>
<td>(12.1±0.4)</td>
<td>(14.4±0.4)</td>
<td>(14.0±0.4)</td>
</tr>
<tr>
<td>High signal box</td>
<td>43.9±0.9</td>
<td>40.4±0.8</td>
<td>44.3±0.9</td>
<td>41.1±0.8</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(1.0±0.1)</td>
<td>(1.4±0.1)</td>
<td>(0.9±0.1)</td>
<td>(0.9±0.1)</td>
</tr>
</tbody>
</table>

Table 6.4: Signal efficiency [%] at each step of selection criteria for $p \to \mu^+e^+e^-$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
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<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>98.9±1.5</td>
<td>99.0±1.5</td>
<td>99.1±1.5</td>
<td>98.9±1.5</td>
</tr>
<tr>
<td>Number of rings</td>
<td>67.0±1.2</td>
<td>65.0±1.3</td>
<td>67.5±1.2</td>
<td>68.2±1.2</td>
</tr>
<tr>
<td>PID</td>
<td>62.3±1.1</td>
<td>58.9±1.1</td>
<td>63.5±1.1</td>
<td>63.2±1.1</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>50.8±1.0</td>
<td>47.3±0.9</td>
<td>52.3±1.0</td>
<td>60.5±1.1</td>
</tr>
<tr>
<td>$\pi^0$ veto</td>
<td>46.8±0.9</td>
<td>43.9±0.9</td>
<td>48.3±0.9</td>
<td>55.6±1.0</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>42.0±0.8</td>
<td>39.5±0.8</td>
<td>43.5±0.8</td>
<td>50.3±0.9</td>
</tr>
<tr>
<td>Neutron veto</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>47.8±0.9</td>
</tr>
<tr>
<td>Low signal box</td>
<td>15.0±0.5</td>
<td>13.5±0.4</td>
<td>16.3±0.5</td>
<td>17.6±0.5</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(9.2±0.3)</td>
<td>(8.3±0.3)</td>
<td>(9.9±0.3)</td>
<td>(10.9±0.4)</td>
</tr>
<tr>
<td>High signal box</td>
<td>27.0±0.6</td>
<td>26.0±0.6</td>
<td>27.2±0.6</td>
<td>30.2±0.7</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(0.3±0.1)</td>
<td>(0.7±0.1)</td>
<td>(0.3±0.1)</td>
<td>(0.3±0.1)</td>
</tr>
</tbody>
</table>
Table 6.5: Signal efficiency [%] at each step of selection criteria for $p \rightarrow \mu^- e^+ e^+$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
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<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>98.9±1.5</td>
<td>98.9±1.5</td>
<td>99.1±1.5</td>
<td>98.9±1.5</td>
</tr>
<tr>
<td>Number of rings</td>
<td>66.1±1.2</td>
<td>64.8±1.1</td>
<td>68.1±1.2</td>
<td>68.1±1.2</td>
</tr>
<tr>
<td>PID</td>
<td>61.7±1.1</td>
<td>58.7±1.1</td>
<td>64.0±1.1</td>
<td>63.2±1.1</td>
</tr>
<tr>
<td>$\pi^0$ veto</td>
<td>39.9±0.8</td>
<td>37.5±0.8</td>
<td>42.0±0.8</td>
<td>51.3±1.0</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>32.7±0.7</td>
<td>30.9±0.7</td>
<td>34.9±0.7</td>
<td>42.4±0.8</td>
</tr>
<tr>
<td>Neutron veto</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>40.7±0.8</td>
</tr>
<tr>
<td>Low signal box</td>
<td>11.9±0.4</td>
<td>11.1±0.4</td>
<td>12.6±0.4</td>
<td>14.8±0.5</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(7.7±0.3)</td>
<td>(6.3±0.3)</td>
<td>(7.7±0.3)</td>
<td>(9.3±0.4)</td>
</tr>
<tr>
<td>High signal box</td>
<td>20.8±0.6</td>
<td>19.8±0.5</td>
<td>22.3±0.6</td>
<td>25.8±0.6</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(0.2±0.0)</td>
<td>(0.5±0.1)</td>
<td>(0.2±0.1)</td>
<td>(0.3±0.1)</td>
</tr>
</tbody>
</table>

Table 6.6: Signal efficiency [%] at each step of selection criteria for $p \rightarrow e^+ \mu^+ \mu^-$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>99.0±1.5</td>
<td>99.1±1.5</td>
<td>99.1±1.5</td>
<td>99.0±1.5</td>
</tr>
<tr>
<td>Number of rings</td>
<td>52.7±1.0</td>
<td>51.1±1.0</td>
<td>53.9±1.0</td>
<td>52.5±1.0</td>
</tr>
<tr>
<td>PID</td>
<td>49.6±0.9</td>
<td>46.2±0.9</td>
<td>50.5±1.0</td>
<td>49.4±0.9</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>27.5±0.7</td>
<td>24.6±0.6</td>
<td>28.0±0.7</td>
<td>37.3±0.8</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>25.1±0.6</td>
<td>22.2±0.5</td>
<td>25.3±0.6</td>
<td>34.0±0.7</td>
</tr>
<tr>
<td>Neutron veto</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>32.5±0.7</td>
</tr>
<tr>
<td>Low signal box</td>
<td>9.2±0.3</td>
<td>8.1±0.3</td>
<td>9.1±0.3</td>
<td>11.7±0.4</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(6.0±0.3)</td>
<td>(5.0±0.3)</td>
<td>(5.9±0.3)</td>
<td>(7.3±0.3)</td>
</tr>
<tr>
<td>High signal box</td>
<td>15.8±0.5</td>
<td>14.1±0.3</td>
<td>16.2±0.5</td>
<td>20.9±0.6</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(0.1±0.0)</td>
<td>(0.3±0.1)</td>
<td>(0.1±0.0)</td>
<td>(0.2±0.1)</td>
</tr>
</tbody>
</table>
### 6.1. EVENT SELECTION

Table 6.7: Signal efficiency [%] at each step of selection criteria for $p \rightarrow e^- \mu^+ \mu^+$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>98.9±1.5</td>
<td>98.9±1.5</td>
<td>99.2±1.5</td>
<td>98.9±1.5</td>
</tr>
<tr>
<td>Number of rings</td>
<td>52.4±1.0</td>
<td>51.8±1.0</td>
<td>52.7±1.0</td>
<td>53.1±1.0</td>
</tr>
<tr>
<td>PID</td>
<td>49.6±1.0</td>
<td>47.0±0.9</td>
<td>49.9±0.9</td>
<td>49.8±1.0</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>34.1±0.7</td>
<td>32.2±0.7</td>
<td>34.8±0.8</td>
<td>44.5±0.9</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>31.0±0.7</td>
<td>29.2±0.6</td>
<td>31.8±0.7</td>
<td>40.5±0.8</td>
</tr>
<tr>
<td>Neutron veto</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>38.6±0.8</td>
</tr>
<tr>
<td>Low signal box</td>
<td>11.1±0.4</td>
<td>10.9±0.4</td>
<td>11.9±0.4</td>
<td>14.4±0.4</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(7.1±0.3)</td>
<td>(6.9±0.3)</td>
<td>(7.4±0.3)</td>
<td>(9.3±0.4)</td>
</tr>
<tr>
<td>High signal box</td>
<td>19.9±0.5</td>
<td>18.2±0.5</td>
<td>20.0±0.5</td>
<td>24.2±0.6</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(0.1±0.0)</td>
<td>(0.2±0.1)</td>
<td>(0.1±0.0)</td>
<td>(0.2±0.0)</td>
</tr>
</tbody>
</table>

Table 6.8: Signal efficiency [%] at each step of selection criteria for $p \rightarrow \mu^+ \mu^+ \mu^-$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>98.8±1.5</td>
<td>98.4±1.5</td>
<td>98.7±1.5</td>
<td>98.6±1.5</td>
</tr>
<tr>
<td>Number of rings</td>
<td>38.2±0.8</td>
<td>35.4±0.8</td>
<td>40.3±0.8</td>
<td>38.7±0.8</td>
</tr>
<tr>
<td>PID</td>
<td>37.4±0.8</td>
<td>33.8±0.7</td>
<td>39.2±0.8</td>
<td>37.6±0.8</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>32.9±0.7</td>
<td>29.7±0.7</td>
<td>34.9±0.8</td>
<td>36.4±0.8</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>30.7±0.7</td>
<td>27.6±0.6</td>
<td>32.3±0.7</td>
<td>34.0±0.7</td>
</tr>
<tr>
<td>Neutron veto</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>32.6±0.7</td>
</tr>
<tr>
<td>Low signal box</td>
<td>10.8±0.4</td>
<td>10.4±0.4</td>
<td>12.0±0.4</td>
<td>12.2±0.4</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(6.5±0.3)</td>
<td>(6.5±0.3)</td>
<td>(7.5±0.3)</td>
<td>(7.6±0.3)</td>
</tr>
<tr>
<td>High signal box</td>
<td>19.9±0.5</td>
<td>17.2±0.5</td>
<td>20.4±0.5</td>
<td>24.2±0.5</td>
</tr>
<tr>
<td>(Free proton)</td>
<td>(&lt;0.1)</td>
<td>(0.1±0.0)</td>
<td>(0.1±0.0)</td>
<td>(0.1±0.0)</td>
</tr>
</tbody>
</table>

Table 6.9: Expected background events at each step of selection criteria for $p \rightarrow e^+e^+e^-$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>17,380±12</td>
<td>9,225±6</td>
<td>6,138±4</td>
<td>38,627±26</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>12,097±10</td>
<td>6,546±5</td>
<td>4,285±3</td>
<td>26,563±21</td>
</tr>
<tr>
<td>Number of rings</td>
<td>966±3</td>
<td>555±2</td>
<td>340±1</td>
<td>2,247±6</td>
</tr>
<tr>
<td>PID</td>
<td>460±2</td>
<td>243±1</td>
<td>157±1</td>
<td>1,038±4</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>270±1</td>
<td>152±1</td>
<td>97±1</td>
<td>568±3</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>0.18±0.04</td>
<td>0.11±0.02</td>
<td>0.05±0.01</td>
<td>0.58±0.10</td>
</tr>
<tr>
<td>Neutron veto</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.23±0.06</td>
</tr>
<tr>
<td>Low signal box</td>
<td>&lt;0.01</td>
<td>0.01±0.01</td>
<td>&lt;0.01</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>High signal box</td>
<td>0.18±0.04</td>
<td>0.10±0.02</td>
<td>0.05±0.01</td>
<td>0.23±0.06</td>
</tr>
</tbody>
</table>
### Table 6.10: Expected background events at each step of selection criteria for $p \to \mu^+e^+e^-$ and $p \to \mu^-e^+e^+$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>17,380±12</td>
<td>9,225±6</td>
<td>6,138±4</td>
<td>38,627±26</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>12,097±10</td>
<td>6,546±5</td>
<td>4,285±3</td>
<td>26,563±21</td>
</tr>
<tr>
<td>Number of rings</td>
<td>966±3</td>
<td>555±2</td>
<td>340±1</td>
<td>2,247±6</td>
</tr>
<tr>
<td>PID</td>
<td>372±2</td>
<td>218±1</td>
<td>135±1</td>
<td>879±4</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>166±1</td>
<td>90±1</td>
<td>61±0</td>
<td>440±3</td>
</tr>
<tr>
<td>$\pi^0$ mass</td>
<td>81±1</td>
<td>47±0</td>
<td>30±0</td>
<td>202±2</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>0.15±0.03</td>
<td>0.12±0.02</td>
<td>0.06±0.01</td>
<td>0.56±0.10</td>
</tr>
<tr>
<td>Neutron veto</td>
<td></td>
<td></td>
<td></td>
<td>0.18±0.06</td>
</tr>
</tbody>
</table>

### Table 6.11: Expected background events at each step of selection criteria for $p \to e^+\mu^+\mu^-$ and $p \to e^-\mu^+\mu^+$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>17,380±12</td>
<td>9,225±6</td>
<td>6,138±4</td>
<td>38,627±26</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>12,097±10</td>
<td>6,546±5</td>
<td>4,285±3</td>
<td>26,563±21</td>
</tr>
<tr>
<td>Number of rings</td>
<td>966±3</td>
<td>555±2</td>
<td>340±1</td>
<td>2,247±6</td>
</tr>
<tr>
<td>PID</td>
<td>108±1</td>
<td>78±1</td>
<td>38±0</td>
<td>265±2</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>17±0</td>
<td>11±0</td>
<td>6±0</td>
<td>58±1</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>0.09±0.02</td>
<td>0.07±0.01</td>
<td>0.03±0.01</td>
<td>0.30±0.07</td>
</tr>
<tr>
<td>Neutron veto</td>
<td></td>
<td></td>
<td></td>
<td>0.07±0.03</td>
</tr>
</tbody>
</table>

### Table 6.12: Expected background events at each step of selection criteria for $p \to \mu^+\mu^+\mu^-$ mode. The error values correspond to the statistic uncertainty of MC sample.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>17,380±12</td>
<td>9,225±6</td>
<td>6,138±4</td>
<td>38,627±26</td>
</tr>
<tr>
<td>FC &amp; FV</td>
<td>12,097±10</td>
<td>6,546±5</td>
<td>4,285±3</td>
<td>26,563±21</td>
</tr>
<tr>
<td>Number of rings</td>
<td>966±3</td>
<td>555±2</td>
<td>340±1</td>
<td>2,247±6</td>
</tr>
<tr>
<td>PID</td>
<td>8±0</td>
<td>5±0</td>
<td>3±0</td>
<td>28±1</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>0.10±0.03</td>
<td>0.05±0.01</td>
<td>0.03±0.01</td>
<td>0.47±0.08</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td></td>
<td></td>
<td></td>
<td>0.21±0.06</td>
</tr>
<tr>
<td>Neutron veto</td>
<td></td>
<td></td>
<td></td>
<td>0.21±0.06</td>
</tr>
<tr>
<td>Low signal box</td>
<td>0.01±0.01</td>
<td>&lt;0.01</td>
<td>&lt;0.01</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>High signal box</td>
<td>0.09±0.03</td>
<td>0.05±0.01</td>
<td>0.03±0.01</td>
<td>0.21±0.06</td>
</tr>
</tbody>
</table>
Table 6.13: The expected fraction of the interaction modes [%] for background events remained in signal box.

<table>
<thead>
<tr>
<th>Interaction Mode</th>
<th>p → e⁺e⁻e⁻</th>
<th>p → μ⁺e⁺e⁻</th>
<th>p → e⁺μ⁺μ⁻</th>
<th>p → μ⁺μ⁺μ⁻</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCQE</td>
<td>12.9</td>
<td>7.1</td>
<td>9.7</td>
<td>21.4</td>
</tr>
<tr>
<td>CC single π</td>
<td>41.0</td>
<td>32.2</td>
<td>34.0</td>
<td>58.2</td>
</tr>
<tr>
<td>CC multi π</td>
<td>11.8</td>
<td>27.2</td>
<td>42.5</td>
<td>12.5</td>
</tr>
<tr>
<td>CC η</td>
<td>9.0</td>
<td>6.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CC K</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>CC DIS</td>
<td>3.9</td>
<td>6.2</td>
<td>4.7</td>
<td>3.4</td>
</tr>
<tr>
<td>NC single π</td>
<td>1.2</td>
<td>1.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NC multi π</td>
<td>3.8</td>
<td>6.7</td>
<td>2.4</td>
<td>2.0</td>
</tr>
<tr>
<td>NC η</td>
<td>0</td>
<td>1.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NC DIS</td>
<td>16.3</td>
<td>11.9</td>
<td>6.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Figure 6.14: Simulated event displays of signal events in $p \rightarrow e^+e^+e^-$ mode. The left figure shows true information and the right figure shows reconstructed information. Type of true particles are described in left figure. In right figure, blue circle shows TDC fit result, red and light blue bold circles show ring fit result for $e$-like and $\mu$-like, respectively.
Figure 6.15: Simulated event displays of signal events in $p \rightarrow \mu^+e^+e^-$ mode. The left figure shows true information and the right figure shows reconstructed information. Type of true particles are described in left figure. In right figure, blue circle shows TDC fit result, red and light blue bold circles show ring fit result for $e$-like and $\mu$-like, respectively.

Figure 6.16: Simulated event displays of signal events in $p \rightarrow e^+\mu^+\mu^-$ mode. The left figure shows true information and the right figure shows reconstructed information. Type of true particles are described in left figure. In right figure, blue circle shows TDC fit result, red and light blue bold circles show ring fit result for $e$-like and $\mu$-like, respectively.
6.1. EVENT SELECTION

Figure 6.17: Simulated event displays of signal events in $p \rightarrow \mu^+ \mu^+ \mu^-$ mode. The left figure shows true information and the right figure shows reconstructed information. Type of true particles are described in left figure. In right figure, blue circle shows TDC fit result, red and light blue bold circles show ring fit result for $e$-like and $\mu$-like, respectively.

Figure 6.18: Simulated event displays of typical background events in $p \rightarrow e^+ e^- e^-$ mode. The left figure shows true information and the right figure shows reconstructed information. Type of true particles are described in left figure. In right figure, blue circle shows TDC fit result, red and light blue bold circles show ring fit result for $e$-like and $\mu$-like, respectively. The interaction mode of this event is CC single $\pi$ production ($\nu_e n \rightarrow e^- n \pi^0$).
Figure 6.19: Simulated event displays of typical background events in $p \rightarrow \mu^+ e^+ e^-$ mode. The left figure shows true information and the right figure shows reconstructed information. Type of true particles are described in left figure. In right figure, blue circle shows TDC fit result, red and light blue bold circles show ring fit result for $e$-like and $\mu$-like, respectively. The interaction mode of this event is CC single $\pi$ production ($\nu_\mu n \rightarrow \mu^- p\pi^0$).

Figure 6.20: Simulated event displays of typical background events in $p \rightarrow e^+ \mu^+ \mu^-$ mode. The left figure shows true information and the right figure shows reconstructed information. Type of true particles are described in left figure. In right figure, blue circle shows TDC fit result, red and light blue bold circles show ring fit result for $e$-like and $\mu$-like, respectively. The interaction mode of this event is CC multi $\pi$ production ($\nu_\mu p \rightarrow \mu^- p\pi^+ \pi^0$).
6.2. SYSTEMATIC UNCERTAINTY

Estimation for systematic uncertainties for signal and background samples with respect to each SK period is described in this section. Since observation time is well defined and SK tank is always full of water, systematic uncertainty for the detector exposure is negligible. But 1% error for the detector exposure is assigned to be conservative.

6.2.1 Correlated decay

In the theory, correlated decay is considered to be happened with about 10% probability[37]. Since this effect is not understood well, 100% source error is considered to be conservative. Therefore, signal samples with 20% probability of correlated decay and 0% probability are created with respect to each mode and compared with nominal sample of 10% probability. Total mass and momentum distributions are mainly changed depending on the probability of correlated decay. Comparison of total mass and momentum with respect to each probability of correlated decay for \( p \rightarrow e^-e^+\mu^- \) are shown in Fig.6.22. Ratios of signal efficiency in low or high signal box between nominal 10% probability and 0% or 20% probability is calculated. Then larger value with respect to each box is assigned as the systematic uncertainty of the correlated decay only for the signal.

6.2.2 Fermi motion

In the signal MC sample, Fermi motion is simulated based on the electron-\(^{12}\)C scattering experiment[107]. On the other hand, in the background MC sample it’s based on Fermi gas
model. Hence this model difference can be a source of systematic uncertainty. In order to estimate the error value, a signal sample simulated with Fermi gas model is created. Then the signal efficiencies in low or high signal box is compared with the nominal signal sample. Initial proton momentum distributions of the two models are shown in Fig. 6.23. The difference is mainly shown in total momentum distribution as shown in Fig. 6.24. The ratio between two different model is assigned as the systematic uncertainty of the Fermi motion only for the signal.

Figure 6.23: Comparison of initial proton momentum between Fermi gas model and experimental spectrum[107]. The vertical error bars correspond to the statistical uncertainty of the MC sample.

Figure 6.22: Total mass (left) and total momentum (right) for \( p \to e^+\mu^+\mu^- \) (left) signal after the number of ring and PID cuts. Each histogram corresponds to the samples with each probability of correlated decay. The vertical error bars correspond to the statistical uncertainty of the MC sample.
6.2. SYSTEMATIC UNCERTAINTY

Figure 6.24: Comparison of total momentum after number of ring and PID cuts between Fermi gas model and experimental spectrum for $p \rightarrow e^+ e^+ e^-$. The vertical error bars correspond to the statistical uncertainty of the MC sample.

6.2.3 Neutrino flux & cross section

The systematic errors for atmospheric neutrino flux and cross section are estimated by event-by-event weighting method. The source errors for this method are already estimated in the latest neutrino oscillation analysis in SK[108]. The ratio of background events remaining in the signal boxes between default and weighted samples is assigned as the systematic error. In order to cover the low statistics in the signal box, 2,000 years MC is used for estimation. Each systematic error for neutrino flux and cross section is summarized in Table 6.14, 6.15.

6.2.4 Pion FSI/SI

A neutral pion generated by neutrino interaction in oxygen can interact with nucleus in that oxygen. By this effect so called pion final state interaction (FSI), pion can be scattered, be absorbed or exchange its charge. Generated pion can also interact with other nucleus in water after escaping from the original nucleus. This effect is called as pion secondary interaction (SI). These interactions are simulated by the NEUT pion cascade model described in section 3.2.3. 17 parameter sets are prepared and the number of events are calculated with respect to each set. For the systematic uncertainty for Pion FSI/SI, RMS/Mean value of remaining background events for 17 parameter sets is used.

6.2.5 Fiducial volume

A systematic uncertainty for the fiducial volume is considered as the difference of entry or exit events at the boundary of FV ($D_{\text{wall}} = 200$ cm) between data and MC. FC samples of 3 rings events are used for this estimation. $D_{\text{wall}}$ distributions are compared between data and background MC which is area normalized to data. In order to determine the area for MC normalization, true $D_{\text{wall}}$ value is smeared by gaussian with $\sigma = 40$ cm, 100 cm, then compared with reconstructed $D_{\text{wall}}$ in background MC. Fig.6.25 shows a comparison of smeared
Table 6.14: Systematic sources and their estimated error values [%] for neutrino flux in each mode. The detail of each systematics are summarized in [108].

<table>
<thead>
<tr>
<th>Systematic Source</th>
<th>$p \rightarrow e^+e^-$</th>
<th>$p \rightarrow \mu^+e^-$</th>
<th>$p \rightarrow e^+\mu^+\mu^-$</th>
<th>$p \rightarrow \mu^+\mu^+\mu^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute normalization</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(E_\nu &lt; 1 \text{ GeV})$</td>
<td>0.6</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>0.7</td>
</tr>
<tr>
<td>$(E_\nu &gt; 1 \text{ GeV})$</td>
<td>6.7</td>
<td>7.1</td>
<td>7.0</td>
<td>6.4</td>
</tr>
<tr>
<td>$(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ ratio</td>
<td>&lt;0.1</td>
<td>0.2</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>$(1 &lt; E_\nu &lt; 10 \text{ GeV})$</td>
<td>0.2</td>
<td>0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>$(E_\nu &gt; 10 \text{ GeV})$</td>
<td>0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>$\nu_e/\nu_\mu$ ratio</td>
<td>1.7</td>
<td>1.1</td>
<td>0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>$(E_\nu &lt; 1 \text{ GeV})$</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>$(1 &lt; E_\nu &lt; 10 \text{ GeV})$</td>
<td>0.2</td>
<td>1.1</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$(E_\nu &gt; 10 \text{ GeV})$</td>
<td>0.3</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Relative normalization FC multi-GeV</td>
<td>0.1</td>
<td>0.9</td>
<td>3.3</td>
<td>4.3</td>
</tr>
<tr>
<td>Up/down asymmetry</td>
<td>&lt;0.1</td>
<td>0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Horizontal/vertical ratio</td>
<td>&lt;0.1</td>
<td>0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>$K/\pi$ production ratio</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Neutrino flight length</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Total</td>
<td>7.0</td>
<td>7.3</td>
<td>8.2</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Table 6.15: Systematic sources and their estimated error values [%] for neutrino cross section in each mode. The detail of each systematics are summarized in [108].

<table>
<thead>
<tr>
<th>Systematic Source</th>
<th>$p \rightarrow e^+e^-$</th>
<th>$p \rightarrow \mu^+e^-$</th>
<th>$p \rightarrow e^+\mu^+\mu^-$</th>
<th>$p \rightarrow \mu^+\mu^+\mu^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_A$ in QE</td>
<td>5.4</td>
<td>3.4</td>
<td>4.8</td>
<td>8.8</td>
</tr>
<tr>
<td>Single $\pi$ production, Axial coupling</td>
<td>6.7</td>
<td>6.9</td>
<td>10.8</td>
<td>11.0</td>
</tr>
<tr>
<td>Single $\pi$ production, $C_{A5}$</td>
<td>2.7</td>
<td>2.8</td>
<td>3.1</td>
<td>4.9</td>
</tr>
<tr>
<td>Single $\pi$ production, Background</td>
<td>1.6</td>
<td>1.7</td>
<td>1.1</td>
<td>0.7</td>
</tr>
<tr>
<td>CCQE $\nu/\nu$ ratio</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>CCQE $\mu/\nu$ ratio</td>
<td>0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>DIS model difference</td>
<td>9.5</td>
<td>13.8</td>
<td>19.8</td>
<td>6.7</td>
</tr>
<tr>
<td>DIS cross section</td>
<td>1.8</td>
<td>2.6</td>
<td>2.8</td>
<td>1.0</td>
</tr>
<tr>
<td>DIS $Q^2$ distribution (high $W$)</td>
<td>&lt;0.1</td>
<td>0.2</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>DIS $Q^2$ distribution (low $W$)</td>
<td>&lt;0.1</td>
<td>0.2</td>
<td>1.7</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>Coherent $\pi^+$ cross section</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>2.1</td>
</tr>
<tr>
<td>NC/CC ratio</td>
<td>4.3</td>
<td>4.3</td>
<td>1.8</td>
<td>0.9</td>
</tr>
<tr>
<td>Total</td>
<td>14.0</td>
<td>16.9</td>
<td>23.6</td>
<td>16.6</td>
</tr>
</tbody>
</table>
true $D_{\text{wall}}$ and reconstructed $D_{\text{wall}}$. In $D_{\text{wall}}$ between 600 cm and 1,000 cm, reconstructed and each smeared distributions agree well. This means that, the difference of the accuracy for the vertex reconstruction between data and MC does not affect on the normalization in this area. Therefore, $600 < D_{\text{wall}} < 1,000$ cm is defined as the area for the MC normalization.

Fig.6.26,6.27 shows reconstructed $D_{\text{wall}}$ for data and MC after area normalization with respect to each period. Ratio of the number of events in fiducial volume ($D_{\text{wall}} > 200$ cm) between data and MC is assigned as the systematic uncertainty of fiducial volume.

![Graph](image)

Figure 6.25: Comparison of reconstructed, true and smeared true distance to wall of background MC.

![Graph](image)

Figure 6.26: Distance to wall for data and MC after area normalization in SK-I (left) and SK-II (right). MC is normalized to data in the region $600 < D_{\text{wall}} < 1,000$ cm.

### 6.2.6 Detector non-uniformity

Energy scale of the Cherenkov rings depends on the position they are detected. This effect is called the detector non-uniformity and its difference between data and MC can be a source of
systematic uncertainty. This is already estimated in the latest neutrino oscillation analysis\cite{108} by using Michel electron sample. The error values are estimated to be 0.6%, 0.6%, 1.3% and 0.5% for SK-I, II, III and IV respectively (section\ref{subsec:7.9}). In case two charged particles decay back to back, detector non-uniformity error for total momentum of these two particles should be twice of the source error. On the other hand, now there are isotropically decayed three charged leptons in this analysis. This non-uniformity error should be less than twice of the source error. But to be conservative, twice of the detector non-uniformity error is also considered as the source error in this analysis. Therefore, the range of total momentum cut is shifted up and down by the twice of source error, and the ratio of remained events in signal box is assigned as the systematic uncertainty. In order to cover the low statistics in the signal box, sum of SK-I to SK-IV samples (2000 years MC) is used for estimation.

6.2.7 Energy scale

Energy scale error estimated in the latest neutrino oscillation analysis\cite{108} is used as the source error of this systematic uncertainty. They are estimated to be 3.3%, 2.8%, 2.4% and 2.1% respectively (section\ref{subsec:7.9}). The range of $\pi^0$ mass, total mass and momentum cuts are shifted up and down by these source error and the ratio of remained events in signal box is assigned as the systematic uncertainty. In order to cover the low statistics in the signal box, 2,000 years MC is used for estimation.

6.2.8 Ring counting & PID

Difference of data and MC in ring counting and PID distribution can be a source of systematic uncertainty. In order to get the source error for them, the PID values in MC are scaled and shifted and compared with data by using sub-GeV multi-ring samples. Then the scale and shift parameters with the smallest $\chi^2$ are obtained. Comparison of PID is performed for 1st, 2nd and 3rd reconstructed rings. Comparison plots for default, scaled and shifted MC events and data with respect to each ring in SK-IV are shown in Fig.\ref{fig:6.28}, \ref{fig:6.29}. Scaled and shifted parameters for PID are summarized in Table\ref{tab:6.16}.

For the ring counting, ring counting likelihood for 3 or 4 rings is used. Comparison plots

![Figure 6.27: Distance to wall for data and MC after area normalization in SK-III (left) and SK-IV (right). MC is normalized to data in the region $600 < D_{\text{wall}} < 1,000$ cm.](image)
Table 6.16: Scale and shift parameters for PID value.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Scale</td>
<td>0.98</td>
<td>1.02</td>
<td>0.98</td>
<td>1.04</td>
</tr>
<tr>
<td>Shift</td>
<td>0.01</td>
<td>0.08</td>
<td>-0.02</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Figure 6.28: PID value with respect to default MC events, scaled and shifted MC events (labeled as Tuned MC) and data for 1st (left) and 2nd (right) rings in SK-IV

Figure 6.29: PID value with respect to default MC events, scaled and shifted MC events (labeled as Tuned MC) and data for 3rd ring in SK-IV
CHAPTER 6. PROTON DECAY SEARCH

of ring counting likelihood for 3 or 4 rings between scaled and shifted MC events and data in SK-IV are shown in Fig. 6.30. (Scale, Shift) parameters for ring counting are (1.0,-0.34),(1.02,-0.02),(1.05,0) and (0.98,-0.02) for SKI,II,III,IV respectively.

Finally these scaled and shifted parameters are propagated to signal and background samples. Then the ratios of remaining events in the signal boxes between nominal and scaled and shifted events are assigned as the systematic uncertainties.

Figure 6.30: Ring counting likelihood for 3 or 4 rings with respect to default MC events, scaled and shifted MC events (labeled as Tuned MC) and data in SK-IV

6.2.9 Decay electron tagging

The difference of the number of decay electrons between data and MC after the area normalization is considered as the systematic error for decay electron tagging. Almost the same signal selection, but vetoing the signal boxes, is applied to each analysis mode. Data and MC comparison plots for the number of decay electrons with respect to each mode in SK-IV are shown in Fig. 6.31, 6.32. The ratios of events between data and MC are used for the errors where the number of decay electrons is 0,1,2 for $p \rightarrow e^+e^-$, $p \rightarrow \mu^+e^-$ and $p \rightarrow e^+\mu^+\mu^-$, respectively. For $p \rightarrow \mu^+\mu^+\mu^-$ mode, the ratio where the number of decay electrons is 2 or 3 is used for the error. In order to cover the low statistics, weighted average values are used for SK-I,II,III.

6.2.10 Neutron tagging

In order to estimate the systematic uncertainty for neutron tagging, pseudo distributions are created from the number of true neutrons remaining in the signal boxes by using the background MC. The number of true neutron for background events remained in signal box is shown in Fig. 6.33. Uniformly distributed random numbers between 0 and 1 are applied to each true neutron in certain event. If the random number is below the neutron tagging efficiency, the neutron is assumed as tagged neutron. In order to cover the small statistics this procedure is iterated 50 times. For the first test, exact multiplicity of tagged neutrons and
Figure 6.31: Data and MC comparison for the multiplicity of decay electrons in $p \rightarrow e^+e^+e^-$ (left) and $p \rightarrow \mu^+\mu^+\mu^-$ (right) modes in SK-IV.

Figure 6.32: Data and MC comparison for the multiplicity of decay electrons in $p \rightarrow e^+\mu^+\mu^-$ (left) and $p \rightarrow \mu^+\mu^+\mu^-$ (right) modes in SK-IV.
the one created from the true neutron under the neutron tagging efficiency 20% or 30% are shown in Fig. 6.33. When the exact neutron tagging efficiency of 25.2% is considered, a pseudo distribution created from true information agrees well with the reconstructed one. Changing the assumption of tagging efficiency for true neutron, we get the correlation plots between neutron tagging efficiency and background rate as shown in Fig. 6.34. Now the background rate for tagging efficiency of 0% is defined as 1. The plot is fitted by exponential function. We get the background rate at estimated tagging efficiency of 25.2% from the fitted function. Then the estimated source error of neutron tagging technique of 6% is considered. We can also get the background rates at the efficiency 23.7% and 26.7%. These efficiencies correspond to the 6% shifted up and down values of 25.2%. Finally the larger value of the difference between default background rate and those with shifted tagging efficiencies is assigned as the systematic uncertainty of the neutron tagging.

![Figure 6.33](image1.png)

**Figure 6.33:** Number of true neutron for background events remained in the signal box (left). Number of tagged neutron in background MC, estimated from true neutron under the tagging efficiency of 20% and 30%.

![Figure 6.34](image2.png)

**Figure 6.34:** Correlation plot between neutron tagging efficiency and background rate.
6.3. DATA AND MC AGREEMENT

6.2.11 Total systematic uncertainty

Systematic uncertainties for signal and background are summarized in Table 6.17 and Table 6.18. The detail of each systematic uncertainty for signal and background of each mode are summarized in Table 6.19-6.28.

Table 6.17: Summary of systematic uncertainty [%] for signal. Each error is averaged by live time of each period. Detector & Reconstruction is the quadratic sum of the errors 6.2.5-6.2.10.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Correlated decay</th>
<th>Fermi momentum</th>
<th>Detector &amp; Reconstruction</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \to e^+e^-e^-$</td>
<td>4.0</td>
<td>10.4</td>
<td>5.9</td>
<td>12.6</td>
</tr>
<tr>
<td>(Low)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(High)</td>
<td>9.3</td>
<td>3.1</td>
<td>4.4</td>
<td>10.8</td>
</tr>
<tr>
<td>$p \to \mu^+e^-e^-$</td>
<td>3.9</td>
<td>10.3</td>
<td>8.1</td>
<td>13.7</td>
</tr>
<tr>
<td>(Low)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(High)</td>
<td>9.4</td>
<td>3.4</td>
<td>7.5</td>
<td>12.6</td>
</tr>
<tr>
<td>$p \to \mu^-e^+e^+$</td>
<td>3.9</td>
<td>10.3</td>
<td>8.1</td>
<td>13.7</td>
</tr>
<tr>
<td>(Low)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(High)</td>
<td>9.6</td>
<td>3.0</td>
<td>7.6</td>
<td>12.5</td>
</tr>
<tr>
<td>$p \to e^+\mu^+\mu^-$</td>
<td>3.7</td>
<td>10.1</td>
<td>8.3</td>
<td>13.6</td>
</tr>
<tr>
<td>(Low)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(High)</td>
<td>9.5</td>
<td>3.5</td>
<td>7.2</td>
<td>12.7</td>
</tr>
<tr>
<td>$p \to e^-\mu^+\mu^+$</td>
<td>3.7</td>
<td>9.4</td>
<td>8.0</td>
<td>13.1</td>
</tr>
<tr>
<td>(Low)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(High)</td>
<td>8.8</td>
<td>5.6</td>
<td>7.2</td>
<td>12.9</td>
</tr>
<tr>
<td>$p \to \mu^+\mu^+\mu^-$</td>
<td>3.8</td>
<td>10.5</td>
<td>18.9</td>
<td>22.1</td>
</tr>
<tr>
<td>(Low)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(High)</td>
<td>9.7</td>
<td>6.5</td>
<td>18.6</td>
<td>22.1</td>
</tr>
</tbody>
</table>

Table 6.18: Summary of systematic uncertainty [%] for background. Each error is averaged by live time of each period. Detector & Reconstruction is the quadratic sum of the errors 6.2.5-6.2.10.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Neutrino flux</th>
<th>Neutrino cross section</th>
<th>Pion FSI/SI</th>
<th>Detector &amp; Reconstruction</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \to e^+e^-e^-$</td>
<td>7.0</td>
<td>14.1</td>
<td>1.9</td>
<td>32.4</td>
<td>36.1</td>
</tr>
<tr>
<td>$p \to \mu^+e^-e^- (p \to \mu^-e^+e^-)$</td>
<td>7.3</td>
<td>16.9</td>
<td>1.9</td>
<td>19.3</td>
<td>26.7</td>
</tr>
<tr>
<td>$p \to e^+\mu^-\mu^- (p \to e^-\mu^+\mu^+)$</td>
<td>8.2</td>
<td>23.6</td>
<td>3.3</td>
<td>19.6</td>
<td>32.0</td>
</tr>
<tr>
<td>$p \to \mu^+\mu^+\mu^-$</td>
<td>8.3</td>
<td>16.6</td>
<td>1.8</td>
<td>32.4</td>
<td>37.4</td>
</tr>
</tbody>
</table>

6.3 Data and MC agreement

Data and MC agreements for basic cut parameters are checked first. Comparison for the number of rings or e-like rings is shown in Fig. 6.35. Comparisons for number of decay electrons with respect to each mode are shown in Fig. 6.36,6.37. Comparison for mass of 2 e-like rings
### Table 6.19: Systematic uncertainty [\%] for signal in $p \rightarrow e^+e^-e^-$ mode.

<table>
<thead>
<tr>
<th></th>
<th>SK-I Low</th>
<th>SK-I High</th>
<th>SK-II Low</th>
<th>SK-II High</th>
<th>SK-III Low</th>
<th>SK-III High</th>
<th>SK-IV Low</th>
<th>SK-IV High</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Model)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlated decay</td>
<td>4.0</td>
<td>9.3</td>
<td>4.2</td>
<td>9.5</td>
<td>4.0</td>
<td>9.4</td>
<td>4.0</td>
<td>9.3</td>
</tr>
<tr>
<td>Fermi motion</td>
<td>10.1</td>
<td>2.9</td>
<td>10.3</td>
<td>2.9</td>
<td>10.4</td>
<td>2.9</td>
<td>10.5</td>
<td>3.2</td>
</tr>
<tr>
<td>(Detector &amp; Reconstruction)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiducial volume</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detector non-uniformity</td>
<td>1.5</td>
<td>0.2</td>
<td>1.9</td>
<td>0.1</td>
<td>3.3</td>
<td>0.4</td>
<td>1.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Energy scale</td>
<td>4.5</td>
<td>0.8</td>
<td>4.6</td>
<td>1.1</td>
<td>2.9</td>
<td>0.6</td>
<td>2.5</td>
<td>0.3</td>
</tr>
<tr>
<td>PID</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Ring counting</td>
<td>2.5</td>
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<td>&lt;0.1</td>
<td>0.3</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Decay electron</td>
<td>2.0</td>
<td>-</td>
<td>2.0</td>
<td>-</td>
<td>2.0</td>
<td>-</td>
<td>2.4</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.1</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>12.2</td>
<td>10.3</td>
<td>12.4</td>
<td>10.2</td>
<td>12.2</td>
<td>10.1</td>
<td>12.9</td>
<td>11.3</td>
</tr>
</tbody>
</table>

### Table 6.20: Systematic uncertainty [\%] for background in $p \rightarrow e^+e^-e^-$ mode.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Model)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutrino flux</td>
<td>7.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutrino cross section</td>
<td>14.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pion FSI/SI</td>
<td>1.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Detector &amp; Reconstruction)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiducial volume</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
</tr>
<tr>
<td>Detector non-uniformity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy scale</td>
<td>12.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>2.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ring counting</td>
<td>&lt;0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay electron</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.4</td>
</tr>
<tr>
<td>Neutron tagging</td>
<td>-</td>
<td>-</td>
<td></td>
<td>4.1</td>
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<tr>
<td>Total</td>
<td>35.8</td>
<td>35.8</td>
<td>35.8</td>
<td>36.2</td>
</tr>
</tbody>
</table>
Table 6.21: Systematic uncertainty [%] for signal in $p \rightarrow \mu^+ e^+ e^-$ mode.

<table>
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<tr>
<th></th>
<th>SK-I Low</th>
<th>SK-I High</th>
<th>SK-II Low</th>
<th>SK-II High</th>
<th>SK-III Low</th>
<th>SK-III High</th>
<th>SK-IV Low</th>
<th>SK-IV High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlated decay</td>
<td>4.0</td>
<td>9.5</td>
<td>3.9</td>
<td>9.0</td>
<td>4.0</td>
<td>9.4</td>
<td>3.9</td>
<td>9.5</td>
</tr>
<tr>
<td>Fermi motion</td>
<td>10.0</td>
<td>2.4</td>
<td>10.3</td>
<td>2.5</td>
<td>10.8</td>
<td>4.4</td>
<td>10.4</td>
<td>4.0</td>
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<tr>
<td>Detector non-uniformity</td>
<td>1.9</td>
<td>0.4</td>
<td>1.6</td>
<td>0.1</td>
<td>3.0</td>
<td>0.4</td>
<td>1.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Energy scale</td>
<td>3.6</td>
<td>1.0</td>
<td>3.1</td>
<td>1.3</td>
<td>2.4</td>
<td>0.6</td>
<td>1.9</td>
<td>0.6</td>
</tr>
<tr>
<td>PID</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>&lt;0.1</td>
<td>0.9</td>
<td>&lt;0.1</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Ring counting</td>
<td>1.8</td>
<td>1.9</td>
<td>0.1</td>
<td>0.3</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Decay electron</td>
<td>7.1</td>
<td></td>
<td>7.1</td>
<td></td>
<td>7.1</td>
<td></td>
<td></td>
<td>4.5</td>
</tr>
<tr>
<td>Neutron tagging</td>
<td>-</td>
<td></td>
<td>-</td>
<td></td>
<td>-</td>
<td></td>
<td></td>
<td>5.5</td>
</tr>
<tr>
<td>Total</td>
<td>13.7</td>
<td>12.3</td>
<td>13.6</td>
<td>11.8</td>
<td>14.1</td>
<td>12.6</td>
<td>13.7</td>
<td>12.9</td>
</tr>
</tbody>
</table>

Table 6.22: Systematic uncertainty [%] for signal in $p \rightarrow \mu^- e^+ e^+$ mode.

<table>
<thead>
<tr>
<th></th>
<th>SK-I Low</th>
<th>SK-I High</th>
<th>SK-II Low</th>
<th>SK-II High</th>
<th>SK-III Low</th>
<th>SK-III High</th>
<th>SK-IV Low</th>
<th>SK-IV High</th>
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<tbody>
<tr>
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<td>3.8</td>
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<td>9.4</td>
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<tr>
<td>Fermi motion</td>
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<td>3.6</td>
<td>11.4</td>
<td>2.5</td>
<td>9.9</td>
<td>3.7</td>
<td>10.5</td>
<td>2.7</td>
</tr>
<tr>
<td>Detector non-uniformity</td>
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<td>0.5</td>
<td>1.5</td>
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<td>0.5</td>
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<td>&lt;0.1</td>
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<td>14.5</td>
<td>12.2</td>
<td>13.2</td>
<td>12.5</td>
<td>13.9</td>
<td>12.5</td>
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Table 6.23: Systematic uncertainty [%] for background in $p \rightarrow \mu^+ e^+ e^-$ and $p \rightarrow \mu^- e^+ e^+$ mode.

<table>
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<th>SK-III</th>
<th>SK-IV</th>
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<tr>
<td>Neutrino flux</td>
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<td>Neutrino cross section</td>
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<td>Pion FSI/SI</td>
<td>1.9</td>
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<tr>
<td>(Detector &amp; Reconstruction)</td>
<td></td>
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</tr>
<tr>
<td>Fiducial volume</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
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<td>Energy scale</td>
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<tr>
<td>PID</td>
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<td>Ring counting</td>
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<td>26.8</td>
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Table 6.24: Systematic uncertainty [%] for signal in $p \rightarrow e^+ \mu^+ \mu^-$ mode.

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<th>SK-II Low</th>
<th>SK-II High</th>
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<th>SK-IV Low</th>
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<td>9.3</td>
<td>3.9</td>
<td>9.4</td>
</tr>
<tr>
<td>Fermi motion</td>
<td>9.0</td>
<td>4.8</td>
<td>10.7</td>
<td>1.3</td>
<td>9.8</td>
<td>6.1</td>
<td>10.5</td>
<td>3.0</td>
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<tr>
<td>(Detector &amp; Reconstruction)</td>
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<tr>
<td>Fiducial volume</td>
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<td>0.7</td>
<td>3.0</td>
<td>0.9</td>
<td>3.2</td>
<td>0.4</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>Energy scale</td>
<td>3.7</td>
<td>1.1</td>
<td>3.9</td>
<td>0.9</td>
<td>2.7</td>
<td>0.5</td>
<td>2.4</td>
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</tr>
<tr>
<td>PID</td>
<td>0.3</td>
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<td>0.3</td>
<td>&lt;0.1</td>
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<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Ring counting</td>
<td>3.2</td>
<td>2.0</td>
<td>&lt;0.1</td>
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<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
<td>0.3</td>
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<tr>
<td>Decay electron</td>
<td>3.7</td>
<td></td>
<td>3.7</td>
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<td>3.7</td>
<td></td>
<td>7.1</td>
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</tr>
<tr>
<td>Neutron tagging</td>
<td>-</td>
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<td>-</td>
<td></td>
<td>-</td>
<td></td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11.4</td>
<td>11.5</td>
<td>13.1</td>
<td>9.7</td>
<td>11.9</td>
<td>11.8</td>
<td>15.1</td>
<td>13.9</td>
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</table>
6.3. DATA AND MC AGREEMENT

Table 6.25: Systematic uncertainty [%] for signal in $p \rightarrow e^-\mu^+\mu^+$ mode.

<table>
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<tr>
<th>(Model)</th>
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<th>SK-IV Low</th>
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<td>3.8</td>
<td>3.8</td>
<td>3.7</td>
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<td>Fermi motion</td>
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<td>(Detector &amp; Reconstruction)</td>
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<tr>
<td>Fiducial volume</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
</tr>
<tr>
<td>Detector non-uniformity</td>
<td>1.3</td>
<td>1.7</td>
<td>3.5</td>
<td>1.4</td>
</tr>
<tr>
<td>Energy scale</td>
<td>3.4</td>
<td>3.4</td>
<td>3.1</td>
<td>2.7</td>
</tr>
<tr>
<td>PID</td>
<td>&lt;0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Ring counting</td>
<td>1.6</td>
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<td>&lt;0.1</td>
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<td>3.7</td>
<td>3.7</td>
<td>7.1</td>
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<td>Neutron tagging</td>
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<tr>
<td>Total</td>
<td>11.3</td>
<td>12.5</td>
<td>12.3</td>
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Table 6.26: Systematic uncertainty [%] for background in $p \rightarrow e^+\mu^+\mu^-$ and $p \rightarrow e^-\mu^+\mu^+$ mode.

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<th>SK-IV</th>
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<tr>
<td>Neutrino cross section</td>
<td>23.6</td>
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<tr>
<td>Pion FSI/SI</td>
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<td>(Detector &amp; Reconstruction)</td>
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<td></td>
</tr>
<tr>
<td>Fiducial volume</td>
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<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
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<tr>
<td>Detector non-uniformity</td>
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<td></td>
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<tr>
<td>Energy scale</td>
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<tr>
<td>PID</td>
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<td>Decay electron</td>
<td>3.7</td>
<td>3.7</td>
<td>3.7</td>
<td>7.1</td>
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<tr>
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### Table 6.27: Systematic uncertainty [%] for signal in $p \rightarrow \mu^+ \mu^+ \mu^-$ mode.

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</tr>
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<td>1.6</td>
<td>0.5</td>
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<tr>
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<td>0.5</td>
<td>3.4</td>
<td>0.9</td>
</tr>
<tr>
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<td>0.1</td>
<td>&lt;0.1</td>
<td>&lt;0.1</td>
</tr>
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<td>0.6</td>
<td>0.1</td>
<td>&lt;0.1</td>
</tr>
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<td>23.5</td>
<td>23.5</td>
<td>23.5</td>
</tr>
<tr>
<td>Neutron tagging</td>
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<tr>
<td>Total</td>
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<td>26.6</td>
<td>26.4</td>
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### Table 6.28: Systematic uncertainty [%] for background in $p \rightarrow \mu^+ \mu^+ \mu^-$ mode.

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<tr>
<td>Neutrino cross section</td>
<td></td>
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<tr>
<td>Pion FSI/SI</td>
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</tr>
<tr>
<td>Fiducial volume</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>3.0</td>
</tr>
<tr>
<td>Detector non-uniformity</td>
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<tr>
<td>Energy scale</td>
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<td></td>
</tr>
<tr>
<td>PID</td>
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</tr>
<tr>
<td>Ring counting</td>
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<tr>
<td>Decay electron</td>
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<tr>
<td>Neutron tagging</td>
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<td>40.2</td>
<td>40.2</td>
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</table>
with respect to $p \rightarrow \mu^+ e^+ e^-$ mode is shown in Fig.6.38. Comparisons for the number of tagged neutrons in each mode are shown in Fig.6.39,6.40.

![Data and MC Agreement](image)

Figure 6.35: Data and MC comparison for number of ring (left) and number of $e$-like ring.

![Data and MC Agreement](image)

Figure 6.36: Data and MC comparison for number of decay electron with respect to $p \rightarrow e^+ e^+ e^-$ (left) and $p \rightarrow \mu^+ e^+ e^-$ (right) modes.

In order to check data and MC agreement in total mass and total momentum distribution before opening the signal box, a control region outside the signal box is defined to avoid the contamination of the signal events. First the difference of total mass and momentum between true and reconstructed value in signal MC sample are checked as shown in Fig.6.41. Considering the fluctuation for total mass and momentum, the validation region is defined 200 MeV far from the signal box as $M < 600$ MeV or $M > 1,250$ MeV or $P > 450$ MeV. The definition of the validation region is shown in Fig.6.42. Comparisons for total mass and momentum distributions in the validation region in each mode are shown in Fig.6.43-6.46. A small excess of data is seen in total mass around $750 < M < 900$ MeV in $p \rightarrow e^+ e^+ e^-$ mode. In this region, the number of expected background events is 81.2 and the number of observed events is 111. Then the significance of this excess is calculated to be $(111 - 81.2)/\sqrt{81.2} = 3.3$. Generally, the significance of 3.3 is not a significant excess. Comparison of total momentum focussing on total mass between 750 and 900 MeV in the control region is also shown in Fig.6.47. A small excess is also seen in total momentum between 600 and 800 MeV or 1,000 and 1,200 MeV. Such kinematic region of $750 < M <$
Figure 6.37: Data and MC comparison for number of decay electron with respect to \( p \rightarrow e^+\mu^+\mu^- \) (left) and \( p \rightarrow \mu^+\mu^+\mu^- \) (right) modes. The black point shows the data and red line shows the background MC.

Figure 6.38: Data and MC comparison for mass of 2 \( e^- \)-like rings with respect to \( p \rightarrow \mu^+e^+e^- \) (left) mode. The black point shows the data and red line shows the background MC.
6.3. DATA AND MC AGREEMENT

Figure 6.39: Data and MC comparison for number of tagged neutron with respect to $p \rightarrow e^+e^+e^-$ (left) and $p \rightarrow \mu^+\mu^+\mu^-$ (right) modes in SK-IV. The black point shows the data and red line shows the background MC.

Figure 6.40: Data and MC comparison for number of tagged neutron with respect to $p \rightarrow e^+\mu^+\mu^-$ (left) and $p \rightarrow \mu^+\mu^+\mu^-$ (right) modes in SK-IV. The black point shows the data and red line shows the background MC.

Figure 6.41: The difference of reconstructed and true value for total mass and total momentum with respect to $p \rightarrow e^+e^+e^-$ signal MC.
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900 MeV and $600 < P < 800$ MeV or $1,000 < P < 1,200$ MeV is far from typical $p \to e^+e^-e^-$ signal events as shown in Fig. 6.49. The event displays of data in the region of excess are also checked. There are basically typical background events and no abnormal events found. Some of event displays in this region are shown in Fig. 6.48. As a conclusion, this excess is attributed to a statistical fluctuation.

The period by period comparisons for these parameters are summarized in Appendix A.

6.4 Result

6.4.1 Data open results

Since data and background MC agree well in the validation region as described in section 6.3, we opened the data in signal box. As a result, 1 signal candidate is observed in both $p \to$
Figure 6.44: Data and MC comparison for total mass (left) and total momentum (right) distributions in the validation region with respect to $p \rightarrow \mu^+ e^+ e^-$ mode.

Figure 6.45: Data and MC comparison for total mass (left) and total momentum (right) distributions in the validation region with respect to $p \rightarrow e^+ \mu^+ \mu^-$ mode.

Figure 6.46: Data and MC comparison for total mass (left) and total momentum (right) distributions in the validation region with respect to $p \rightarrow \mu^+ \mu^+ \mu^-$ mode.
CHAPTER 6. PROTON DECAY SEARCH

Figure 6.47: Data and background mc comparison for total momentum distributions focusing on total mass between 750 and 900 MeV in the validation region.

Figure 6.48: Examples of the event displays of data in the region of excess in $p \to e^+ e^+ e^-$. The blue circles show TDC fit result, red circles show ring fit result by the APfit. The total mass is 874 MeV and total momentum is 728 MeV in the left event. The total mass is 769 MeV and total momentum is 1,069 MeV in the right event.
6.4. RESULT

$e^+\mu^+\mu^- (p \rightarrow e^-\mu^+\mu^+)$ and $p \rightarrow \mu^+\mu^+\mu^-$ modes, respectively. 2 dimensional plots for total mass and momentum for signal MC, background MC and data for each mode are shown in Fig.6.49-6.52. Comparison of data and background MC around the signal boxes for total mass and momentum distribution is also shown in Fig.6.53-6.56. Period by period comparisons for these plots are also summarized in Appendix A. The observed data are consistent with the prediction of atmospheric background events. Cut flow for data with respect to each mode are summarized in Table 6.29-6.32. The observed number of events at each cut step is consistent with the expected number of background events.

![Figure 6.49: 2 dimensional plots of total mass and momentum for signal (left), background (center) and measured data (right) in $p \rightarrow e^+e^+e^-$ mode after all selections are applied. Light blue shows free proton and dark blue shows bound proton in signal plot. Two black squares show the low and high signal boxes. The exposure of background MC is 500 years, which corresponds to 30 times that of data.](image)

![Figure 6.50: 2 dimensional plots of total mass and momentum for signal MC (left), background MC (center) and measured data (right) in $p \rightarrow \mu^+e^+e^-$ mode after all selections are applied. Light blue shows free proton and dark blue shows bound proton in signal plot. Two black squares show the low and high signal boxes. The exposure of background MC is 500 years, which corresponds to 30 times that of data.](image)

6.4.2 Observed candidates

1 event is observed in the high signal box in $p \rightarrow e^+\mu^+\mu^- (p \rightarrow e^-\mu^+\mu^+)$ mode and 1 event is also observed in the high signal box in $p \rightarrow \mu^+\mu^+\mu^-$ mode. Detail about both events are
Figure 6.51: 2 dimensional plots of total mass and momentum for signal MC (left), background MC (center) and measured data (right) in $p \rightarrow e^+ \mu^+ \mu^-$ mode after all selections are applied. Light blue shows free proton and dark blue shows bound proton in signal plot. Two black squares show the low and high signal boxes. The exposure of background MC is 500 years, which corresponds to 30 times that of data.

Figure 6.52: 2 dimensional plots of total mass and momentum for signal MC (left), background MC (center) and measured data (right) in $p \rightarrow \mu^+ \mu^+ \mu^-$ mode after all selections are applied. Light blue shows free proton and dark blue shows bound proton in signal plot. Two black squares show the low and high signal boxes. The exposure of background MC is 500 years, which corresponds to 30 times that of data.
Figure 6.53: Data and background comparison for total mass (left) and momentum (right) in $p \rightarrow e^+e^+e^-$ mode after all selections are applied. Black point shows data, red line shows background, dark blue line shows total and filled light blue shows free proton of signal. Dotted black line show the signal box.

Figure 6.54: Data and background comparison for total mass (left) and momentum (right) in $p \rightarrow \mu^+\mu^+\mu^-$ mode after all selections are applied. Black point shows data, red line shows background, dark blue line shows total and filled light blue shows free proton of signal. Dotted black line show the signal box.
Figure 6.55: Data and background comparison for total mass (left) and momentum (right) in $p \rightarrow e^+\mu^+\mu^-$ mode after all selections are applied. Black point shows data, red line shows background, dark blue line shows total and filled light blue shows free proton of signal. Dotted black line show the signal box.

Figure 6.56: Data and background comparison for total mass (left) and momentum (right) in $p \rightarrow \mu^+\mu^+\mu^-$ mode after all selections are applied. Black point shows data, red line shows background, dark blue line shows total and filled light blue shows free proton of signal. Dotted black line show the signal box.
### Table 6.29: Observed events at each step of selection criteria for $p \rightarrow e^+e^+e^-$ mode. The expected number of background events at each step is also shown in brackets. The error values correspond to the statistic uncertainty.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FC &amp; FV</strong></td>
<td>12,178±110</td>
<td>6,626±81</td>
<td>4,347±66</td>
<td>26,542±163</td>
</tr>
<tr>
<td></td>
<td>(12,097±10)</td>
<td>(6546±5)</td>
<td>(4285±3)</td>
<td>(26,563±21)</td>
</tr>
<tr>
<td>Number of rings</td>
<td>995±32</td>
<td>551±23</td>
<td>368±19</td>
<td>2,282±48</td>
</tr>
<tr>
<td></td>
<td>(966±3)</td>
<td>(555±2)</td>
<td>(340±1)</td>
<td>(2,247±6)</td>
</tr>
<tr>
<td><strong>PID</strong></td>
<td>458±21</td>
<td>231±15</td>
<td>191±14</td>
<td>1,053±32</td>
</tr>
<tr>
<td></td>
<td>(460±2)</td>
<td>(243±1)</td>
<td>(157±1)</td>
<td>(1,038±4)</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>256±16</td>
<td>137±12</td>
<td>113±11</td>
<td>570±24</td>
</tr>
<tr>
<td></td>
<td>(270±1)</td>
<td>(152±1)</td>
<td>(97±1)</td>
<td>(568±3)</td>
</tr>
<tr>
<td><strong>Total mass &amp; momentum</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.18±0.04)</td>
<td>(0.11±0.02)</td>
<td>(0.05±0.01)</td>
<td>(0.58±0.10)</td>
</tr>
<tr>
<td><strong>Neutron veto</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td><strong>Low signal box</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.01)</td>
<td>(0.01±0.01)</td>
<td>(&lt;0.01)</td>
<td>(&lt;0.01)</td>
</tr>
<tr>
<td><strong>High signal box</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.18±0.04)</td>
<td>(0.10±0.02)</td>
<td>(0.05±0.01)</td>
<td>(0.23±0.06)</td>
</tr>
</tbody>
</table>

### Table 6.30: Observed events at each step of selection criteria for $p \rightarrow \mu^+\mu^-\mu^-$ and $p \rightarrow \mu^-e^+e^+$ mode. The expected number of background events at each step is also shown in brackets. The error values correspond to the statistic uncertainty.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FC &amp; FV</strong></td>
<td>12,178±110</td>
<td>6,626±81</td>
<td>4,347±66</td>
<td>26,542±163</td>
</tr>
<tr>
<td></td>
<td>(12,097±10)</td>
<td>(6546±5)</td>
<td>(4285±3)</td>
<td>(26,563±21)</td>
</tr>
<tr>
<td>Number of rings</td>
<td>995±32</td>
<td>551±23</td>
<td>368±19</td>
<td>2,282±48</td>
</tr>
<tr>
<td></td>
<td>(966±3)</td>
<td>(555±2)</td>
<td>(340±1)</td>
<td>(2,247±6)</td>
</tr>
<tr>
<td><strong>PID</strong></td>
<td>458±21</td>
<td>231±15</td>
<td>191±14</td>
<td>1,053±32</td>
</tr>
<tr>
<td></td>
<td>(460±2)</td>
<td>(243±1)</td>
<td>(157±1)</td>
<td>(1,038±4)</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>182±13</td>
<td>94±10</td>
<td>57±8</td>
<td>473±22</td>
</tr>
<tr>
<td></td>
<td>(166±1)</td>
<td>(90±1)</td>
<td>(61±0)</td>
<td>(440±3)</td>
</tr>
<tr>
<td>$\pi^0$ veto</td>
<td>86±9</td>
<td>49±7</td>
<td>26±5</td>
<td>227±15</td>
</tr>
<tr>
<td></td>
<td>(81±1)</td>
<td>(47±0)</td>
<td>(30±0)</td>
<td>(202±2)</td>
</tr>
<tr>
<td><strong>Total mass &amp; momentum</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.15±0.03)</td>
<td>(0.12±0.02)</td>
<td>(0.06±0.01)</td>
<td>(0.56±0.10)</td>
</tr>
<tr>
<td><strong>Neutron veto</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td><strong>Low signal box</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.01)</td>
<td>(0.01±0.01)</td>
<td>(&lt;0.01)</td>
<td>(&lt;0.01)</td>
</tr>
<tr>
<td><strong>High signal box</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.13±0.03)</td>
<td>(0.11±0.02)</td>
<td>(0.05±0.01)</td>
<td>(0.18±0.06)</td>
</tr>
</tbody>
</table>
Table 6.31: Observed events at each step of selection criteria for \( p \rightarrow e^+\mu^+\mu^- \) and \( p \rightarrow e^-\mu^+\mu^- \) mode. The expected number of background events at each step is also shown in brackets. The error values correspond to the statistic uncertainty.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
<th>SK-II</th>
<th>SK-III</th>
<th>SK-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC &amp; FV</td>
<td>12,178±110</td>
<td>6,626±81</td>
<td>4347±66</td>
<td>26,542±163</td>
</tr>
<tr>
<td></td>
<td>(12,097±10)</td>
<td>(6546±5)</td>
<td>(4285±3)</td>
<td>(26,563±21)</td>
</tr>
<tr>
<td>Number of rings</td>
<td>995±32</td>
<td>551±23</td>
<td>368±19</td>
<td>2,282±48</td>
</tr>
<tr>
<td></td>
<td>(966±3)</td>
<td>(555±2)</td>
<td>(340±1)</td>
<td>(2,247±6)</td>
</tr>
<tr>
<td>PID</td>
<td>127±11</td>
<td>94±10</td>
<td>32±6</td>
<td>257±16</td>
</tr>
<tr>
<td></td>
<td>(108±1)</td>
<td>(78±1)</td>
<td>(38±0)</td>
<td>(265±2)</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>27±5</td>
<td>15±4</td>
<td>5±2</td>
<td>56±7</td>
</tr>
<tr>
<td></td>
<td>(17±0)</td>
<td>(11±0)</td>
<td>(6±0)</td>
<td>(58±1)</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
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<td>1±1</td>
</tr>
<tr>
<td></td>
<td>(0.09±0.02)</td>
<td>(0.07±0.01)</td>
<td>(0.03±0.01)</td>
<td>(0.30±0.07)</td>
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<tr>
<td>Neutron veto</td>
<td>-</td>
<td>-</td>
<td>1±1</td>
<td></td>
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<tr>
<td></td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(0.07±0.03)</td>
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<tr>
<td>Low signal box</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.01)</td>
<td>(&lt;0.01)</td>
<td>(&lt;0.01)</td>
<td>(&lt;0.01)</td>
</tr>
<tr>
<td>High signal box</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1±1</td>
</tr>
<tr>
<td></td>
<td>(0.09±0.02)</td>
<td>(0.07±0.01)</td>
<td>(0.03±0.01)</td>
<td>(0.07±0.03)</td>
</tr>
</tbody>
</table>

Table 6.32: Observed events at each step of selection criteria for \( p \rightarrow \mu^+\mu^+\mu^- \) mode. The expected number of background events at each step is also shown in brackets. The error values correspond to the statistic uncertainty.

<table>
<thead>
<tr>
<th></th>
<th>SK-I</th>
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<th>SK-III</th>
<th>SK-IV</th>
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<tbody>
<tr>
<td>FC &amp; FV</td>
<td>12,178±110</td>
<td>6,626±81</td>
<td>4347±66</td>
<td>26,542±163</td>
</tr>
<tr>
<td></td>
<td>(12,097±10)</td>
<td>(6546±5)</td>
<td>(4285±3)</td>
<td>(26,563±21)</td>
</tr>
<tr>
<td>Number of rings</td>
<td>995±32</td>
<td>551±23</td>
<td>368±19</td>
<td>2,282±48</td>
</tr>
<tr>
<td></td>
<td>(966±3)</td>
<td>(555±2)</td>
<td>(340±1)</td>
<td>(2,247±6)</td>
</tr>
<tr>
<td>PID</td>
<td>28±5</td>
<td>22±5</td>
<td>10±3</td>
<td>70±8</td>
</tr>
<tr>
<td></td>
<td>(26±0)</td>
<td>(17±0)</td>
<td>(9±0)</td>
<td>(64±1)</td>
</tr>
<tr>
<td>Number of decay electrons</td>
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<td>6±2</td>
<td>1±1</td>
<td>28±5</td>
</tr>
<tr>
<td></td>
<td>(8±0)</td>
<td>(5±0)</td>
<td>(3±0)</td>
<td>(28±1)</td>
</tr>
<tr>
<td>Total mass &amp; momentum</td>
<td>1±1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.10±0.03)</td>
<td>(0.05±0.01)</td>
<td>(0.03±0.01)</td>
<td>(0.47±0.08)</td>
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<td>Neutron veto</td>
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<td>-</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(0.21±0.06)</td>
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<tr>
<td>Low signal box</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.01±0.01)</td>
<td>(&lt;0.01)</td>
<td>(&lt;0.01)</td>
<td>(&lt;0.01)</td>
</tr>
<tr>
<td>High signal box</td>
<td>1±1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.09±0.03)</td>
<td>(0.05±0.01)</td>
<td>(0.03±0.01)</td>
<td>(0.21±0.06)</td>
</tr>
</tbody>
</table>
6.4. RESULT

discussed in this section.

Considering the number of expected background to be 0.27 in \( p \rightarrow e^+ \mu^+ \mu^- \) mode, the poisson probability of observing greater than or equal to 1 event is calculated to be 18.4%. This probability means that 1 observed event is not significant excess compared to the number of expected background events in this mode. For \( p \rightarrow e^- \mu^+ \mu^- \) mode, the poisson probability of observing greater than or equal to 1 event is 25.8% with respect to the number of expected background events to be 0.40. This is not a significant excess in this mode either. These values are summarized in Table 6.33.

Table 6.33: Summary of background events, the number of candidate, Poisson probability to observe events greater than or equal to the number of data candidates. The probability is calculated for the total signal box.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Background [events]</th>
<th>Candidate [events]</th>
<th>Probability [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>Low</td>
<td>Total</td>
<td>p \rightarrow e^+e^-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>p \rightarrow \mu^+e^-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( p \rightarrow e^+e^- )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( p \rightarrow e^-e^+ )</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Event displays for both candidates are shown in Fig. 6.57, Fig. 6.58. Information of both candidates is summarized in Table 6.34. Reconstructed values of each ring for both candidates are also summarized in Table 6.35, 6.36. For the \( p \rightarrow e^+\mu^+\mu^- \) candidate, 2 rings near Ring1 are clearly not reconstructed by APfit. Such events are also often seen in typical background events like Fig. 6.20. For the \( p \rightarrow \mu^+\mu^+\mu^- \) candidate, 1 ring near Ring1 is clearly not reconstructed by APfit either. This event is also similar to the typical background events like Fig. 6.21. As a conclusion, these observed candidates are typical atmospheric neutrino background events like single or multi \( \pi \) production events.

Table 6.34: Information of candidates in \( p \rightarrow e^+\mu^+\mu^- \) (\( p \rightarrow e^-\mu^+\mu^+ \)) and \( p \rightarrow \mu^+\mu^+\mu^- \) modes.

<table>
<thead>
<tr>
<th>Period</th>
<th>( p \rightarrow e^+\mu^+\mu^- )</th>
<th>( p \rightarrow \mu^+\mu^+\mu^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK-IV</td>
<td>SK-I</td>
<td>SK-IV</td>
</tr>
<tr>
<td>Run # / Event #</td>
<td>70589 / 37485314</td>
<td>8854 / 36673156</td>
</tr>
<tr>
<td>Vertex X/Y/X [cm]</td>
<td>-400.1/1004.7/-54.5</td>
<td>-357.3/1118.0/-1270.3</td>
</tr>
<tr>
<td>Distance to wall [cm]</td>
<td>608.6</td>
<td>516.3</td>
</tr>
<tr>
<td>Number of rings (e-like, ( \mu )-like)</td>
<td>3 (1,2)</td>
<td>3 (0,3)</td>
</tr>
<tr>
<td>Number of decay electrons</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Number of tagged neutron</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Total mass/momentum [MeV]</td>
<td>882.4/159.8</td>
<td>835.1/170.0</td>
</tr>
</tbody>
</table>
Figure 6.57: Event displays of the observed candidate in $p \to e^+ \mu^+ \mu^-$ ($p \to e^- \mu^+ \mu^+$) mode before (left) and after (right) ring reconstruction. Blue circle shows TDC fit result, red and purple bold circles show ring fit result for $e$-like and $\mu$-like respectively in right figure. Index of each ring is also described in right figure for Table 6.35

Figure 6.58: Event displays of the observed candidate in $p \to \mu^+ \mu^+ \mu^-$ ($p \to \mu^- \mu^+ \mu^+$) mode before (left) and after (right) ring reconstruction. Blue circle shows TDC fit result, red and purple bold circles show ring fit result for $e$-like and $\mu$-like respectively in right figure. Index of each ring is also described in right figure for Table 6.36
Table 6.35: Reconstructed values of each ring for candidate observed in \( p \rightarrow e^+ \mu^+ \mu^- \) (\( p \rightarrow e^- \mu^+ \mu^+ \)) mode. Definition of ring index is described in Fig. 6.57.

<table>
<thead>
<tr>
<th>Ring1</th>
<th>Ring2</th>
<th>Ring3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction from vertex X/Y/Z (vector)</td>
<td>0.15/0.28/-0.95</td>
<td>-0.30/0.49/0.82</td>
</tr>
<tr>
<td>PID likelihood value</td>
<td>0.82 ((\mu)-like)</td>
<td>1.87 ((\mu)-like)</td>
</tr>
<tr>
<td>Reconstructed opening angle [degree]</td>
<td>40.3</td>
<td>33.8</td>
</tr>
<tr>
<td>Momentum [MeV]</td>
<td>412.3</td>
<td>296.7</td>
</tr>
<tr>
<td>Expected opening angle [degree]</td>
<td>39.2</td>
<td>37.2</td>
</tr>
</tbody>
</table>

Table 6.36: Reconstructed values of each ring for the candidate observed in \( p \rightarrow \mu^+ \mu^+ \mu^- \) mode. Definition of ring index is described in Fig. 6.58.

<table>
<thead>
<tr>
<th>Ring1</th>
<th>Ring2</th>
<th>Ring3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction from vertex X/Y/Z (vector)</td>
<td>-0.51/0.37/0.77</td>
<td>0.28/0.67/-0.69</td>
</tr>
<tr>
<td>PID likelihood value</td>
<td>2.21 ((\mu)-like)</td>
<td>2.55 ((\mu)-like)</td>
</tr>
<tr>
<td>Reconstructed opening angle [degree]</td>
<td>37.1</td>
<td>36.8</td>
</tr>
<tr>
<td>Momentum [MeV]</td>
<td>277.0</td>
<td>289.7</td>
</tr>
<tr>
<td>Expected opening angle [degree]</td>
<td>36.6</td>
<td>37.0</td>
</tr>
</tbody>
</table>

### 6.5 Lifetime limit

There are no significant excess of data compared to the number of expected background events in all modes. Two events are observed in the signal box, which are considered as typical atmospheric neutrino background events. As a conclusion, any events of proton decay into three charged leptons are not discovered in this analysis. Finally, we set lower limits for the proton lifetime with respect to each decay mode by using Bayesian method. Now we have 8 signal regions (4 periods x 2 boxes). The probability density function (PDF) is calculated for each region. Then all PDFs are combined to estimate the lifetime limit for each decay mode.

The probability of detecting \(n_i\) events under the assumption of expected signal and background events \(\Gamma \lambda_i \epsilon_i \epsilon_i + b_i\) by Poisson statistics is defined as follow. \(i\) is the index of each signal box, \(\Gamma\) is the decay rate, \(\lambda_i\) is the exposure (unit is the number of protons\(^1\) multiplied by live time [year]), \(\epsilon_i\) is the signal efficiency and \(b_i\) is expected background events.

\[
P(n_i | \Gamma \lambda_i \epsilon_i \epsilon_i, b_i) = \frac{e^{-(\Gamma \lambda_i \epsilon_i \epsilon_i + b_i)} (\Gamma \lambda_i \epsilon_i \epsilon_i + b_i)^{n_i}}{n_i!}
\]  
(6.4)

When we follow the Bayesian theory this probability is described as below.

\[
P(n_i) = P(n_i | \Gamma \lambda_i \epsilon_i \epsilon_i, b_i)P(\Gamma \lambda_i \epsilon_i \epsilon_i, b_i)
= P(n_i | \Gamma \lambda_i \epsilon_i \epsilon_i)P(\Gamma)P(\lambda_i)P(\epsilon_i)P(b_i)
\]  
(6.5)

Here we assume the probabilities of decay rate, exposure, signal efficiency and background are independent. Then the PDF for \(n_i\) observed events as the function of decay rate \(P(\Gamma | n_i)\)

\(^1\)Note that there are 10 protons in a molecule of water, two free and eight bound protons in Oxygen.
is calculated as follow.

\[ P(\Gamma|n_i) = \int \int \int P(n_i) d\epsilon_i d\lambda_i db_i \]

\[ = \frac{1}{A_i} \int \int \int e^{-(\Gamma \lambda_i + b_i)} (\Gamma \lambda_i + b_i)^{n_i} n_i! P(\Gamma) P(\lambda_i) P(\epsilon_i) P(b_i) d\epsilon_i d\lambda_i db_i \]  

(6.6)

Then \( A_i \) is the normalization constant defined as below.

\[ A_i = \int_0^\infty P(\Gamma|n_i) d\Gamma \]  

(6.7)

We assume the probabilities for decay rate is uniform as below.

\[ P(\Gamma) = 1 \]  

(6.8)

We also assume the probabilities for exposure and signal efficiency follow Gaussian as below.

\[ P(\lambda_i) = \exp\left\{ -(\lambda_i - \lambda_{0,i})^2 / 2\sigma_{\lambda_i}^2 \right\} \]  

(6.9)

\[ P(\epsilon_i) = \exp\left\{ -(\epsilon_i - \epsilon_{0,i})^2 / 2\sigma_{\epsilon_i}^2 \right\} \]  

(6.10)

Here \( \lambda_{0,i} (\sigma_{\lambda_i}) \) and \( \epsilon_{0,i} (\sigma_{\epsilon_i}) \) are estimations (systematic errors) for exposure and signal efficiency, respectively. Here the normalization factors for \( P(\Gamma) \), \( P(\lambda_i) \) and \( P(\epsilon_i) \) are not described in each expression. Since the number of background events is very small, the probability for background is described as the convolution of Gaussian and Poisson distributions as below.

\[ P(b_i) \propto \int_0^\infty \frac{e^{-B} B^{n_{bi}}}{n_{bi}!} \exp\left\{ -(C_i b_i - B)^2 / 2\sigma_{b_i}^2 \right\} dB \]  

(6.11)

Here \( n_{bi} \) is the number of expected background events in 500 years MC, \( C_i \) is the constant value to normalize MC live time to data live time and \( \sigma_{b_i} \) is the systematic error for the background. The upper limit of the decay rate \( \Gamma \) at 90% confidence level is calculated from following equation.

\[ \int_{\Gamma=0}^{\Gamma_{\text{limit}}} \prod_{i=1}^{N=8} P(\Gamma|n_i) d\Gamma = 0.9 \]  

(6.12)

Finally the lower limit of partial lifetime is calculated as below.

\[ \tau / B = \frac{1}{\Gamma_{\text{limit}}} \]  

(6.13)

Here \( B \) is the branching ratio of each decay mode.

The lower limit of partial lifetime for each decay mode at 90% confidence level is calculated as below.

\[ \tau / B_{p \to e^+e^+e^-} > 3.4 \times 10^{34} \text{ years} \]
\[ \tau / B_{p \to \mu^+e^+e^-} > 2.3 \times 10^{34} \text{ years} \]
\[ \tau / B_{p \to \mu^-e^-e^+} > 1.9 \times 10^{34} \text{ years} \]
\[ \tau / B_{p \to e^+e^-\mu^+\mu^-} > 9.2 \times 10^{33} \text{ years} \]
\[ \tau / B_{p \to e^- \mu^+ \mu^+} > 1.1 \times 10^{34} \text{ years} \]
\[ \tau / B_{p \to \mu^+ e^+ e^-} > 1.0 \times 10^{34} \text{ years} \]

Comparison of current most stringent limit by IMB-3\textsuperscript{20} and HPW\textsuperscript{21} experiments are summarized in Fig.6.59 and Table\textsuperscript{??}. The partial lifetime limit for proton decay into three charged leptons are largely improved and the first result for \( p \rightarrow \mu^- e^+ e^+ \) mode is obtained by this analysis.

\textbf{Lifetime Limit [years] (90\%CL)}

![Diagram showing lifetime limit for each mode of proton decay into three charged leptons.](image)

Figure 6.59: Estimated partial lifetime limit for each mode of proton decay into three charged leptons by this analysis (red histogram), IMB (star)\textsuperscript{20} and HPW (circle)\textsuperscript{21} experiments.
Chapter 7

Discussion

The observed data were consistent with the atmospheric neutrino background prediction for all modes of proton decay into three charged leptons. Therefore, these decay modes predicted by the model (section 1.2.4) at the energy scale below 100 TeV could not be verified. In this model, \( p \rightarrow \mu^- e^+ e^+ \) and \( p \rightarrow e^- \mu^+ \mu^+ \) can be dominant in all proton decay modes by assuming some symmetries. Such scenario below 100 TeV energy scale could not be verified either by this analysis. The decay rates of these modes are proportional to the \( \Lambda^{12} \) (\( \Lambda \) is the energy scale) according to the equation (1.41). Then proton decay of longer lifetime than calculated limits is still considerable at higher energy scale. We need to improve the sensitivity of the analysis to discover such longer lifetime proton decay events.

New operation period SK-V already started in Super-Kamiokande. This new period simply increases the statistics. Furthermore, SK-Gd project will start soon in this period. In this project, Gadolinium (Gd) is dissolved in water, which increase the capturing efficiency for thermal neutron. Then the performance of neutron tagging will be improved, which can decrease the atmospheric neutrino background. These improvements leads to better sensitivity of proton decay analysis.

Future plan, Hyper-Kamiokande (HK) project[111] can further increase the statistics. The HK detector plans to be cylindrical tank of 74 m diameter and 60 m height. 40,000 PMTs will be arranged on the tank wall. The tank will be filled with 26 Mton ultra pure water. FV will be 19 Mton, which is about 10 times larger volume than SK FV. Schematic view of the HK detector is shown in Fig.7.1. HK can collect 10 times larger exposure of data than SK during same live time. In the proton decay analysis with such larger statistics, background rejection is more important. One of the way to reject more background events is improvement of the ring counting. In this analysis, background MC events remained in signal box mainly consist of the events that some rings are not reconstructed, mainly due to the rings close to each other. 2 observed events also have same features. The improvements of ring fitter for such kind of close ring events can reject more background events.
Figure 7.1: Schematic view of the HK detector[111].
Chapter 8

Conclusion

Proton decays into three charged leptons is predicted in GUT theory of $d = 10$ operators. Following decay modes without any correlation between charged leptons were searched by using 0.37 Mton-years exposure of data collected by Supker-Kamiokande.

- $p \rightarrow e^+e^+e^-$
- $p \rightarrow \mu^+e^+e^-$
- $p \rightarrow \mu^-e^+e^+$
- $p \rightarrow e^+\mu^+\mu^-$
- $p \rightarrow e^-\mu^+\mu^+$
- $p \rightarrow \mu^+\mu^+\mu^-$

The observed data were consistent with the atmospheric neutrino background prediction and no clear signal for proton decay was observed. Therefore, the model of these decay modes at the energy scale below 100 TeV could not be verified in this analysis. The lower partial lifetime limits at 90% CL were calculated as below.

$$\tau / B_{p \rightarrow e^+e^+e^-} (SK) > 3.4 \times 10^{34} \text{ years}$$
$$\tau / B_{p \rightarrow \mu^+e^+e^-} (SK) > 2.3 \times 10^{34} \text{ years}$$
$$\tau / B_{p \rightarrow \mu^-e^+e^+} (SK) > 1.9 \times 10^{34} \text{ years}$$
$$\tau / B_{p \rightarrow e^+\mu^+\mu^-} (SK) > 9.2 \times 10^{33} \text{ years}$$
$$\tau / B_{p \rightarrow e^-\mu^+\mu^+} (SK) > 1.1 \times 10^{34} \text{ years}$$
$$\tau / B_{p \rightarrow \mu^+\mu^+\mu^-} (SK) > 1.0 \times 10^{34} \text{ years}$$

Comparing with the previous stringent limits by IMB and HPW experiments, each limit was largely improved by this analysis. The lower limit for $p \rightarrow \mu^-e^+e^+$ mode was obtained for the first time. These are the most stringent limits in the world.
Appendix A

Period by Period Comparisons

Data and background MC comparison plots with respect to SK-I to SK-IV are shown in this section. They also agree well in each period.

Figure A.1: Data and MC comparison for the number of ring with respect to each period. Same plot for all periods is shown in Fig.6.35 left.
Figure A.2: Data and MC comparison for the number of $e$-like ring with respect to each period. Same plot for all periods is shown in Fig. 6.35 right.
Figure A.3: Data and MC comparison for the number of decay electron in $p \rightarrow e^+ e^+ e^-$ mode with respect to each period. Same plot for all periods is shown in Fig. 6.36 left.
Figure A.4: Data and MC comparison for the number of decay electron in $p \rightarrow \mu^+ e^+ e^- \rightarrow (p \rightarrow \mu^- e^+ e^+)$ mode with respect to each period. Same plot for all periods is shown in Fig. 6.36 right.
Figure A.5: Data and MC comparison for the number of decay electron in $p \rightarrow e^+\mu^+\mu^-$ ($p \rightarrow e^-\mu^+\mu^+$) mode with respect to each period. Same plot for all periods is shown in Fig. 6.37 left.
Figure A.6: Data and MC comparison for the number of decay electron in $p \rightarrow \mu^+ \mu^- \mu^-$ mode with respect to each period. Same plot for all periods is shown in Fig. 6.37 right.
Figure A.7: Data and MC comparison for mass of 2 e-like rings in $p \rightarrow \mu^+ e^+ e^-$ mode with respect to each period. Same plot for all periods is shown in Fig.6.38.
Figure A.8: Data and MC comparison for total mass in validation region of $p \rightarrow e^+e^-\nu$ mode with respect to each period. Same plot for all periods is shown in Fig. 6.43 left.
Figure A.9: Data and MC comparison for total momentum in validation region of $p \rightarrow e^+ e^- e^-$ mode with respect to each period. Same plot for all periods is shown in Fig.6.43 right.
Figure A.10: Data and MC comparison for total mass in validation region of $p \rightarrow \mu^+ e^+ e^-$ ($p \rightarrow \mu^- e^+ e^+$) mode with respect to each period. Same plot for all periods is shown in Fig. 6.44.
Figure A.11: Data and MC comparison for total momentum in validation region of $p \rightarrow \mu^+ e^+ e^-$ ($p \rightarrow \mu^- e^+ e^+$) mode with respect to each period. Same plot for all periods is shown in Fig. 6.44 right.
Figure A.12: Data and MC comparison for total mass in validation region of $p \rightarrow e^+\mu^+\mu^-$ ($p \rightarrow e^-\mu^+\mu^+$) mode with respect to each period. Same plot for all periods is shown in Fig. 6.45 left.
Figure A.13: Data and MC comparison for total momentum in validation region of $p \rightarrow e^+\mu^+\mu^- (p \rightarrow e^-\mu^+\mu^+)$ mode with respect to each period. Same plot for all periods is shown in Fig.6.45 right.
Figure A.14: Data and MC comparison for total mass in validation region of $p \rightarrow \mu^+ \mu^+ \mu^-$ mode with respect to each period. Same plot for all periods is shown in Fig. 6.46 left.
Figure A.15: Data and MC comparison for total momentum in validation region of $p \rightarrow \mu^{+}\mu^{+}\mu^{-}$ mode with respect to each period. Same plot for all periods is shown in Fig. 6.46 right.
Figure A.16: 2 dimensional plots of total mass and momentum for signal MC (left), background MC (center) and measured data (right) in $p \rightarrow e^+e^-e^-$ mode after all selections are applied. Top to bottom plots correspond to SK-I to SK-IV periods. Light blue shows free proton and dark blue shows bound proton in signal plot. Two black squares show the low and high signal boxes. Same plot for all periods is shown in Fig. 6.49.
Figure A.17: 2 dimensional plots of total mass and momentum for signal MC (left), background MC (center) and measured data (right) in $p \rightarrow \mu^+e^+e^-$ mode after all selections are applied. Top to bottom plots correspond to SK-I to SK-IV periods. Light blue shows free proton and dark blue shows bound proton in signal plot. Two black squares show the low and high signal boxes. Same plot for all periods is shown in Fig. 6.50.
Figure A.18: 2 dimensional plots of total mass and momentum for signal MC (left), background MC (center) and measured data (right) in $p \rightarrow e^\pm \mu^\pm \mu^-$ mode after all selections are applied. Top to bottom plots correspond to SK-I to SK-IV periods. Light blue shows free proton and dark blue shows bound proton in signal plot. Two black squares show the low and high signal boxes. Same plot for all periods is shown in Fig. 6.51.
Figure A.19: 2 dimensional plots of total mass and momentum for signal MC (left), background MC (center) and measured data (right) in $p \rightarrow \mu^+ \mu^+ \mu^-$ mode after all selections are applied. Top to bottom plots correspond to SK-I to SK-IV periods. Light blue shows free proton and dark blue shows bound proton in signal plot. Two black squares show the low and high signal boxes. Same plot for all periods is shown in Fig. 6.52.
Figure A.20: Data and background comparison for total mass (left) and momentum (right) in $p \rightarrow e^+e^+e^-$ mode after all selections are applied. Top to bottom plots correspond to SK-I to SK-IV periods. Black point shows data, red line shows background, dark blue line shows total and filled light blue shows free proton of signal. Dotted black line show the signal box. Same plot for all periods is shown in Fig. 6.53.
Figure A.21: Data and background comparison for total mass (left) and momentum (right) in $p \rightarrow \mu^+ e^+ e^-$ mode after all selections are applied. Top to bottom plots correspond to SK-I to SK-IV periods. Black point shows data, red line shows background, dark blue line shows total and filled light blue shows free proton of signal. Dotted black line show the signal box. Same plot for all periods is shown in Fig. 6.54.
Figure A.22: Data and background comparison for total mass (left) and momentum (right) in $p \rightarrow e^+ \mu^+ \mu^-$ mode after all selections are applied. Top to bottom plots correspond to SK-I to SK-IV periods. Black point shows data, red line shows background, dark blue line shows total and filled light blue shows free proton of signal. Dotted black line show the signal box. Same plot for all periods is shown in Fig. 6.55.
Figure A.23: Data and background comparison for total mass (left) and momentum (right) in $p \rightarrow \mu^+ \mu^- \mu^-$ mode after all selections are applied. Top to bottom plots correspond to SK-I to SK-IV periods. Black point shows data, red line shows background, dark blue line shows total and filled light blue shows free proton of signal. Dotted black line show the signal box. Same plot for all periods is shown in Fig. 6.56.
Appendix B

Previous Experiments

B.1 IMB-3

The IMB-3 detector is the water Cherenkov detector located \( \sim 600 \text{ m} \) underground in Fairport Salt Mine. 3.3 kton of water filled in a cube \( \sim 15 \text{ m} \) on a side is defined as a FV. Since the neutral particles generated with the proton decay events travel about 2.5 m in water, an additional 2 m veto region is needed outside the FV. Total detector size is \( 22.5 \times 17 \times 18 \text{ m}^3 \) rectangular filled with 8 kton water. 2,048 20 cm PMTs with wavelength shifters are arranged on the detector wall. 935 events were collected during 851 days from 1986 to 1991. Schematic view of the IMB-3 detector is shown in Fig.B.1. Selection criteria for proton decay into three charged leptons

Figure B.1: Schematic view of the IMB-3 detector[23].
basically consist with following requirements.

1. Visible energy cut.
2. Anisotropy cut which basically corresponds to the total momentum cut in this analysis.
3. The number of muon decay signals.
4. The number of tracks found by multiple track fitter.
5. Invariant mass cut which basically corresponds to the total mass cut in this analysis.
6. Human scan for track verification.

Cuts, signal efficiency, expected background events, observed events and calculated limits at 90% CL for each proton decay mode are summarized in Table.

Table B.1: Cuts, signal efficiency, expected background events, observed events and calculated limits at 90% CL for each proton decay mode. The numbers in cuts correspond to the each requirement described in this section.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Cuts (years)</th>
<th>Cuts (years)</th>
<th>Expected</th>
<th>Observed</th>
<th>Limit [years]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow e^+e^-e^-$</td>
<td>12346</td>
<td>71</td>
<td>0.5</td>
<td>0</td>
<td>$7.9 \times 10^{32}$</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+e^-e^-$</td>
<td>12345</td>
<td>47</td>
<td>1.0</td>
<td>0</td>
<td>$5.3 \times 10^{32}$</td>
</tr>
<tr>
<td>$p \rightarrow e^+\mu^-\mu^-$</td>
<td>123</td>
<td>47</td>
<td>0.9</td>
<td>1</td>
<td>$3.6 \times 10^{32}$</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+\mu^-\mu^-$</td>
<td>123</td>
<td>60</td>
<td>0.3</td>
<td>0</td>
<td>$6.8 \times 10^{32}$</td>
</tr>
</tbody>
</table>

B.2 HPW

The HPW detector is also a water Cherenkov detector located at a depth of 1,500 m of water equivalent. 680 Mton water is filled in a cylindrical tank of 5.6 m in radius and 7 m deep. 704 five-inch PMTs are arranged on the tank wall. The inside wall of the tank are covered with mirrors to increase the light collection efficiency. The detector is surrounded by wire chambers to record the position of charged particles which pass into or go outside the detector. Schematic view of the HPW detector is shown in Fig. B.2. The entire data set consists of $1.7 \times 10^6$ events collected in 282 days from 1983 to 1984. The maximum numbers of muon decay signal are required to be 2 for $p \rightarrow e^+\mu^-\mu^-$ and $p \rightarrow e^-\mu^+\mu^+$, and 3 for $p \rightarrow \mu^+\mu^-\mu^-$. Cherenkov equivalent energy ranges are required to be [540,800] MeV for $p \rightarrow e^+\mu^-\mu^-$ and $p \rightarrow e^-\mu^+\mu^+$, and [320,540] MeV for $p \rightarrow \mu^+\mu^-\mu^-$. Signal efficiencies are 5.6-7.9%, 6.6-9.4% and 11.8-16.2% for $p \rightarrow e^+\mu^-\mu^-$, $p \rightarrow e^-\mu^+\mu^+$, and $p \rightarrow \mu^+\mu^-\mu^-$ modes, respectively. 2 candidates were observed, but they were determined to be caused by entering particles. The expected number of background events from neutrino interaction is estimated to be $0.7 \pm 0.2$ events.
Figure B.2: Schematic view of the HPW detector[21].
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