Texture Zeros and CP-violating Phases in the Neutrino Mass Matrix

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OUTLINE

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★ Classification and Specific Analyses
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Motivation and General Comments

Strong experimental evidence in favor of neutrino oscillations (SK, SNO, KamLAND, K2K, ...) implies that ★ neutrinos are massive

\[ \Delta m^2_{\text{sun}} \sim 10^{-5} \text{ eV}^2, \quad \Delta m^2_{\text{atm}} \sim 10^{-3} \text{ eV}^2, \quad (m_i < 0.24 \text{ eV}) \]

but the exact mass spectrum \((m_1, m_2, m_3)\) remains unknown; ★ lepton flavors are mixed

\[
V = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} \\
0 & e^{-i\delta} & 0 \\
-s_{13} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
e^{i\rho} & 0 & 0 \\
0 & e^{i\sigma} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

with \(\theta_{23} \sim 45^\circ, \quad \theta_{12} \sim 33^\circ, \quad \theta_{13} < 13^\circ\)

but \(\theta_{13}\) and CP-violating phases \((\delta, \rho, \sigma)\) remain unknown.

In the lack of a convincing flavor theory, five approaches have been tried towards deeper understanding of fermion masses:
<table>
<thead>
<tr>
<th>Topic</th>
<th>Authors and Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiative Mechanism</td>
<td>S. Weinberg 1972; A. Zee 1980</td>
</tr>
<tr>
<td>Texture Zeros</td>
<td>S. Weinberg; F. Wilczek, A. Zee; H. Fritzsch 1977</td>
</tr>
<tr>
<td>Family Symmetries</td>
<td>H. Harari et al 1978; C. Froggatt, H. Nielsen 1979</td>
</tr>
<tr>
<td>Seesaw Mechanism</td>
<td>T. Yanagida; S. Glashow; M. Gell-Mann et al 1979</td>
</tr>
<tr>
<td>Extra Dimensions</td>
<td>K. Dienes et al; G. Dvali, A. Smirnov 1999</td>
</tr>
</tbody>
</table>

**Texture zeros** of a fermion mass matrix may dynamically mean strongly suppressed (in comparison with weakly suppressed or unsuppressed elements) and stem from a new flavor symmetry.

**Lesson:** Reasonable zeros of quark mass matrices allow us to establish simple and testable relations between flavor mixing angles and quark mass ratios — the same or similar relations should be predicted by the underlying (true) theory with much fewer fundamental parameters. A phenomenological study of possible texture zeros does make some sense to get useful hint about flavor dynamics responsible for quark mass generation.

**Expectation:** Plausible zeros of lepton mass matrices may lead to something interesting and testable in neutrino oscillations.
Phenomenology of lepton masses and mixing at low energies:

$M_l$—charged lepton mass matrix; $M_\nu$—effective neutrino mass matrix

$$U_l^\dagger M_l U_l' = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad U_\nu^\dagger M_\nu U_\nu' = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad V = U_l^\dagger U_\nu$$

Without loss of generality, $M_l$ can be taken to be diagonal, real and positive. Then $M_\nu$ can be expressed in terms of 9 physical parameters as follows:

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T = U \begin{pmatrix} m_1 e^{2i\rho} & 0 & 0 \\ 0 & m_2 e^{2i\sigma} & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T$$

$$U = \begin{pmatrix} c_x c_z & s_x c_z & s_x \\ -c_x s_y s_z - s_x c_y e^{-i\delta} & s_x s_y s_z + c_x c_y e^{i\delta} & s_y c_z \\ -c_x s_y s_z + s_x c_y e^{-i\delta} & -s_x s_y s_z - c_x c_y e^{i\delta} & c_y c_z \end{pmatrix} \text{ (Dirac-like)}$$

<table>
<thead>
<tr>
<th>Parameter counting</th>
<th>Observables (I)</th>
<th>Observables (II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutrino masses: $m_1, m_2, m_3$</td>
<td>$\Delta m^2_{\text{sun}}, \Delta m^2_{\text{atm}}$</td>
<td>$\langle m \rangle^2 = \sum m_i^2</td>
</tr>
<tr>
<td>Mixing angles: $\theta_x, \theta_y, \theta_z$</td>
<td>$\theta_{\text{sun}}, \theta_{\text{atm}}, \theta_{\text{chz}}$</td>
<td>$M_{ee} = \sum m_i V^2_{ei}$</td>
</tr>
<tr>
<td>CP-violating phases: $\delta, \rho, \sigma$</td>
<td>(sign of $m_3 - m_2$)</td>
<td>$J = s_x c_x s_y c_y s_z c_z^2 \sin \delta$</td>
</tr>
</tbody>
</table>
★ It seems impossible to fully determine $M_\nu$ from the feasible experiments at present or in the near future (in particular, $\rho$ and $\sigma$).

★ Texture zeros of $M_\nu$ may lead to simple and testable relations between unknown and known parameters of neutrino oscillations.

★ Texture zeros of $M_\nu$ may not be preserved to all orders or at any scales in the unspecified interactions from which neutrino masses are generated. At the one-loop level, however, the RGE evolution of $M_\nu$ from the seesaw scale ($M_1$ — mass of the lightest heavy right-handed neutrino) to low energy scales allow texture zeros to preserve:

$$M_\nu(M_Z) \propto \begin{pmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{pmatrix} M_\nu(M_1) \begin{pmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{pmatrix}$$

where $I_e$ etc denote evolution functions of Yukawa constants of electron etc.

★ Texture zeros, once combined with the seesaw mechanism, may allow us to simultaneously study leptogenesis and neutrino oscillations.

★ This talk is subject to a phenomenological classification and analyses of texture zeros and CP-violating phases of $M_\nu$ at low energy scales.
It is obvious that the four-zero, five-zero and six-zero textures of $M_\nu$ have no chance to account for current neutrino mass & mixing data. Our analyses will therefore focus on the left three cases: one-zero, two-zero and three-zero textures of $M_\nu$. 

**Definition and counting of texture zeros:**

★ because $M_\nu$ is symmetric, a pair of off-diagonal zeros of $M_\nu$ is commonly counted as one zero.

★ the number of possible textures of $3 \times 3$ $M_\nu$ with $N$ zeros can be given as $6!/\left[N! \times (6-N)!ight]$.

★ $N=1$: there are totally 6 one-zero textures of $M_\nu$;

★ $N=2$: there are totally 15 two-zero textures of $M_\nu$;

★ $N=3$: there are totally 20 three-zero textures of $M_\nu$;

★ $N=4$: there are totally 15 four-zero textures of $M_\nu$;

★ $N=5$: there are totally 6 five-zero textures of $M_\nu$;

★ $N=6$: there is totally 1 six-zero texture of $M_\nu$. 

Classification and Specific Analyses
None of 20 possibilities are allowed by current neutrino data.
Comments: some of the three-zero patterns of $M_\nu$ can survive, if $M_l$ is taken to be non-diagonal but as simple as possible.

Fritzsch texture of $M_\nu$ and $M_l$ (pattern 12) is allowed by current data:

$$M_\nu = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} \sim 4 \times 10^{-2} \text{eV} \times \begin{pmatrix} 0 & 0.24 & 0 \\ 0.24 & 0 & 0.55 \\ 0 & 0.55 & 1 \end{pmatrix}$$

$$M_l = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} \sim 1.67 \text{ GeV} \times \begin{pmatrix} 0 & 0.0045 & 0 \\ 0.0045 & 0 & 0.26 \\ 0 & 0.26 & 1 \end{pmatrix}$$

With $m_1 : m_2 : m_3 \sim 1 : 3 : 10$ and $m_e : m_\mu : m_\tau \sim 1 : 210 : 3500$ (Z.Z.X. 02).

Fritzsch texture and seesaw (M. Fukugita, M. Tanimoto, T. Yanagida 93/03)

$$M_l = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad M_R = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then $M_\nu$ can be calculated with seesaw. The results are favored by data.
Two-zero Textures of $M_\nu$

★ Classification: P. Frampton, S. Glashow, D. Marfatia 02
★ Majorana phases and Absolute neutrino masses: Z.Z.X. 02
★ Seesaw realization and Leptogenesis: M. Tanimoto and his collaborators (M. Honda, A. Kageyama, S. Kaneko, N. Simoyama, et al.) 02, 03, 04
★ Other relevant works: P. Frampton, M. Oh, T. Yoshikawa 02; B. Desai, D. Roy, A. Vaucher 02; G. Bhattacharyya, A. Raychaudhuri, A. Sil 02; R. Barbieri, T. Hambye, A. Romanino 03; K. Hasegawa, C.S. Lim, K. Ogure 03

Two independent zeros of $M_\nu$ lead to two constraints:

$$m_1 U_{a1} U_{b1} e^{2i\rho} + m_2 U_{a2} U_{b2} e^{2i\sigma} + m_3 U_{a3} U_{b3} = 0$$

$$m_1 U_{p1} U_{q1} e^{2i\rho} + m_2 U_{p2} U_{q2} e^{2i\sigma} + m_3 U_{p3} U_{q3} = 0$$

Then two mass ratios and Majorana phases can be determined:

$$\frac{m_1 e^{2i\rho}}{m_3} = \frac{U_{a3} U_{b3} U_{p2} U_{q2} - U_{a2} U_{b2} U_{p3} U_{q3}}{U_{a2} U_{b2} U_{p1} U_{q1} - U_{a1} U_{b1} U_{p2} U_{q2}}$$

$$\frac{m_2 e^{2i\sigma}}{m_3} = \frac{U_{a1} U_{b1} U_{p3} U_{q3} - U_{a3} U_{b3} U_{p1} U_{q1}}{U_{a2} U_{b2} U_{p1} U_{q1} - U_{a1} U_{b1} U_{p2} U_{q2}}$$
Main criterion to accept or reject a two-zero texture of $M_\nu$ is to see whether its prediction for $R_\nu$ is compatible with data:

$$
R_\nu \equiv \frac{|m_2^2 - m_1^2|}{|m_3^2 - m_2^2|} = \frac{\Delta m^2_{\text{sun}}}{\Delta m^2_{\text{atm}}} \quad \text{Then we obtain} \quad m_3 = \frac{\sqrt{\Delta m^2_{\text{atm}}}}{\sqrt{1 - (m_2/m_3)^2}}
$$

The absolute values of three neutrino masses are determinable!

A positive example — $A_1$ (see numerical results in the following page):

$$
M = \begin{pmatrix}
0 & 0 & x \\
0 & x & x \\
x & x & x
\end{pmatrix} \Rightarrow \begin{cases}
\frac{m_1}{m_3} \approx t_xt_ys_z \\
\frac{m_2}{m_3} \approx t_ys_z \\
\frac{m_3}{t_x} \approx s_z \\
\frac{\rho}{2} \approx \delta \\
\frac{\sigma}{2} \pm \frac{\pi}{2} \approx \delta
\end{cases} \quad \begin{aligned}
R_\nu &\approx \frac{t_x^2}{t_x^2} \left| 1 - t_x^4 \right| s_z^2 \\
m_3 &\approx \sqrt{\Delta m^2_{\text{atm}}}
\end{aligned}
$$

Typically inputting $\theta_x = 30^\circ$, $\theta_y = 45^\circ$, $\theta_z = 5^\circ$, $\delta = 90^\circ$, we obtain

$$
\frac{m_1}{m_3} \approx 0.04, \quad \frac{m_2}{m_3} \approx 0.13, \quad \rho \approx 45^\circ, \quad \sigma \approx 135^\circ \text{ (or } -45^\circ) \quad R_\nu \approx 0.014, \quad M_{ee} = 0
$$

Totally 15 two-zero patterns of $M_\nu$: 7 of them are accepted; 2 of them are marginally allowed; 6 of them are excluded by the present data of neutrino oscillations (W.L. Guo, Z.Z.X. 03)
<table>
<thead>
<tr>
<th>Pattern A₁</th>
<th>Pattern A₂</th>
<th>Pattern B₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 0 ×)</td>
<td>(0 × 0)</td>
<td>(× × 0)</td>
</tr>
<tr>
<td>(0 × ×)</td>
<td>(× × ×)</td>
<td>(× 0 ×)</td>
</tr>
<tr>
<td>(× × 0)</td>
<td>(0 × ×)</td>
<td>(0 × ×)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pattern B₂</th>
<th>Pattern B₃</th>
<th>Pattern B₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>(× 0 ×)</td>
<td>(× 0 ×)</td>
<td>(× × 0)</td>
</tr>
<tr>
<td>(0 × ×)</td>
<td>(0 0 ×)</td>
<td>(× × ×)</td>
</tr>
<tr>
<td>(× × 0)</td>
<td>(× × ×)</td>
<td>(0 × 0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pattern C</th>
<th>Pattern D₁</th>
<th>Pattern D₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>(× × ×)</td>
<td>(× × ×)</td>
<td>(× × ×)</td>
</tr>
<tr>
<td>(× 0 ×)</td>
<td>(× 0 0)</td>
<td>(× × 0)</td>
</tr>
<tr>
<td>(× × 0)</td>
<td>(× 0 ×)</td>
<td>(× × 0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pattern E₁</th>
<th>Pattern E₂</th>
<th>Pattern E₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 × ×)</td>
<td>(0 × ×)</td>
<td>(0 × ×)</td>
</tr>
<tr>
<td>(× 0 ×)</td>
<td>(× × ×)</td>
<td>(× × 0)</td>
</tr>
<tr>
<td>(× × ×)</td>
<td>(× × 0)</td>
<td>(× 0 ×)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pattern F₁</th>
<th>Pattern F₂</th>
<th>Pattern F₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>(× 0 0)</td>
<td>(× 0 ×)</td>
<td>(× × 0)</td>
</tr>
<tr>
<td>(0 × ×)</td>
<td>(0 × 0)</td>
<td>(× × 0)</td>
</tr>
<tr>
<td>(0 × ×)</td>
<td>(× 0 ×)</td>
<td>(0 0 ×)</td>
</tr>
</tbody>
</table>

Patterns in each category (A, B, C, D, E, F) have similar PH consequences.
“Predictions” for $\theta_{13}$ or $\theta_{z}$ (A. Joshipura, this workshop):

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Texture of $M_\nu$</th>
<th>Prediction for $\theta_{13}$</th>
</tr>
</thead>
</table>
| A₁      | \[
\begin{pmatrix}
0 & 0 & \times \\
0 & \times & \times \\
\times & \times & \times 
\end{pmatrix}\] | $\sin \theta_z \approx \frac{R_\nu \tan^2 \theta_x}{\tan^2 \theta_y |1 - \tan^4 \theta_x|}$ |
| A₂      | \[
\begin{pmatrix}
0 & \times & \times \\
\times & \times & \times \\
0 & \times & \times 
\end{pmatrix}\] | $\sin \theta_z \approx \frac{R_\nu \tan^2 \theta_x \tan^2 \theta_y}{|1 - \tan^4 \theta_x|}$ |
| B₁      | \[
\begin{pmatrix}
\times & \times & \times \\
\times & 0 & \times \\
0 & \times & \times 
\end{pmatrix}\] | $\sin \theta_z \approx \frac{R_\nu \tan \theta_x}{(1 + \tan^2 \theta_x) |\tan 2 \theta_y \cos \delta|}$ |
| B₂      | \[
\begin{pmatrix}
\times & 0 & \times \\
\times & \times & \times \\
\times & \times & 0 
\end{pmatrix}\] | $\sin \theta_z \approx \frac{R_\nu \tan \theta_x}{(1 + \tan^2 \theta_x) |\tan 2 \theta_y \cos \delta|}$ |
| B₃      | \[
\begin{pmatrix}
\times & 0 & \times \\
0 & \times & \times \\
\times & \times & \times 
\end{pmatrix}\] | $\sin \theta_z \approx \frac{R_\nu \tan \theta_x}{(1 + \tan^2 \theta_x) \tan^2 \theta_y |\tan 2 \theta_y \cos \delta|}$ |
| B₄      | \[
\begin{pmatrix}
\times & \times & \times \\
\times & \times & \times \\
0 & \times & 0 
\end{pmatrix}\] | $\sin \theta_z \approx \frac{R_\nu \tan \theta_x \tan^2 \theta_y}{(1 + \tan^2 \theta_x) |\tan 2 \theta_y \cos \delta|}$ |
| C       | \[
\begin{pmatrix}
\times & \times & \times \\
\times & 0 & \times \\
\times & \times & 0 
\end{pmatrix}\] | $\sin \theta_z \sim \frac{1}{\tan 2 \theta_x \tan 2 \theta_y \cos \delta}$ |

With reasonable inputs, always possible to get $\theta_{13} \sim 5^\circ$ or $\sin^2 2 \theta_{13} \sim 3\%$. 
Possible **seesaw** realization of $M_\nu$ — typical examples with diagonal $M_D$ for illustration (A. Kageyama, S. Kaneko, N. Shimoyama, M. Tanimoto 02):

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Texture of $M^T$</th>
<th>Dirac $m_D$</th>
<th>Majorana $M_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$\begin{pmatrix} 0 &amp; 0 &amp; x \ x &amp; 0 &amp; x \ x &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; 0 &amp; 0 \ 0 &amp; x &amp; 0 \ x &amp; 0 &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; x &amp; x \ x &amp; 0 &amp; x \ x &amp; x &amp; 0 \end{pmatrix}$, $\begin{pmatrix} 0 &amp; x &amp; x \ x &amp; 0 &amp; x \ x &amp; 0 &amp; x \end{pmatrix}$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$\begin{pmatrix} x &amp; x &amp; 0 \ 0 &amp; x &amp; x \ x &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; 0 &amp; 0 \ 0 &amp; x &amp; 0 \ x &amp; 0 &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; x &amp; x \ x &amp; 0 &amp; x \ x &amp; 0 &amp; x \end{pmatrix}$, $\begin{pmatrix} 0 &amp; x &amp; x \ x &amp; 0 &amp; x \ x &amp; 0 &amp; x \end{pmatrix}$</td>
</tr>
<tr>
<td>$H_1$</td>
<td>$\begin{pmatrix} x &amp; x &amp; 0 \ 0 &amp; 0 &amp; x \ 0 &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 &amp; x &amp; x \ 0 &amp; x &amp; x \ 0 &amp; x &amp; x \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$H_2$</td>
<td>$\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; x \ x &amp; x &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; x &amp; 0 \ x &amp; 0 &amp; x \end{pmatrix}$, $\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; 0 \end{pmatrix}$</td>
</tr>
<tr>
<td>$H_3$</td>
<td>$\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; x \ x &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; x &amp; 0 \ x &amp; 0 &amp; x \end{pmatrix}$, $\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; 0 \end{pmatrix}$</td>
</tr>
<tr>
<td>$H_4$</td>
<td>$\begin{pmatrix} x &amp; x &amp; 0 \ 0 &amp; x &amp; x \ x &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; x &amp; 0 \ x &amp; 0 &amp; x \end{pmatrix}$, $\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; 0 \end{pmatrix}$</td>
</tr>
<tr>
<td>$C$</td>
<td>$\begin{pmatrix} x &amp; x &amp; 0 \ 0 &amp; x &amp; x \ x &amp; x &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; x \end{pmatrix}$</td>
<td>$\begin{pmatrix} x &amp; x &amp; 0 \ x &amp; 0 &amp; x \end{pmatrix}$, $\begin{pmatrix} x &amp; 0 &amp; x \ 0 &amp; x &amp; 0 \end{pmatrix}$</td>
</tr>
</tbody>
</table>
One-zero Textures of $M_\nu$

Totally 6 possible patterns. Of particular interest is $M_{ee} = 0$ — no observable neutrinoless double beta decay (S. Pascoli, S. Petcov 02, this workshop)
★ It does not necessarily mean that neutrinos are Dirac particles;
★ It might imply that two Majorana phases lie in a special region.

The following conditions must be satisfied for $M_{ee} = 0$ to hold:

\[
\text{Re}(M_{ee}) = m_1 |V_{e1}|^2 \cos 2\rho + m_2 |V_{e2}|^2 \cos 2\sigma + m_3 |V_{e3}|^2 = 0
\]

\[
\text{Im}(M_{ee}) = m_1 |V_{e1}|^2 \sin 2\rho + m_2 |V_{e2}|^2 \sin 2\sigma = 0
\]

Two neutrino mass ratios are then correlated with two CP-violating phases:

\[
\frac{m_1}{m_2} = - \frac{|V_{e2}|^2}{|V_{e1}|^2} \cdot \frac{\sin 2\sigma}{\sin 2\rho}, \quad \frac{m_2}{m_3} = + \frac{|V_{e3}|^2}{|V_{e2}|^2} \cdot \frac{\sin 2\rho}{\sin 2(\sigma - \rho)},
\]

where $0 \leq m_1/m_2 < 1$ and $0 < m_2/m_3 < 1$. We arrive at (Z.Z.X. 03):

\[
R_\nu = \frac{|V_{e3}|^4}{|V_{e1}|^4} \cdot \frac{|V_{e1}|^4 \sin^2 2\rho - |V_{e2}|^4 \sin^2 2\sigma}{|V_{e2}|^4 \sin^2 2(\sigma - \rho) - |V_{e3}|^4 \sin^2 2\rho}
\]

Constraints on $\rho$ & $\sigma$ can be obtained.
Comments and discussions:

★ ρ & σ may take nontrivial values, which guarantee $M_{ee} = 0$. For example,

\[
\left(\rho, \sigma\right) = \left(\frac{\pi}{4}, \frac{2\pi}{3}\right) \quad \text{or} \quad \left(\frac{3\pi}{4}, \frac{\pi}{3}\right) \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{\sqrt{3}}{2} \frac{|V_{e2}|^2}{|V_{e1}|^2} \quad \text{and} \quad \frac{m_2}{m_3} = \frac{2}{|V_{e3}|^2}
\]

★ $m_2/m_3$ has a lower limit, corresponding to $|V_{e3}| \sim 0.07$ or $\sin^2 2\theta_{13} \sim 2\%$.

★ A definite signal of neutrinoless double beta decay will rule out $M_{ee} = 0$. Otherwise, $M_{ee} = 0$ could be taken as a prerequisite for model building.
One-zero Textures of $\mathbf{M}_\nu$ with one vanishing mass eigenvalue

Because $m_1 < m_2$ must hold, one has $m_1 = 0$ (normal) or $m_3 = 0$ (inverted).

Note that $M_{ab} = 0$ implies the following relation:

$$m_1 V_{a1} V_{b1} + m_2 V_{a2} V_{b2} + m_3 V_{a3} V_{b3} = 0 \quad (a, b = e, \mu, \tau)$$

★ If $m_1 = 0$ holds, we obtain

$$\xi \equiv \frac{m_2}{m_3} = \frac{|V_{a3} V_{b3}|}{|V_{a2} V_{b2}|}, \quad \arg \left( \frac{V_{a3} V_{b3}}{V_{a2} V_{b2}} \right) = \pm \pi, \quad R_\nu = \frac{\xi^2}{|1 - \xi^2|}.$$

$R_\nu \sim 0.03$ means that $\xi \sim O(0.1)$ — normal mass hierarchy.

★ If $m_3 = 0$ holds, we obtain

$$\xi' \equiv \frac{m_1}{m_2} = \frac{|V_{a2} V_{b2}|}{|V_{a1} V_{b1}|}, \quad \arg \left( \frac{V_{a2} V_{b2}}{V_{a1} V_{b1}} \right) = \pm \pi, \quad R_\nu = 1 - \xi'^2.$$

$R_\nu \sim 0.03$ means that $0.95 < \xi' < 1$ — inverted mass hierarchy.

Results: 3 one-zero patterns of $\mathbf{M}_\nu$ with $m_1 = 0$ and 4 one-zero patterns of $\mathbf{M}_\nu$ with $m_3 = 0$ are compatible with current neutrino data (Z.Z.X. 03).
<table>
<thead>
<tr>
<th>Pattern</th>
<th>Masses $m_2$ and $m_3$</th>
<th>Phase $\sigma$</th>
<th>$\langle m_{ee} \rangle$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{m_1=0}$:</td>
<td>$m_2 = \sqrt{\Delta m_{31}^2}$</td>
<td>$\sigma = \frac{\Delta_2}{2}$</td>
<td>$\langle m_{ee} \rangle = 0$</td>
</tr>
<tr>
<td>$m_3 \approx \sqrt{\Delta m_{32}^2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$B_{m_1=0}$: Similar to $A_{m_1=0}$

$\sigma = \frac{\Delta_1}{2} \pm \frac{\pi}{2}$ with $\tan \Delta_1 = \frac{c_x c_y s_\delta}{c_x c_y c_\delta - s_x s_y s_\delta}$

$\langle m_{ee} \rangle \lesssim \mathcal{O}(10^{-2})$

$C_{m_1=0}$: Similar to $B_{m_1=0}$

$\sigma = \frac{\Delta_1'}{2} \pm \frac{\pi}{2}$ with $\tan \Delta_1' = \frac{c_x s_y s_\delta}{c_x s_y c_\delta + s_x c_y s_\delta}$

Similar to $B_{m_1=0}$

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<table>
<thead>
<tr>
<th>Pattern</th>
<th>Masses $m_1$ and $m_2$</th>
<th>Phase $\rho - \sigma$</th>
<th>$\langle m_{ee} \rangle$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{m_3=0}$:</td>
<td>$m_1 \approx m_2 = \sqrt{\Delta m_{31}^2}$</td>
<td>$\rho - \sigma = \frac{\Delta_2}{2} \pm \frac{\pi}{2}$ with $\tan \Delta_2 = \frac{s_y c_y s_\delta s_\delta}{s_x c_x (s_y^2 s_\delta^2 - c_y^2) + (s_x^2 - c_x^2) s_y c_y s_\delta c_\delta}$</td>
<td>$\langle m_{ee} \rangle \lesssim \mathcal{O}(10^{-2})$</td>
</tr>
<tr>
<td>$C_{m_3=0}$: Similar to $B_{m_3=0}$</td>
<td></td>
<td></td>
<td>Similar to $B_{m_3=0}$</td>
</tr>
<tr>
<td>$\rho - \sigma = \frac{\Delta_2'}{2} \pm \frac{\pi}{2}$ with $\tan \Delta_2' = \frac{s_y c_y s_\delta s_\delta}{s_x c_x (s_y^2 c_\delta^2 - c_y^2) + (s_x^2 - c_x^2) s_y c_y s_\delta c_\delta}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{m_3=0}$: Similar to $B_{m_3=0}$</td>
<td>$\rho - \sigma = \Delta_2 \pm \frac{\pi}{2}$ with $\tan \Delta_2$ given above</td>
<td>Similar to $B_{m_3=0}$</td>
<td></td>
</tr>
<tr>
<td>$E_{m_3=0}$: Similar to $B_{m_3=0}$</td>
<td>$\rho - \sigma = \Delta_2' \pm \frac{\pi}{2}$ with $\tan \Delta_2'$ given above</td>
<td>Similar to $B_{m_3=0}$</td>
<td></td>
</tr>
<tr>
<td>$F_{m_3=0}$: Similar to $B_{m_3=0}$</td>
<td></td>
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</tr>
</tbody>
</table>

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Legend:
- $m_{ee}$: Effective mass
- $\Delta_1, \Delta_1'$: Phase differences
- $\Delta_2, \Delta_2'$: Phase differences
- $\rho, \sigma$: Phase angles
- $c, s$: Cosine and sine functions
- $x, y$: Mass eigenstates
- $\delta$: Mixing angle
The minimal seesaw model of neutrino masses and leptogenesis

**Motivation:** to simultaneously interpret neutrino mixing and cosmological baryon number asymmetry (P. Frampton, S. Glashow, T. Yanagida 02)

\[ -\mathcal{L}_{\text{mass}} = \begin{pmatrix} \nu_e, \nu_\mu, \nu_\tau \end{pmatrix} M_D \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} N_1^c, N_2^c \end{pmatrix} M_R \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + \text{h.c.} \]

In the basis of diagonal $M_l$ and $M_R$, FGY ansatz of $M_D$ gives

\[
M_D = \begin{pmatrix} a & 0 \\ a' & b \\ 0 & b' \end{pmatrix}, \quad M_\nu \approx M_D M_R^{-1} M_D^T = \begin{pmatrix}
\frac{a^2}{M_1} & \frac{aa'}{M_1} & 0 \\
\frac{aa'}{M_1} & \frac{(a')^2}{M_1} + \frac{b^2}{M_2} & \frac{bb'}{M_2} \\
0 & \frac{bb'}{M_2} & \frac{(b')^2}{M_2}
\end{pmatrix}
\]

Because of $|\text{Det}(M_\nu)| = m_1 m_2 m_3 = 0$, $m_1 = 0$ or $m_3 = 0$ must hold.

★ $M_\nu$ is just of the one-zero texture with $m_1 = 0$ or $m_3 = 0$ discussed above;
★ The texture of $M_\nu$ and $m_i = 0$ are stable against radiative corrections;
★ CP-violating phases of $M_\nu$ can be determined in terms of mixing angles.

In the following, we take the FGY ansatz with $m_1 = 0$ for example/illustration.
Two calculable CP-violating phases (W.L. Guo / J.W. Mei, Z.Z.X., 03):

\[
\delta = \arccos \left[ \frac{c_y^2 s_z^2 - R_{\nu} s_x^2 \left( c_x^2 s_y^2 + s_x^2 c_y^2 s_z^2 \right)}{2 R_{\nu} s_x^3 c_x s_y c_y s_z} \right]
\]

\[
\sigma = \frac{1}{2} \arctan \left[ \frac{c_x s_y \sin \delta}{s_x c_y s_z + c_x s_y \cos \delta} \right]
\]

\[|\cos \delta| < 1 \text{ requires } \sin \theta_z \sim 0.075 \text{ or } \sin^2 \theta_{13} \sim 2.2\% , \text{ very restrictive!}\]

Cosmological baryon number asymmetry via leptogenesis \((M_1 \ll M_2)\):

\[
\varepsilon_1 \equiv \frac{\Gamma(N_1 \to l + H) - \Gamma(N_1 \to \bar{l} + H^*)}{\Gamma(N_1 \to l + H) + \Gamma(N_1 \to \bar{l} + H^*)}
\]

\[
\propto M_1 \cdot \sin \left[ \arctan \left( \frac{\sqrt{R_{\nu}} \ s_x^2 c_z^2 \sin 2\sigma}{s_z^2 + \sqrt{R_{\nu}} \ s_x^2 c_z^2 \cos 2\sigma} \right) \right]
\]

\[Y_B \propto \varepsilon_1 \sim 10^{-10} \text{ can be achieved for } M_1 \geq 10^{10} \text{ GeV (SM/SUSY)}\]

Remark: (a) Radiative correction to \(\varepsilon_1\) or \(Y_B\) is of \(O(1)\) [a factor 1.2—1.5];
(b) FGY ansatz can be tested at low energies via a measurement of \(\theta_{13}\).
Vanishing entries of simplified $M_l$ and $M_\nu$ (or $V$) may serve as a symmetry limit of small entries of realistic $M_l$ and $M_\nu$ (or $V$). The latter may come from explicit perturbations or radiative corrections. Model building may start from the possible symmetry limit, in which CP might be conserving.

For example, the bi-large mixing pattern of $V$ with small $|V_{e3}|$ at low energies might result from the bi-maximal mixing pattern of $V$ with vanishing $|V_{e3}|$. Radiative corrections play a role (W. Grimus; A. Joshipura, this workshop).

Lepton and quark mass matrices could have the same texture zeros, arising from a universal flavor symmetry at a certain energy scale.

For example, the four-zero texture is favored for quark mass matrices and is seesaw-invariant for lepton mass matrices (H. Fritzsch, Z.Z.X. 00). It can be applied to a SO(10)-inspired neutrino model (W. Buchmüller, D. Wyler, 01):

**Idea:** SO(10) lepton-quark symmetry $\epsilon_u \sim 0.07$ and $\epsilon_d \sim 0.21$ from quark data

\[
M_D \sim M_u \sim m_t \begin{pmatrix}
0 & O(\epsilon_u^3) & 0 \\
O(\epsilon_u^3) & O(\epsilon_u^2) & O(\epsilon_u^2) \\
0 & O(\epsilon_u^2) & O(1)
\end{pmatrix}, \quad M_l \sim M_d \sim m_b \begin{pmatrix}
0 & O(\epsilon_d^3) & 0 \\
O(\epsilon_d^3) & O(\epsilon_d^2) & O(\epsilon_d^2) \\
0 & O(\epsilon_d^2) & O(1)
\end{pmatrix}
\]
Can zeros be forgotten in analyzing textures of lepton mass matrices? — the idea of anarchy for $M_\nu$ (L. Hall, H. Murayama, N. Weiner 00)

It will be very useful to explore the full parameter space of $M_\nu$ with more accurate data with no assumption of zeros (M. Frigerio, A.Yu. Smirnov 02). Such an analysis might more or less favor certain of texture zeros in $M_\nu$.

Remarks: the mass hierarchies of charged leptons & quarks (and perhaps neutrinos) seem to imply certain textures of fermion mass matrices, in which zeros may (approximately) be present. The study of texture zeros could help establish the true bridge between observables and fundamental parameters.